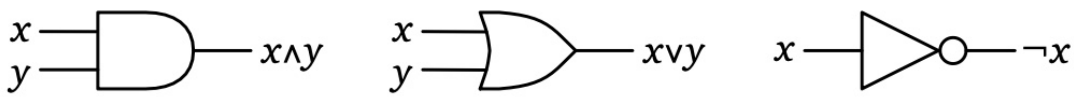
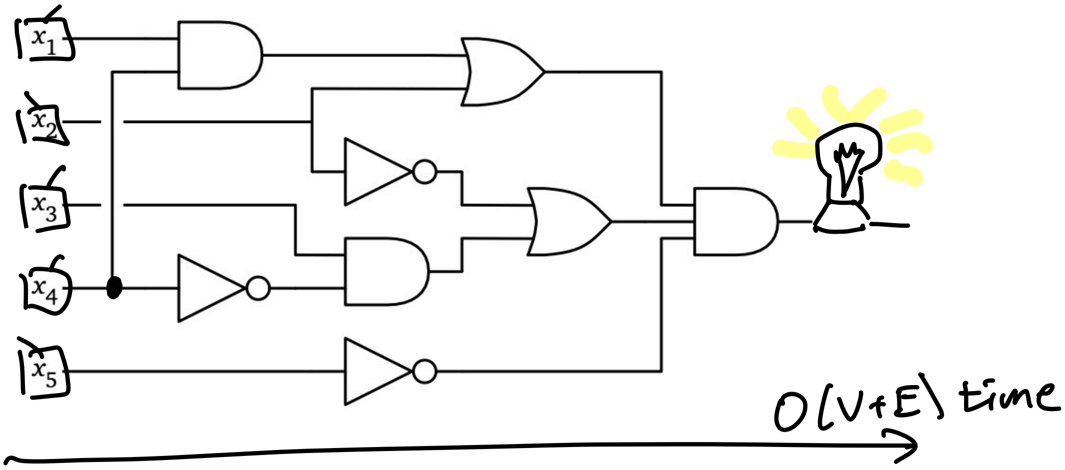




SAT: Given a boolean circuit K, can we turn on the light?

Solvable in polynomial time on a <u>nondeterministic</u> machine
$\iff$ we can <u>verify</u> a YES answer in poly time.

NP = nondeterministic polynomial time
P = deterministic polynomial time

$\boxed{\text{Is } P = NP?}$

Clay Math Inst
$1M

Cook-Levin: IF 3SAT $\in$ P then P=NP.

X is NP-hard: There is a polytime reduction from 3SAT to X

x variable

$x \quad \bar{x}$ literals

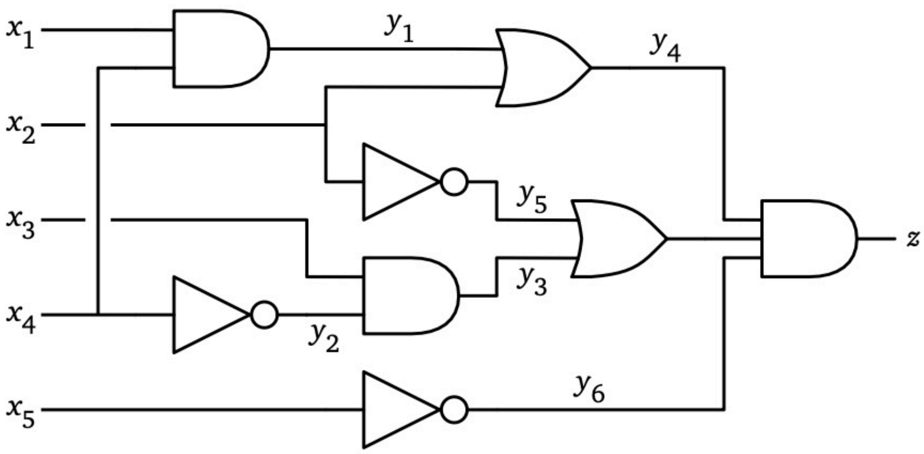
(literal $\vee$ literal $\vee$ literal) clause

clause $\wedge$ clause $\wedge \cdots \wedge$ clause 3SAT formula

$$\underset{T}{(a} \vee \underset{F}{b} \vee \underset{F}{\bar{c}}) \wedge (\underset{F}{b} \vee \underset{T}{\bar{d}} \vee \underset{F}{e}) \wedge (\underset{T}{\bar{b}} \vee \underset{F}{\bar{c}} \vee \underset{T}{\bar{e}}) \wedge \cdots$$

a = TRUE e = FALSE
b = FALSE
c = True
d = False

3SAT: Given 3CNF, can we assign values to vars so that each clause has ≥ 1 true literal?



