CS/ECE 374: Algorithms & Models of Computation, Fall 2017

Midterm 1   October 2: 7–9pm, 2017

A 9am Rucha SC 1404	B 10am Rucha SC 1404	C 11am Srihita SC 1404	D noon Shant DCL 1320	E 1pm Abhishek DCL 1320	F 1pm Xilin DCL 1320	G 2pm Shalan ECE 1002	H 2pm Phillip ECE 1002	J 3pm Vishal ECE 1002	K 3pm Phillip ECE 1002	? Waiting list ECE 1002

Name:

NetID:

#	1	2	3	4	5	Total
Score						
IDK Score						
Max	20	20	20	20	20	100
Grader						

- ***Don't panic!***
- Please print your name and NetID **in each page** in the appropriate fields, and circle your discussion section in the boxes above. We will return your exam at the indicated section.
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original) 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, you please turn off and put away *all* medically unnecessary electronic devices.
    - Submit your cheat sheet together with your exam. An exam without your cheat sheet attached to it will not be graded.
    - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- Please ask for clarification if any question is unclear.
- **This exam lasts 120 minutes.** The clock started when you got the questions.
- If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
- As usual, answering any (sub)problem with I don't know (and nothing else) is worth 25% partial credit. Correct, complete, but sub-optimal solutions are *always* worth more than 25%. A blank answer is not the same as I don't know.
- Total IDK points for the whole exam would not exceed 10.
- **Beware the Three Deadly Sins.** Give complete solutions, not examples. Declare all your variables. If you don't know the answer admit it and use IDK.
- **Style counts.** Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
- Please return ***all*** paper with your answer booklet: your cheat sheet, and all scratch paper. We will return everything with your graded exam.
- ***Good luck!***

1 For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth 2 points; each incorrect answer is worth 0 points; and flipping a coin is (on average) worth 1 point. **There is no IDK for this question**.

1.A. If $L_1, L_2, \ldots$ are all context free languages, then $\bigcup_{i=0}^{\infty} L_i$ is context free.

False: ☐ **True:** ☐

1.B. For all languages L, if L is regular, then L does not have an infinite fooling set.

False: ☐ **True:** ☐

1.C. $\left\{ 0^i 1^j 0^k 1^\ell \mid i, j, k, \ell \geq 0 \right\}$ is not regular.

False: ☐ **True:** ☐

1.D. Consider the logical statement "If the moon is made of silver, then the sun is made of chicken." This expression is:

False: ☐ **True:** ☐

1.E. If a language $L \subseteq \{0\}^*$ then the language L^* is regular.

False: ☐ **True:** ☐

1.F. The strings 010 and 101 are distinguishable by the language

$$L = \left\{ x \in \Sigma^* \mid \text{the number of } 1\text{s in } x \text{ is even} \right\}.$$

False: ☐ **True:** ☐

1.G. Let L be a regular language over alphabet Σ, and consider the language

$$L' = \{xy \mid x, y \in \Sigma^*, \alpha \in \Sigma, \text{ and } x\alpha y \in L\}.$$

The language L' is regular.

False: ☐ **True:** ☐

1.H. For all context-free languages L and L', the language $L \cup L'$ is also context-free.

False: ☐ **True:** ☐

1.I. For all languages $L \subset \Sigma^*$, if L is recognized by a DFA, then $\Sigma^* \setminus L$ can NOT be represented by an NFA.

False: ☐ **True:** ☐

1.J. Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary DFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cup L(M') = \Sigma^*$.

False: ☐ **True:** ☐

2   For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either **_prove_** that the language is regular or **_prove_** that the language is not regular. **_Exactly one of these two languages is regular._** [This is a tricky question.]

2.A. $L = \{xyxy \mid x, y \in \Sigma^+\}$.

2.B. $L = \left\{xyx^R \mid x, y \in \Sigma^+\right\}$, where x^R is the *reverse* string of x.

3 For any language L, let $\text{PS}(L) = \{xz \mid xyz \in L \text{ for some } x, y, z \in \Sigma^*\}$ be the language containing all strings in L after one is allowed to delete some continuous middle portion. For example, if $L = \{000, 10101\}$, then we have the string $10\cancel{10}1 = 101 \in \text{PS}(L)$. In this case, we have

$$\text{PS}(L) = \left\{ \ \varepsilon, 0, 00, 000, 10101, 0101, 1101, 1001, 1011, 1010, 101, 01, 11, 10, 1 \ \right\}.$$

Prove that for any regular language L, the language $\text{PS}(L)$ is also regular. Your proof should be self contained, and not use any languages seen in class/lectures (suggestion: first describe the necessary construction, and then prove the correctness of the construction).

4   In the following, you do **_not_** need to prove that your answers are correct.

4.A.   Provide a DFA and a regular expression for the following language: The set of all strings in $\{0, 1\}^*$ that do not contain the substring 11111.

4.B.   Provide a regular expression for the following language: The set of all strings in $\{0, 1\}^*$ that contain both the substrings 01 and 11.

5 A CFG G is *silly* if all production rules are of the form $A \to cB$ or $A \to \varepsilon$, where $c \in \Sigma$ and A and B are variables (i.e.., non-terminals) of G. For a given silly CFG grammar G, provide a construction that shows that the language of $L(G)$ is regular. Provide a convincing argument why this is true (you do not need to provide a formal proof).

[If you do not know the answer - just use IDK – no need to waste your and our time.]