

Midterm next next Monday
Conflict exam Tuesday
HW4 due Tuesday

DZES

Regular:

sequencing: AB
branching: A+B
repetition: A*

Context-free:
recursion!

$\{0^n 1^n \mid n \geq 0\}$

$\{ [^n]^n \mid n \geq 0 \}$
push pop



The Recursive Mind

*The Origins of Human Language,
Thought, and Civilization*



Michael C. Corballis

[Comment](#) > *Behav Brain Sci.* 2012 Aug;35(4):219-20. doi: 10.1017/S0140525X11001865.

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Evidence of recursion in tool use

Lluís Barceló-Coblijn¹, Antoni Gomila

Affiliations + expand

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Abstract

We discuss the discovery of technologies involving knotted netting, such as textiles, basketry, and cordage, in the Upper Paleolithic. This evidence, in our view, suggests a new way of connecting toolmaking and syntactic structure in human evolution, because these technologies already exhibit an "infinite use of finite means," which we take to constitute the key transition to human cognition.

nature

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LETTERS

Recursive syntactic pattern learning by songbirds

Timothy Q. Gentner^{1†}, Kimberly M. Fenn², Daniel Margoliash^{1,2} & Howard C. Nusbaum²

⟨sentence⟩ → ⟨noun phrase⟩⟨verb phrase⟩⟨noun phrase⟩

⟨noun phrase⟩ → ⟨adjective phrase⟩⟨noun⟩