$$(y_1 = x_1 \wedge x_4) \wedge (y_2 = \overline{x_4}) \wedge (y_3 = x_3 \wedge y_2) \wedge (y_4 = y_1 \vee x_2) \wedge$$
$$(y_5 = \overline{x_2}) \wedge (y_6 = \overline{x_5}) \wedge (y_7 = y_3 \vee y_5) \wedge (z = y_4 \wedge y_7 \wedge y_6) \wedge z$$

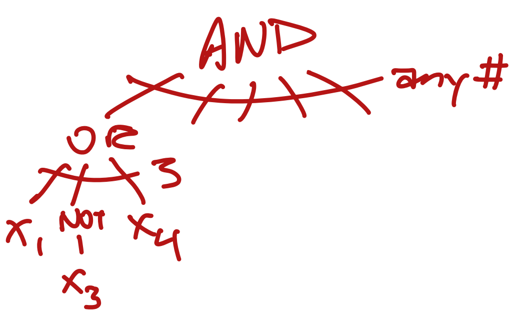
3SAT. Given boolean formula in conjunctive normal form with 3 literals per clause

$$\boxed{(y_1 \vee \overline{x_1} \vee \overline{x_4})} \wedge \boxed{(\overline{y_1} \vee x_1 \vee z_1)} \wedge \boxed{(\overline{y_1} \vee x_1 \vee \overline{z_1})} \wedge (\overline{y_1} \vee x_4 \vee z_2) \wedge (\overline{y_1} \vee x_4 \vee \overline{z_2})$$

clause

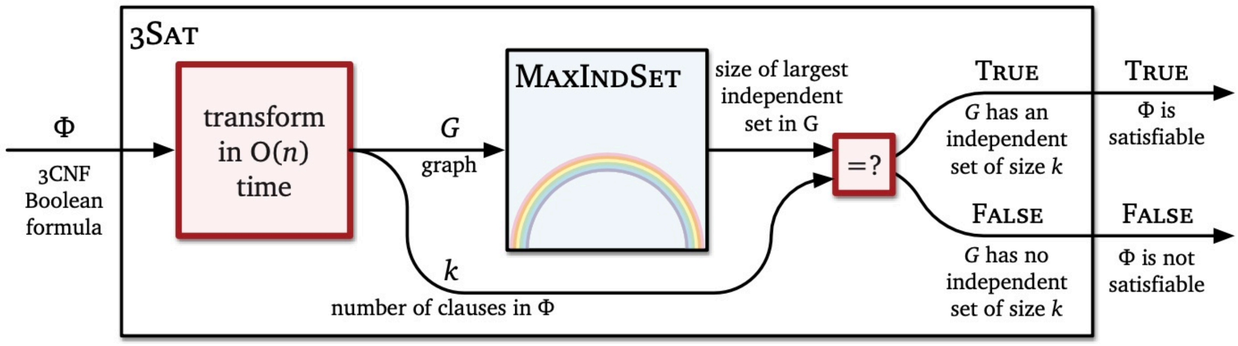
$$\wedge (y_2 \vee x_4 \vee z_3) \wedge (y_2 \vee x_4 \vee \overline{z_3}) \wedge (\overline{y_2} \vee \overline{x_4} \vee z_4) \wedge (\overline{y_2} \vee \overline{x_4} \vee \overline{z_4})$$
$$\wedge (y_3 \vee \overline{x_3} \vee \overline{y_2}) \wedge (\overline{y_3} \vee x_3 \vee z_5) \wedge (\overline{y_3} \vee x_3 \vee \overline{z_5}) \wedge (\overline{y_3} \vee y_2 \vee z_6) \wedge (\overline{y_3} \vee y_2 \vee \overline{z_6})$$
$$\wedge (\overline{y_4} \vee y_1 \vee x_2) \wedge (y_4 \vee \overline{x_2} \vee z_7) \wedge (y_4 \vee \overline{x_2} \vee \overline{z_7}) \wedge (y_4 \vee \overline{y_1} \vee z_8) \wedge (y_4 \vee \overline{y_1} \vee \overline{z_8})$$
$$\wedge (y_5 \vee x_2 \vee z_9) \wedge (y_5 \vee x_2 \vee \overline{z_9}) \wedge (\overline{y_5} \vee \overline{x_2} \vee z_{10}) \wedge (\overline{y_5} \vee \overline{x_2} \vee \overline{z_{10}})$$
$$\wedge (y_6 \vee x_5 \vee z_{11}) \wedge (y_6 \vee x_5 \vee \overline{z_{11}}) \wedge (\overline{y_6} \vee \overline{x_5} \vee z_{12}) \wedge (\overline{y_6} \vee \overline{x_5} \vee \overline{z_{12}})$$
$$\wedge (\overline{y_7} \vee y_3 \vee y_5) \wedge (y_7 \vee \overline{y_3} \vee z_{13}) \wedge (y_7 \vee \overline{y_3} \vee \overline{z_{13}}) \wedge (y_7 \vee \overline{y_5} \vee z_{14}) \wedge (y_7 \vee \overline{y_5} \vee \overline{z_{14}})$$
$$\wedge (y_8 \vee \overline{y_4} \vee \overline{y_7}) \wedge (\overline{y_8} \vee y_4 \vee z_{15}) \wedge (\overline{y_8} \vee y_4 \vee \overline{z_{15}}) \wedge (\overline{y_8} \vee y_7 \vee z_{16}) \wedge (\overline{y_8} \vee y_7 \vee \overline{z_{16}})$$
$$\wedge (y_9 \vee \overline{y_8} \vee \overline{y_6}) \wedge (\overline{y_9} \vee y_8 \vee z_{17}) \wedge (\overline{y_9} \vee y_6 \vee z_{18}) \wedge (\overline{y_9} \vee y_6 \vee \overline{z_{18}}) \wedge (\overline{y_9} \vee y_8 \vee \overline{z_{17}})$$
$$\wedge (y_9 \vee z_{19} \vee z_{20}) \wedge (y_9 \vee \overline{z_{19}} \vee z_{20}) \wedge (y_9 \vee z_{19} \vee \overline{z_{20}}) \wedge (y_9 \vee \overline{z_{19}} \vee \overline{z_{20}})$$

