Hw 1 due 9pm

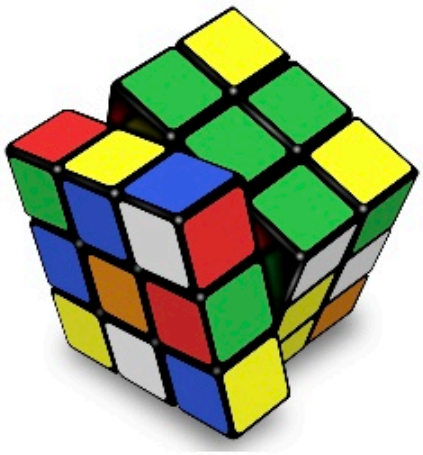
Extension req's due tomorrow 9pm

One submission per group per problem
- identify other group members
- identify pages for each subproblems
- PDF only

Monday is Labor Day → all deadlines +24 hours

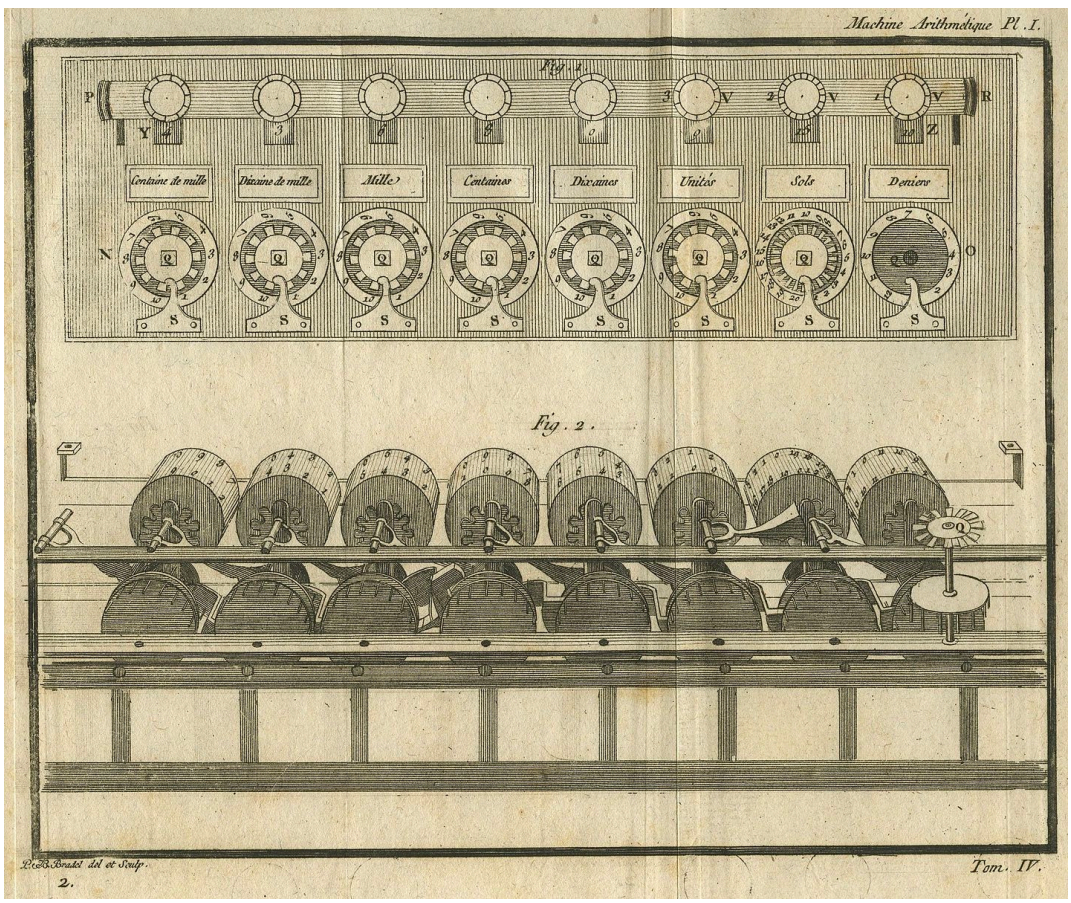
GPS 2 is due Tuesday

Hw 2 Wednesday



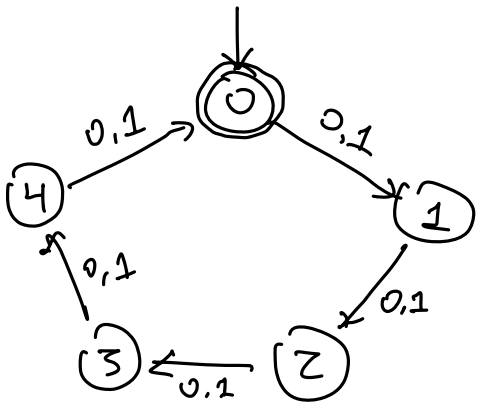
Three finite-state machines.