⟨adj. phrase⟩ → ⟨article⟩ | ⟨possessive⟩ | ⟨adjective phrase⟩⟨adjective⟩

⟨verb phrase⟩ → ⟨verb⟩ | ⟨adverb⟩⟨verb phrase⟩

⟨noun⟩ → dog | trousers | daughter | nose | homework | time lord | pony | …

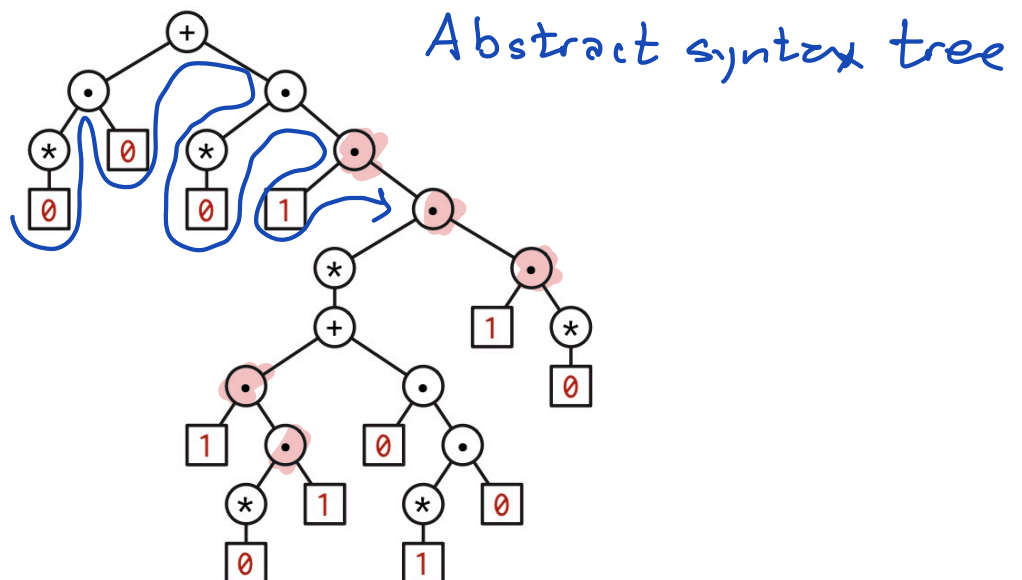
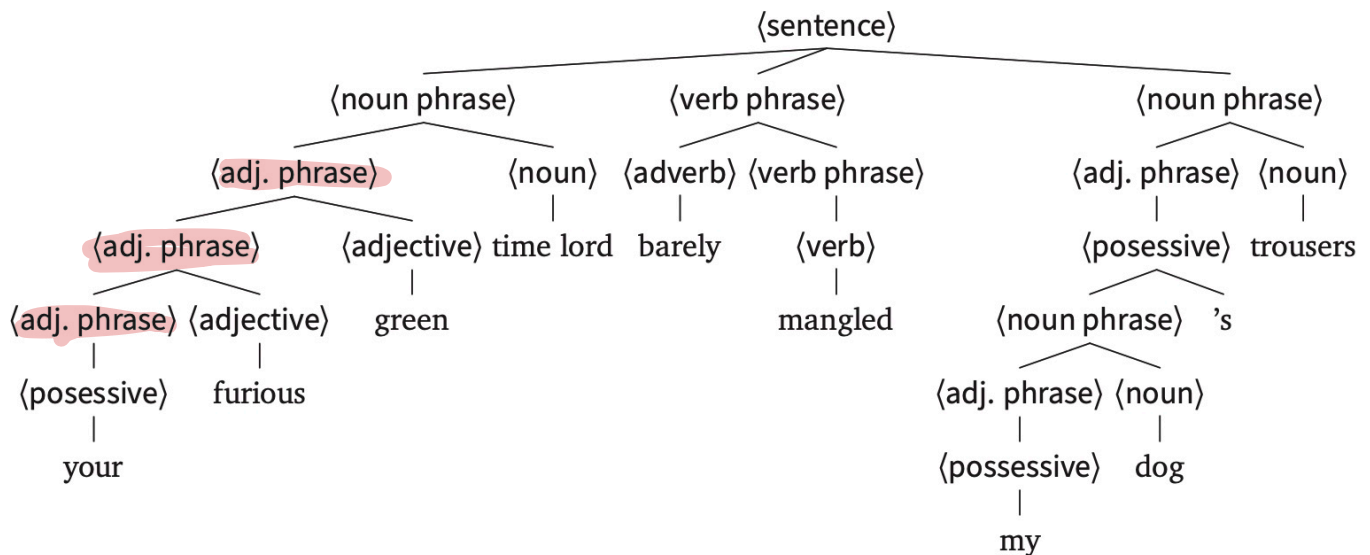
⟨article⟩ → the | a | some | every | that | …

⟨possessive⟩ → ⟨noun phrase⟩'s | my | your | his | her | …

⟨adjective⟩ → friendly | furious | moist | green | severed | timey-wimey | little | …

⟨verb⟩ → ate | found | wrote | killed | mangled | saved | invented | broke | …

⟨adverb⟩ → squarely | incompetently | barely | sort of | awkwardly | totally | …



A regular expression tree for $0^*0 + 0^*1(10^*1 + 01^*0)^*10^*$

$S \rightarrow A$
 $S \rightarrow B$
 $A \rightarrow 0A$
 $A \rightarrow 0C$
 $B \rightarrow B1$
 $B \rightarrow C1$
 $C \rightarrow \epsilon$
 $C \rightarrow 0C1$

Σ alphabet $\{0,1\}$ terminals literals

Γ nonterminals $\{S, A, B, C\}$

Production rules

$A \rightarrow \omega$

$A \in \Gamma \quad \omega \in (\Sigma \cup \Gamma)^*$

Starting non-terminal S

Context-free grammar

$S \rightarrow A \rightarrow 0A \rightarrow 00A \rightarrow 000C \rightarrow 0000C1$
 \downarrow
 $0000011 \leftarrow 00000C11$

$\rightarrow 00A B 11 C 0A \rightarrow 00A B 11 0C 1 0A$

Grammar generates $w \in \Sigma^*$ if there is a sequence of productions $S \rightsquigarrow \dots \rightsquigarrow w$.