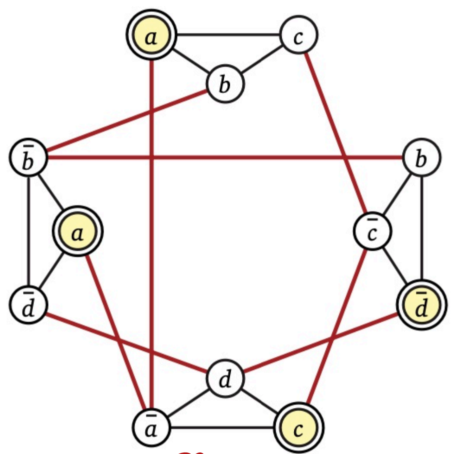
literal literal



$$T_{MaxIndSet} \geq T_{3SAT} - O(n)$$

$$T_{3SAT}(n) \leq T_{MaxIndSet}(O(n)) + O(n)$$





$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

a = T
b = F
c = T
d = F

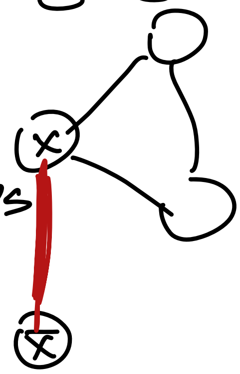
3SAT(Φ):

k ← #clauses in Φ
G ← TRANFORM(Φ)
if MaxIndSet(G) = k
    return True
els
    return False

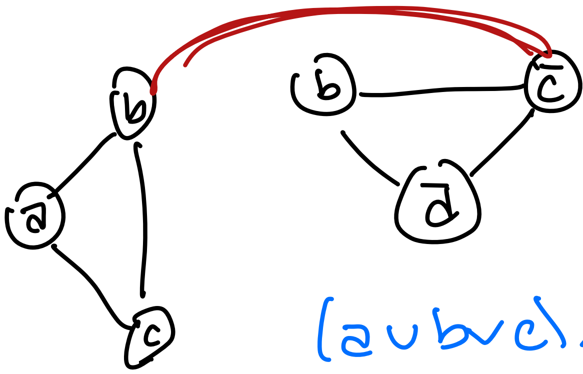
TRANSFORM:

G has 3k vertices
3 per clause
1 per literal

"clause gadges"



Edges between contradicting vertices
"variable gadgets"



