

**Write your answers in the separate answer booklet.**  
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “True” if the statement is *always* true and “False” otherwise. Each correct answer is worth +1 point; each incorrect answer is worth  $-\frac{1}{2}$  point; checking “I don’t know” is worth  $+\frac{1}{4}$  point; and flipping a coin is (on average) worth  $+\frac{1}{4}$  point. You do *not* need to prove your answer is correct.

**Read each statement very carefully.** Some of these are deliberately subtle.

- (a) If  $2 + 2 = 5$ , then Jeff is the Queen of England.
  - (b) For all languages  $L_1$  and  $L_2$ , the language  $L_1 \cup L_2$  is regular.
  - (c) For all languages  $L \subseteq \Sigma^*$ , if  $L$  is not regular, then  $\Sigma^* \setminus L$  cannot be represented by a regular expression.
  - (d) For all languages  $L_1$  and  $L_2$ , if  $L_1 \subseteq L_2$  and  $L_1$  is regular, then  $L_2$  is regular.
  - (e) For all languages  $L_1$  and  $L_2$ , if  $L_1 \subseteq L_2$  and  $L_1$  is not regular, then  $L_2$  is not regular.
  - (f) For all languages  $L$ , if  $L$  is regular, then  $L$  has no infinite fooling set.
  - (g) The language  $\{0^m 1^n \mid 0 \leq m + n \leq 374\}$  is regular.
  - (h) The language  $\{0^m 1^n \mid 0 \leq m - n \leq 374\}$  is regular.
  - (i) For every language  $L$ , if the language  $L^R = \{w^R \mid w \in L\}$  is regular, then  $L$  is also regular. (Here  $w^R$  denotes the reversal of string  $w$ ; for example,  $(\text{BACKWARD})^R = \text{DRAWKCAB}$ .)
  - (j) Every context-free language is regular.
2. Let  $L$  be the set of strings in  $\{0, 1\}^*$  in which every run of consecutive 0s has even length and every run of consecutive 1s has odd length.
- (a) Give a regular expression that represents  $L$ .
  - (b) Construct a DFA that recognizes  $L$ .

You do *not* need to prove that your answers are correct.

3. For each of the following languages over the alphabet  $\{0, 1\}$ , either *prove* that the language is regular or *prove* that the language is not regular. **Exactly one of these two languages is regular.**
- (a) The set of all strings in which the substrings 00 and 11 appear the same number of times.
  - (b) The set of all strings in which the substrings 01 and 10 appear the same number of times.

For example, both of these languages contain the string 1100001101101.

4. Consider the following recursive function:

$$\text{stutter}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \cdot \text{stutter}(x) & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

For example,  $\text{stutter}(00101) = 0000110011$ .

**Prove** that for any regular language  $L$ , the following languages are also regular.

- (a)  $\text{STUTTER}(L) := \{\text{stutter}(w) \mid w \in L\}$ .  
 (b)  $\text{STUTTER}^{-1}(L) := \{w \mid \text{stutter}(w) \in L\}$ .

5. Recall that string concatenation and string reversal are formally defined as follows:

$$w \cdot y := \begin{cases} y & \text{if } w = \varepsilon \\ a \cdot (x \cdot y) & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

$$w^R := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ x^R \cdot a & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

**Prove** that  $(w \cdot x)^R = x^R \cdot w^R$ , for all strings  $w$  and  $x$ . Your proof should be complete, concise, formal, and self-contained.