1. Suppose you are given an array of numbers, some of which are marked as *icky*, and you want to compute the length of the longest increasing subsequence of $A$ that includes at most $k$ icky numbers. Your input consists of the integer $k$, the number array $A[1..n]$, and another boolean array $Icky[1..n]$.

   For example, suppose your input consists of the integer $k = 2$ and the following array (with icky numbers are indicated by stars):

   | $3^\star$ | $1^\star$ | 4 | $1^\star$ | $5^\star$ | 9 | $2^\star$ | 6 | 5 | $3^\star$ | 5 | 9 | 7 | $9^\star$ | 3 | 2 | 3 | $8^\star$ | 4 | $6^\star$ | 2 | $6^\star$ |
   |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

   Then your algorithm should return the integer 5, which is the length of the increasing subsequence $4, 5^\star, 6, 7, 9^\star$.

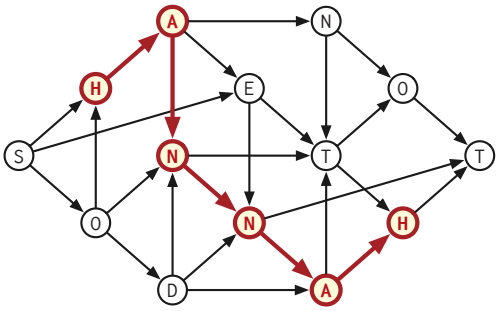
   (a) Describe an algorithm for this problem using dynamic programming.

   (b) Describe an algorithm for this problem by reducing it to a standard graph problem.

**Think about later:**

2. Let $G$ be a directed acyclic graph whose vertices have labels from some fixed alphabet. Any directed path in $G$ has a label, which is a string obtained by concatenating the labels of its vertices. Recall that a *palindrome* is a string that is equal to its reversal.

   Describe and analyze an algorithm to find the length of the longest palindrome that is the label of a path in $G$. For example, given the dag below, your algorithm should return the integer 6, which is the length of the palindrome HANNAH.




   (a) Describe an algorithm for this problem using dynamic programming.

   (b) Describe an algorithm for this problem by reducing it to a standard graph problem.