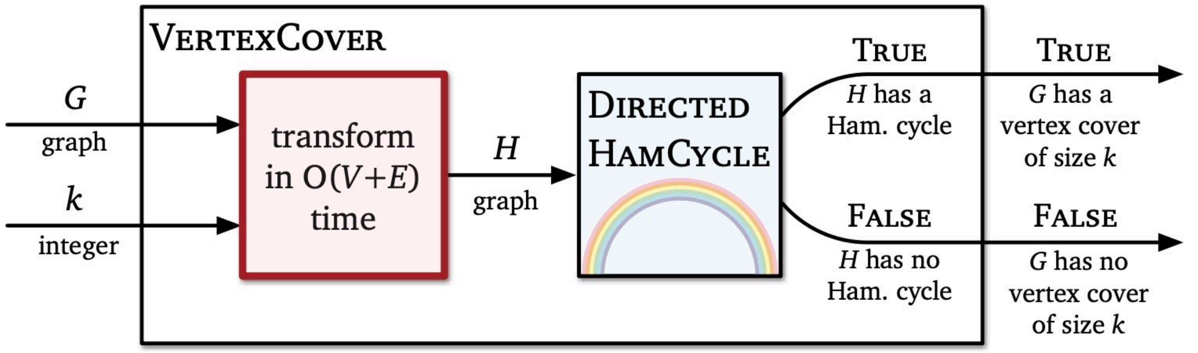
$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge \dots$

Claim:

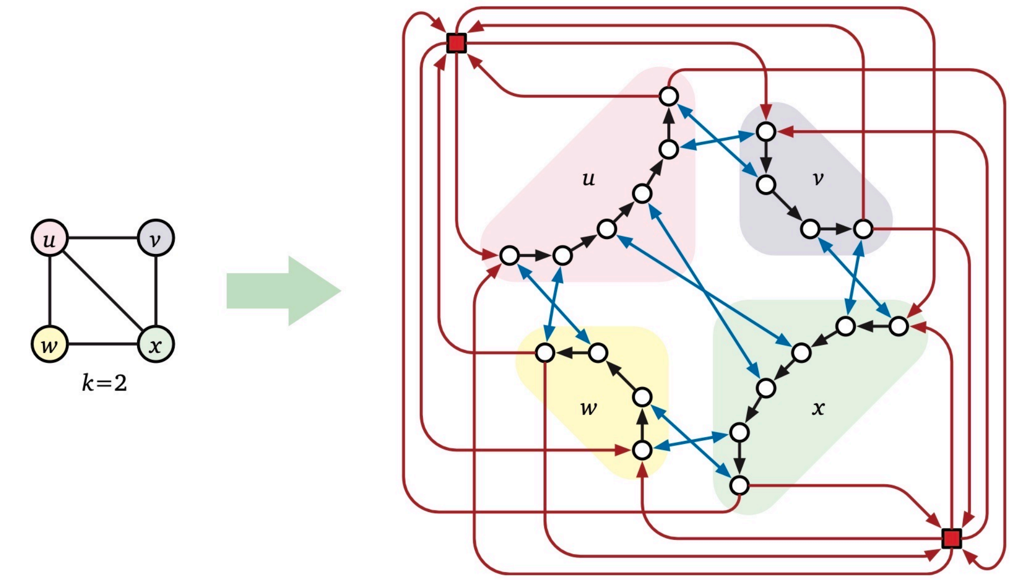
Φ is sastisfiable
iff
G has ind. set of size k.

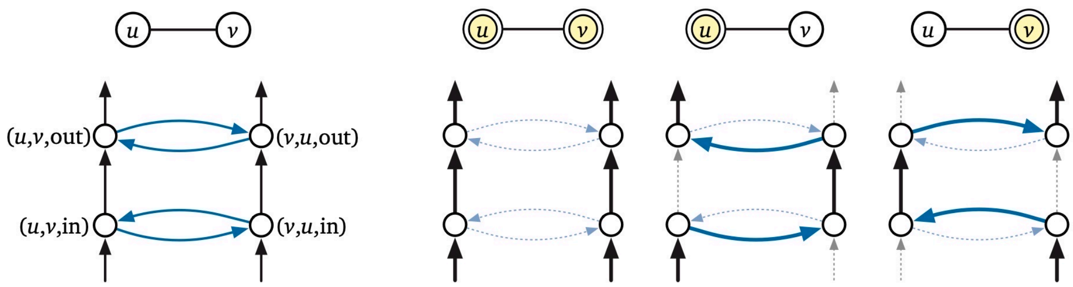
$\Rightarrow$ Spose Φ is satisfiable
Fix a satisfying assignment
Pick one true literal in each clause
Corresponding vertices in G are indep.

