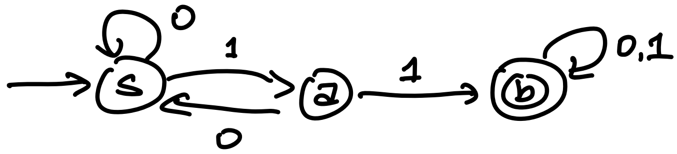


$(0+1)^* 11 (0+1)^*$

Kleene's Thm: automatic = regular

$M = (Q, s, A, \delta)$

- Q — states
- $s \in Q$ — start state
- $A \subseteq Q$ — accepting state
- $\delta : Q \times \Sigma \to Q$ — transition

$\delta^* : Q \times \Sigma^* \to Q$

$$\delta^*(q,w) = \begin{cases} q & \text{if } w = e \\ \delta^*(\delta(q,a), x) & \text{if } w = ax \end{cases}$$

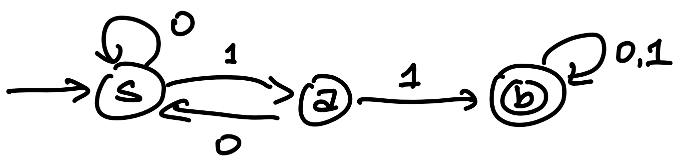
M accepts $w \Leftrightarrow \delta^*(s,w) \in A$

$\boxed{\texttt{def delta(q: state, a: symbol)} \to \texttt{state:}}$

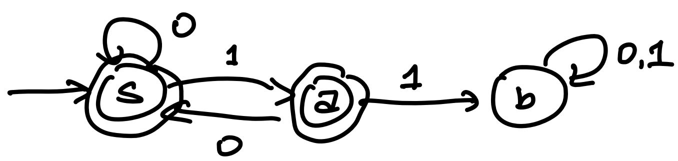
$\delta(p,a) = q$ $\qquad p \xrightarrow{a} q$

$L(M) = \{w \mid M \text{ accepts } w\}$
$= \{w \mid \delta^*(s,w) \in A\}$

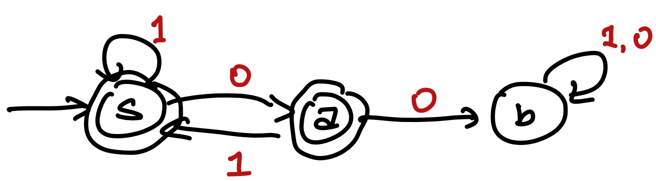
$\delta^*(s, 0010110) = b \in A$ ✓



strings containing 11



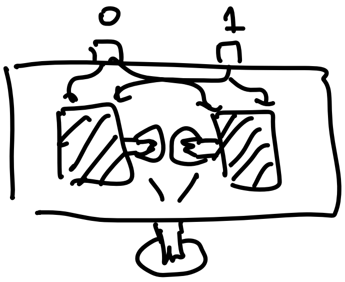
strings not containing 11

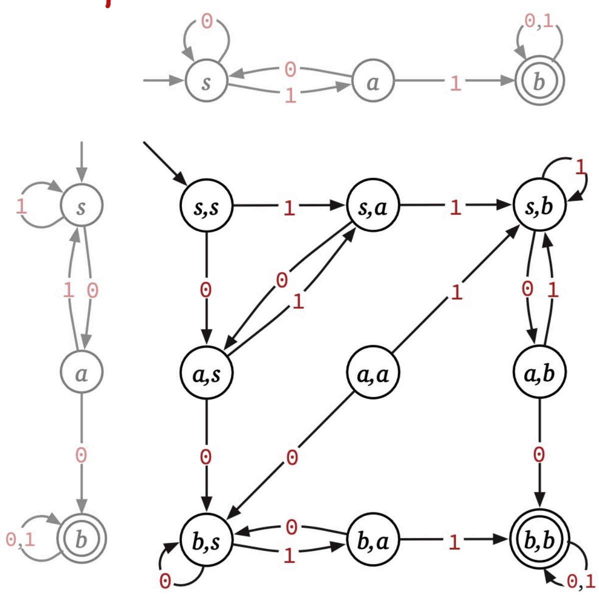


strings not containg 00

Strings containing both 00 and 11

"product construction"





Building a DFA for the language of strings containing both 00 and 11.

