Prove that each of the following languages is ***not*** regular.

1 $\{0^{2^n} \mid n \geq 0\}$.

2 $\{0^{2n}1^n \mid n \geq 0\}$

3 $\{0^m1^n \mid m \neq 2n\}$

4 Strings over $\{0, 1\}$ where the number of 0s is exactly twice the number of 1s.

5 Strings of properly nested parentheses (), brackets [], and braces {}. For example, the string ([]){} is in this language, but the string ([ )] is not, because the left and right delimiters don't match.

6 Strings of the form $w_1\#w_2\#\cdots\#w_n$ for some $n \geq 2$, where each substring w_i is a string in $\{0, 1\}^*$, and some pair of substrings w_i and w_j are equal.

Extra problems

7 $\{0^{n^2} \mid n \geq 0\}$

8 $\{w \in (0 + 1)^* \mid w$ is the binary representation of a perfect square$\}$