1642



$\{ w \in \{0,1\}^* \mid |w| \text{ is divisible by } 5 \}$

```
count ← 0
For i ← 1 to |w|
    count ← count + 1 mod 5
return (count == 0)
```



Deterministic Finite Automaton

(Q, s, A, δ)

Q = set of states — any finite nonempty set

$s \in Q$ start state

$A \subseteq Q$ accepting states

$\delta : Q \times \Sigma \to Q$ transition function

δ	0	1
0	1	1
1	2	2
2	3	3
3	4	4
4	0	0

$Q = \{0,1,2,3,4\}$ ← state q means #symbols mod 5 = q

$s = 0$

$A = \{0\}$

$\delta(q, a) = (q+1) \bmod 5$

next

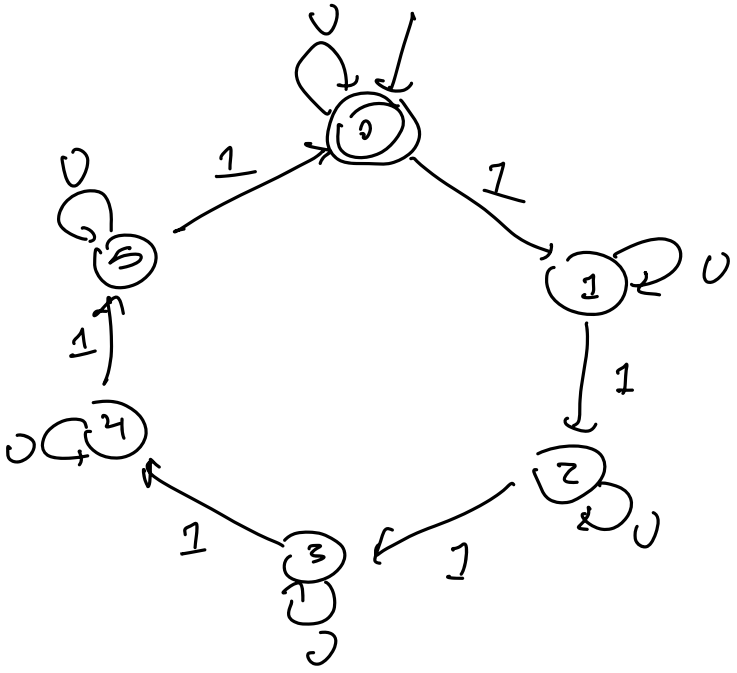
strings where #1s is divisible by 6

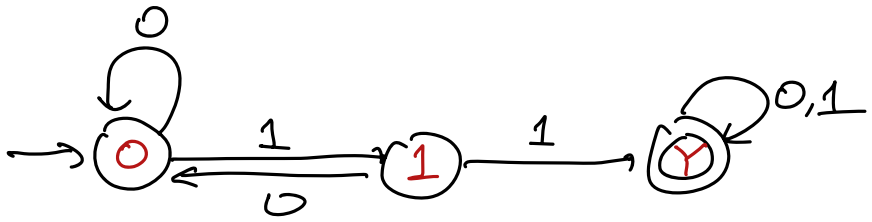
$Q = \{0, 1, 2, 3, 4, 5\}$ — state q means $q = \#1s \bmod 6$

$s = 0$

$A = \{0\}$

$$\delta(q, a) = \begin{cases} q & \text{if } a = 0 \\ (q+1) \bmod 6 & \text{if } a = 1 \end{cases}$$





All strings with 11 substring

state 0: either start or just read 0, haven't seen 11

1: just read 1, haven't seen 11

Y: seen 11

Extended transition function

$$\delta^* : Q \times \Sigma^* \rightarrow Q$$

$$\delta^*(q, w) = \begin{cases} q & \text{if } w = \varepsilon \\ \delta^*(\delta(q, a), x) & \text{if } w = ax \end{cases}$$

