HW 10

Due on **Wednesday, November 20, 2024 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024 Version: **1.0**

Submission instructions as in previous homeworks.

19 Let $\mathsf{G} = (\llbracket n \rrbracket, \mathsf{E})$ be an undirected graph with n vertices and $m \geq n$ edges, and with distinct real weights on the edges. You are also given an MST T of G. Unfortunately, and sadly, the price of k of the edges $e_1, e_2, \ldots, e_k$ in T had dramatically increased after these edges were viciously attacked by the peace loving Houthi rebels. That is, their old price was $w(e_i)$, and the new price is $w(e_i') > w(e_i)$, for all i. Describe an algorithm, as fast as possible, that computes the new MST G under the new pricing.

20 Let $\mathsf{G} = (\mathsf{V}, \mathsf{E})$ be an undirected graph with n vertices and $m \geq n$ edges. Assume $\mathsf{V} = L \cup R$, where L, R are disjoint sets of size at most $n/2 + 1$. Edges between L and R in G are red, while all other edges are blue. The red edges are free, but all the blue edges have a price of one associated with them. Assume you would like to compute a spanning tree of G, of minimum cost, that uses the maximum number of red edges, under the constraint that no two red edges in the spanning tree might share a vertex. For simplicity, assume that for any two vertices in L, there is path connecting them using only blue edges, and the same hold for any pair of vertices in R.

Describe an algorithm, as fast as possible, for solving this problem. Prove the correctness of your algorithm.