HW 3

Due on **Wednesday, September 18, 2024 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024 Version: **1.02**

Submission instructions as in previous homeworks.

5 (100 PTS.) Zip it.

Let $\Sigma = \{0, 1\}$. For a string $s \in \Sigma^*$, let head(s) be the first character of s, or ε if $|s| = 0$. Similarly, if $s = cx$, with $c \in \Sigma$, and $x \in \Sigma^*$, let tail$(s) = x$ (again, its ε if s is the empty string).

For two strings s, and t, their ***zip*** is the set of strings

$$Z(s,t) = \begin{cases} \{s\} & |t| = 0 \\ \{t\} & |s| = 0 \\ \{\mathrm{head}(s)\}Z(tail(s), t) \ \cup \ \{\mathrm{head}(t)\}Z(s, \mathrm{tail}(t)) & \text{otherwise.} \end{cases}$$

Similarly, we define the ***zip*** of two languages L_1, L_2 to be $Z(L_1, L_2) = \bigcup_{s \in L_1, t \in L_2} Z(s,t)$.

You are given two DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$. Describe formally how to construct an automata (i.e., DFA or NFA) N, for the language $L_Z = Z\big(L(M_1), L(M_2)\big)$. This implies that L_Z is regular. Argue that your constructed automata indeed accepts the desired language[1].

6 (100 PTS.) A fool and their languages.

Prove that the following languages are not regular by providing a fooling set. You need to provide an infinite set, and prove that it is a valid fooling set for the given language.

For each part, first state formally and clearly the fooling set. Then, in a new paragraph prove formally that your fooling set is correct.

6.A. (20 PTS.) $L_1 = \{w \mid w \in \{0, 1, 2\}^*, \text{ and } \nabla_0(w) = \nabla_1(w)\}$, where $\nabla_c(w)$ is the number of runs in w made of the character c. Thus $\nabla_0(20100100022) = 3$.

6.B. (20 PTS.) $L_2 = \left\{0^i 1 0^j \;\middle|\; \gcd(i,j) = 1\right\}$.

6.C. (20 PTS.) $L_3 = \left\{a^i b^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } i \text{ divides } j\right\}$.

6.D. (20 PTS.) $L_4 = \left\{0^{i^2} 0^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } j \leq i\right\}$.

6.E. (20 PTS.) $L_5 = \left\{w a^{\#_a(w)} b^{\#_b(w)} \;\middle|\; w \in \{a, b\}^*\right\}$, where $\#_c(w)$ is the number of times the character c appears in w.

[1]Proving it formally is tedious and boring, so you do not have to do it.