Given $M_1 = (Q_1, s_1, A_1, \delta_1)$ and $M_2 = (Q_2, s_2, A_2, \delta_2)$

Define $M = (Q, s, A, \delta)$ as follows

$$Q = Q_1 \times Q_2 = \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

$$s = (s_1, s_2)$$

$$A = \{(p,q) \mid p \in A_1 \ \boxed{\text{and}} \ q \in A_2\}$$

$$\delta((p,q),a) = (\delta_1(p,a), \ \delta_2(q,a))$$

Theorem: $L(M) = L(M_1) \cap L(M_2)$

Key Lemma: $\delta^*((p,q),w) = (\delta_1^*(p,w), \delta_2^*(q,w))$

for all $p \in Q_1$, $q \in Q_2$, $w \in \Sigma^*$.

Proof: Let p, q be arb. states

w be arb string

Assume for all strings x shorter than w

for all states $p' \in Q_1$ and $q' \in Q_2$

$\delta^*((p',q'),x) = (\delta^*(p',x), \delta^*(q',x))$

There are two cases:

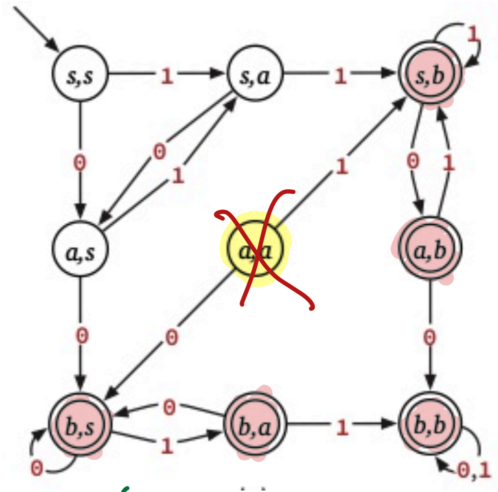
- $w = \varepsilon$

$$\begin{aligned}
\delta^*((p,q),w) &= \delta^*((p,q),\varepsilon) && [w=\varepsilon] \\
&= (p,q) && \text{by def. } \delta^* \\
&= (\delta_1^*(p,\varepsilon), \delta_2^*(q,\varepsilon)) && \text{def } \delta^* \\
&= (\delta_1^*(p,w), \delta_2^*(q,w)) && [w=\varepsilon]
\end{aligned}$$

- $w \neq \varepsilon \Rightarrow w = ax$ for some $a \in \Sigma$ and $x \in \Sigma^*$

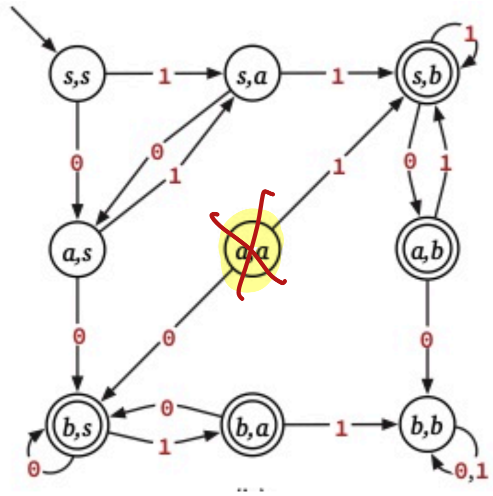
$$\begin{aligned}
\delta^*((p,q),w) &= \delta^*((p,q),ax) && [w=ax] \\
&= \delta^*(\delta((p,q),a), x) && [\text{def } \delta^*] \\
&= \delta^*((\delta_1(p,a), \delta_2(q,a)), x) && [\text{def } \delta] \\
&= (\delta_1^*(\delta_1(p,a),x), \delta_2^*(\delta_2(q,a),x)) && [\text{IH}] \\
&= (\delta_1^*(p,ax), \delta_2^*(q,ax)) && (\text{def } \delta_1^* \ \delta_2^*] \\
&= (\delta_1^*(p,w), \delta_2^*(q,w)) && [w=ax)
\end{aligned}$$

