3.2
Constructing DFAs

DFAs: State = Memory

How do we design a DFA M for a given language L? That is $L(M) = L$.

- DFA is a like a program that has fixed amount of memory independent of input size.
- The memory of a DFA is encoded in its states
- The state/memory must capture enough information from the input seen so far that it is sufficient for the suffix that is yet to be seen (note that DFA cannot go back)

DFA Construction: Examples

Example I: Basic languages

Assume $\Sigma = \{0, 1\}$.

$L = \emptyset$, $L = \Sigma^*$, $L = \{\epsilon\}$, $L = \{0\}$.

DFA Construction: Examples

Example II: Length divisible by 5

Assume $\Sigma = \{0, 1\}$.
$L = \{w \in \{0, 1\}^* \mid |w| \text{ is divisible by } 5\}$

DFA Construction: examples

Example III: Ends with 01

Assume $\Sigma = \{0, 1\}$.

$L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 01\}$

DFA Construction: examples

Example IV: Contains 001

Assume $\Sigma = \{0, 1\}$.

$L = \{w \in \{0, 1\}^* \mid w \text{ contains 001 as substring}\}$

DFA Construction: examples

Example V: Contains 001 or 010

Assume $\Sigma = \{0, 1\}$.

$L = \{w \in \{0, 1\}^* \mid w \text{ contains 001 or 010 as substring}\}$

DFA construction examples

Example VI: Has a 1 exactly k positions from end

Assume $\Sigma = \{0, 1\}$.
$L = \{w \mid w \text{ has a 1 } k \text{ positions from the end}\}$.

DFA Construction: Example

$L = \{\text{Binary numbers congruent to } 0 \mod 5\}$

Example:

1. $1101011_2 = 107_{10} = 2 \mod 5$,
2. $1010_2 = 10 = 0 \mod 5$

Key observation:

$\mathrm{val}(w) \mod 5 = a$ implies

$$\mathrm{val}(w0) \mod 5 = (\mathrm{val}(w) * 2) \mod 5 = 2a \mod 5$$

$$\mathrm{val}(w1) \mod 5 = (\mathrm{val}(w) \cdot 2 + 1) \mod 5 = (2a + 1) \mod 5$$

DFA Construction: Example

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THE END

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(for now)