Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do ***not*** need to prove your answer is correct.

   Read each statement *very* carefully. Some of these are deliberately subtle.

   (a) If 100 is a prime number, then Jeff is the Queen of England.

   (b) The language $\{0^m 0^{n+m} 0^n \mid m, n \geq 0\}$ is regular.

   (c) For all languages L, the language L^* is regular.

   (d) For all languages $L \subset \Sigma^*$, if L can be recognized by a DFA, then $\Sigma^* \setminus L$ cannot be represented by a regular expression.

   (e) For all languages L and L', if $L \subseteq L'$ and L' is regular, then L is regular.

   (f) For all languages L, if L has a finite fooling set, then L is not regular.

   (g) Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **NFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cup L(M') = \Sigma^*$.

   (h) Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **NFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

   (i) For all context-free languages L and L', the language $L \bullet L'$ is also context-free.

   (j) Every regular language is context-free.

2. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.***

   (a) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

   (b) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

   For example, both of these languages contain the string 00110100000110100.

3. Let $L = \{1^m 0^n \mid n \leq m \leq 2n\}$ and let G be the following context free-grammar:

$$S \to 1S0 \mid 11S0 \mid \varepsilon$$

   (a) ***Prove*** that $L(G) \subseteq L$.

   (b) ***Prove*** that $L \subseteq L(G)$.

4. For any language L, let $\textsc{Prefixes}(L) := \{x \mid xy \in L \text{ for some } y \in \Sigma^*\}$ be the language containing all prefixes of all strings in L. For example, if $L = \{000, 100, 110, 111\}$, then $\textsc{Prefixes}(L) = \{\varepsilon, 0, 1, 00, 10, 11, 000, 100, 110, 111\}$.

   Prove that for any regular language L, the language $\textsc{Prefixes}(L)$ is also regular.

5. For each of the following languages L, give a regular expression that represents L ***and*** describe a DFA that recognizes L.

   (a) The set of all strings in $\{0, 1\}^*$ that contain either both or neither of the substrings 01 and 10.

   (b) The set of all strings in $\{0, 1\}^*$ that do not contain the substring 1010.

   You do ***not*** need to prove that your answers are correct.