$\square$



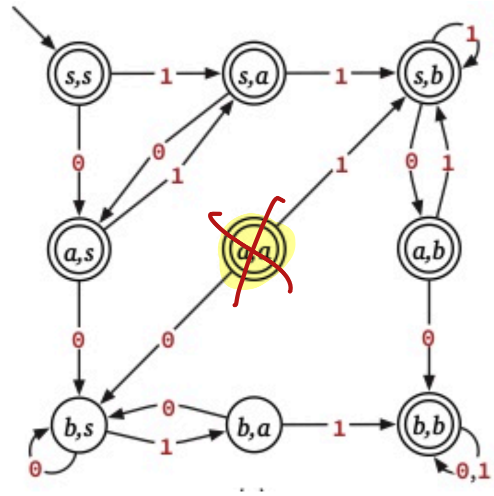
OR

$\cup$

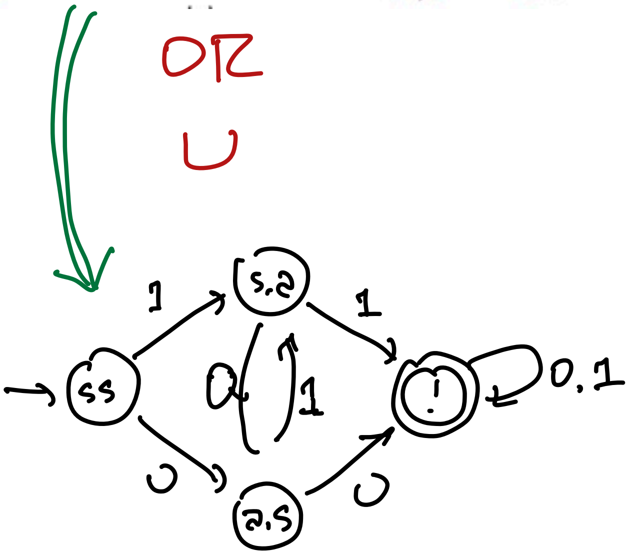


XOR

$\oplus$



$00 \Rightarrow 11$



If L_1 and L_2 are automatic, then
regular

$L_1 \cup L_2$

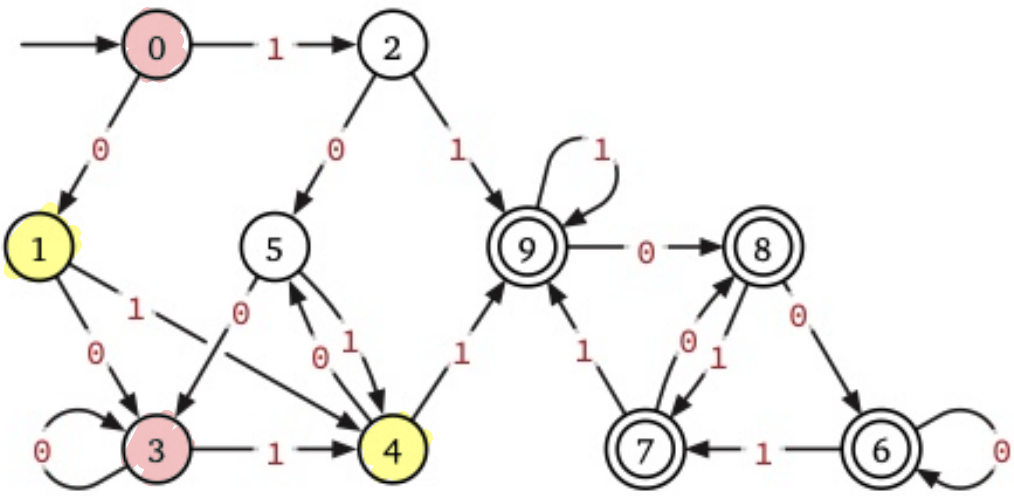
$L_1 \cap L_2$

$L_1 \oplus L_2$

$L_1 \setminus L_2$

$\overline{L_1} = \Sigma^* \setminus L_1$

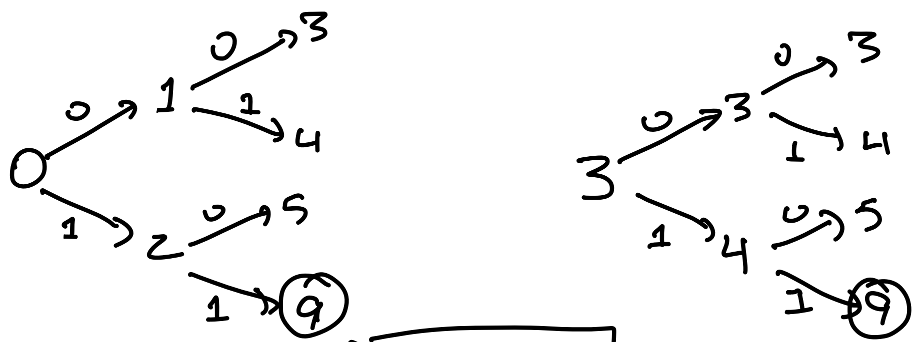
are all automatic
regular



p and q are distinguishable $\iff$

for some string w $\delta^*(p,w) \in A$ and $\delta^*(q,w) \notin A$

or vice versa.



$\boxed{0 \equiv 3}$

$1 \xrightarrow{1} 4$ $\qquad$ $4 \xrightarrow{1} 9$

$\boxed{1 \not\equiv 4}$