Backus-Naur form BNF

$S \rightarrow A \mid B$
 $A \rightarrow 0A \mid 0C$
 $B \rightarrow B1 \mid C1$
 $C \rightarrow \epsilon \mid 0C1$

$\rightarrow \{0^m 1^n \mid m \neq n\}$

$\rightarrow \{0^m 1^n \mid m > n\}$

$\rightarrow \{0^m 1^n \mid n > m\} \leftarrow$

$\rightarrow \{0^n 1^n \mid n \geq 0\}$

\cup or $+$ or

\vee or $|$ or

$S = 0^* 1^* \setminus C$

$C \rightarrow 0C1 \rightarrow 00C11 \rightarrow 000C111 \rightarrow 000111$

$B \rightarrow B1 \rightarrow B11 \rightarrow \dots \rightarrow B111111$

$\rightarrow C111111$

\downarrow
 $0^n 1^n 111111$

\downarrow
 $\{C1^n \mid n > 0\}$

Noam Chomsky

Regular

Cont Free

Cont Sensitive

Recursive

$S \rightarrow OS \mid S1 \mid \epsilon \rightarrow 0^*1^*$

Regular languages are all context-free

Not all languages are context free

$\{0^n 1^n 0^n \mid n \geq 0\}$

$\{ww \mid w \in \Sigma^*\}$

ACKACK

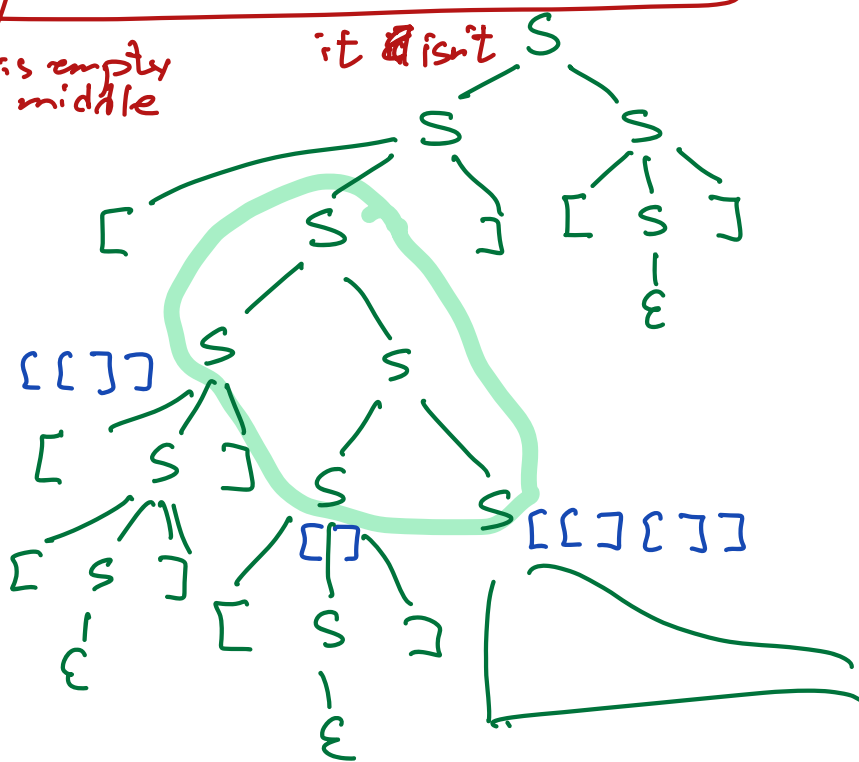
Balanced Brackets

$\{ [[[]] [] [[] []]] \{ [] \}$

$S \rightarrow SS \mid [S] \mid \epsilon$

Stack is empty in middle

~~[] [] [] []~~



$S \rightarrow [S]S \mid \epsilon$

$S \rightarrow [S]S$
 $\rightarrow [] S$
 $\rightarrow []$