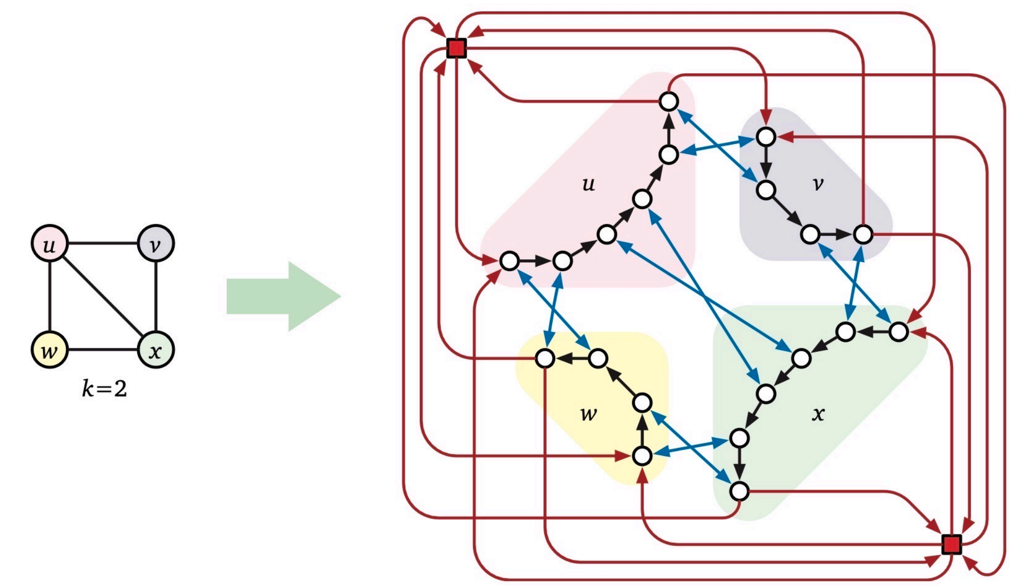
$\Leftarrow$ Spose G has indep set S of size k
Derive assignment to variables
make literals in S true
$\Rightarrow$ each clause gadget has 1 node in S
$\Rightarrow$ each clause in Φ has ≥ 1 true literal.

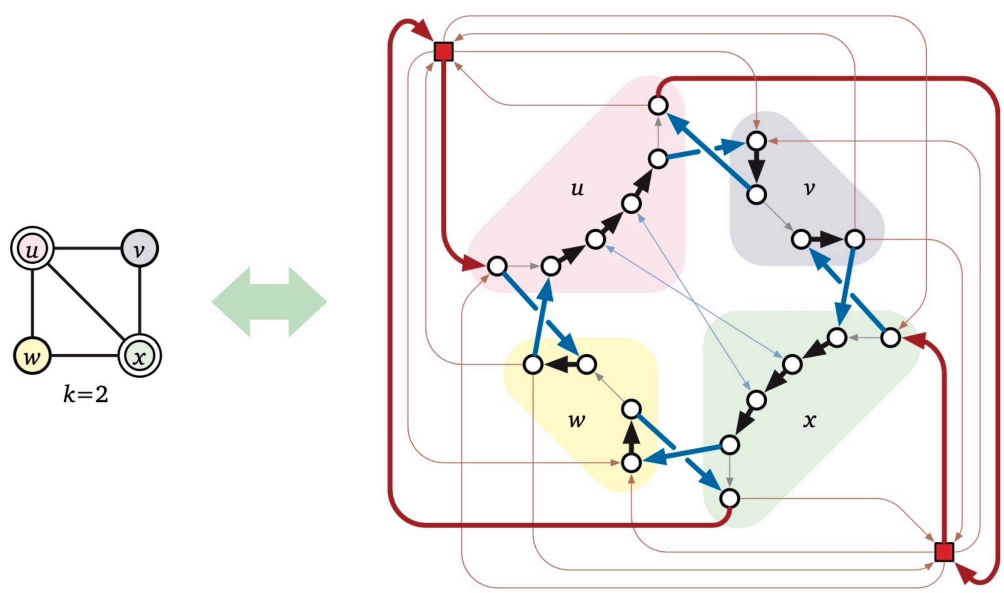


VertexCover
G
graph
k
integer
transform
in O(V+E)
time
H
graph
Directed
HamCycle
True
H has a
Ham. cycle
False
H has no
Ham. cycle
True
G has a
vertex cover
of size k
False
G has no
vertex cover
of size k


u
v
w
x
k=2
u
v
w
x


u
v
(u,v,out)
(v,u,out)
(u,v,in)
(v,u,in)
u
v
u
v
u
v


u
v
w
x
k=2
u
v
w
x


u
v
w
x
k=2
u
v
w
x