$\delta^*(0, 0100101) = 1$

$\delta^*(0, 00101101) = Y$

$$L(M) = \{ w \mid \delta^*(s, w) \in A \}$$

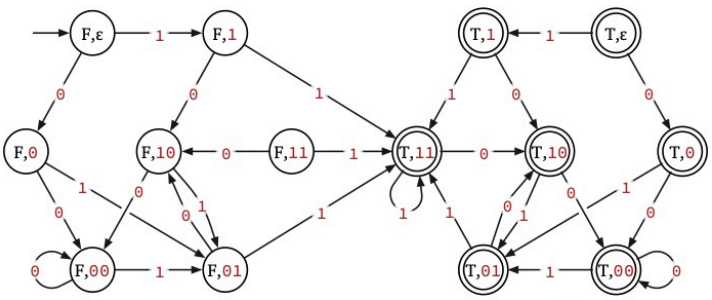
$M = (Q, s, A, \delta)$

$L(M) = (0+1)^* 11 (0+1)^*$

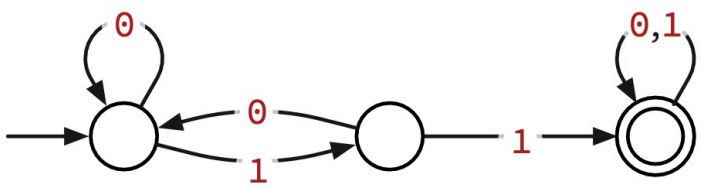
```
CONTAINS11(w[1..n]):
  found ← FALSE        last2 ← ε
  for i ← 1 to n
      if i = 1
          last2 ← w[1]
      else
          last2 ← w[i−1]·w[i]
      if last2 = 11
          found ← TRUE
  return found
```

q	$\delta[q,0]$	$\delta[q,1]$
(FALSE, ε)	(FALSE, 0)	(FALSE, 1)
(FALSE, 0)	(FALSE, 00)	(FALSE, 01)
(FALSE, 1)	(FALSE, 10)	(**TRUE**, 11)
(FALSE, 00)	(FALSE, 00)	(FALSE, 01)
(FALSE, 01)	(FALSE, 10)	(**TRUE**, 11)
(FALSE, 10)	(FALSE, 00)	(FALSE, 01)
(FALSE, 11)	(FALSE, 10)	(**TRUE**, 11)

q	$\delta[q,0]$	$\delta[q,1]$
(TRUE, ε)	(TRUE, 0)	(TRUE, 1)
(TRUE, 0)	(TRUE, 00)	(TRUE, 01)
(TRUE, 1)	(TRUE, 10)	(TRUE, 11)
(TRUE, 00)	(TRUE, 00)	(TRUE, 01)
(TRUE, 01)	(TRUE, 10)	(TRUE, 11)
(TRUE, 10)	(TRUE, 00)	(TRUE, 01)
(TRUE, 11)	(TRUE, 10)	(TRUE, 11)



Our brute-force DFA for strings containing the substring 11



A minimal DFA for superstrings of 11

binary #s divisible by 5

1010

$2^3 + 2^1 = 10_{10}$

$$\text{value}(w) = \begin{cases} 0 & w = \varepsilon \\ 2 \cdot \text{value}(x) + a & w = x \cdot a \end{cases}$$

$$\begin{aligned} \text{value}(1010) &= 2 \cdot \text{value}(101) \\ &= 2(2\,\text{value}(10) + 1) \\ &= 2(2 \cdot 2 + 1) \\ &= 2 \cdot 5 \\ &= 10 \end{aligned}$$

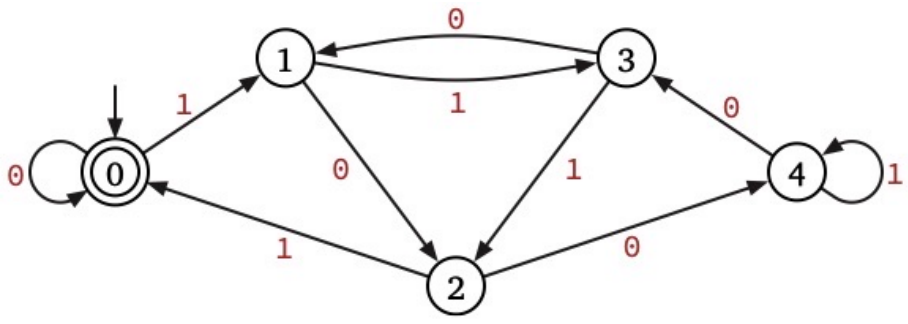
```
MULTIPLEOF5(w[1..n]):
  rem ← 0
  for i ← 1 to n
      rem ← (2 · rem + w[i]) mod 5
  if rem = 0
      return TRUE
  else
      return FALSE
```

i	$w[1..i]$	*value*	*rem*
0	ε	0	0
1	0	0	0
2	00	0	0
3	001	1	1
4	0010	2	2
5	00101	5	0
6	001011	11	1
7	0010111	23	3
8	00101110	46	1
9	001011101	93	3
10	0010111011	187	2
11	00101110110	374	4

```
DOSOMETHINGCOOL(w[1..n]):
  q ← 0
  for i ← 1 to n
      q ← δ[q, w[i]]
  return A[q]
```

q	$\delta[q,0]$	$\delta[q,1]$	$A[q]$
0	0	1	TRUE
1	2	3	FALSE
2	4	0	FALSE
3	1	2	FALSE
4	3	4	FALSE

$\delta(q,a) = (2q+a) \bmod 5$



State-transition graph for MULTIPLEOF5