CS/ECE 374 A ✧ Fall 2023

Practice Midterm 2

November 2, 2023

Name:	
NetID:	

- ***Don't panic!***
- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanner can actually see. Anything you write outside the boxes will be erased before we start grading.
- If you run out of space for an answer, please use the scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. Please ask for more scratch paper if you need it.
- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

CS/ECE 374 A ✧ Fall 2023 Practice Midterm 2 Problem 1	Name:

(a) Write the solution to each of the following recurrences in the box immediately below it. (Use the space below the boxes for scratch work.)

$A(n) = 3A(n/2) + O(n^2)$ $\qquad$ $B(n) = 7B(n/2) + O(n^2)$ $\qquad$ $C(n) = 4C(n/2) + O(n^2)$

(b) Draw a directed acyclic graph with at most ten vertices, exactly one source, exactly one sink, and more than one topological order.

(c) Draw a directed graph with at most ten vertices, with distinct positive edge weights, that has more than one shortest path from some vertex s to some other vertex t.

(d) Describe an appropriate memoization structure and evaluation order for the following (meaningless) recurrence, and give the running time of the resulting iterative algorithm to compute $Huh(1, n)$.

$$Huh(i,k) = \begin{cases} 0 & \text{if } i > n \text{ or } k < 0 \\ \min\begin{Bmatrix} Huh(i+1, k-2) \\ Huh(i+2, k-1) \end{Bmatrix} + A[i,k] & \text{if } A[i,k] \text{ is even} \\ \max\begin{Bmatrix} Huh(i+1, k-2) \\ Huh(i+2, k-1) \end{Bmatrix} - A[i,k] & \text{if } A[i,k] \text{ is odd} \end{cases}$$

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See the question sheet for a detailed description of your game with Elmo.

(a) ***Prove*** that you should not also use the greedy strategy. That is, show that there is a game that you can win, but only if you do *not* follow the same greedy strategy as Elmo. Assume Elmo plays first.

(b) Describe and analyze an algorithm to determine, given the initial sequence of cards, the maximum number of points that you can collect playing against Elmo.

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Suppose you are given a directed graph $G = (V, E)$, whose vertices are either red, green, or blue. Edges in G do not have weights, and G is not necessarily a dag. The ***remoteness*** of a vertex v is the *maximum* of three shortest-path lengths:

- The length of a shortest path to v from the closest red vertex
- The length of a shortest path to v from the closest blue vertex
- The length of a shortest path to v from the closest green vertex

In particular, if v is not reachable from vertices of all three colors, then v is infinitely remote. Describe and analyze an algorithm to find a vertex of G with *minimum* remoteness.

CS/ECE 374 A ✧ Fall 2023 Practice Midterm 2 Problem 4	Name:

Suppose you are given an array $A[1..n]$ of integers such that $A[i]+A[i+1]$ is even for *exactly one* index i. In other words, the elements of A alternate between even and odd, except for exactly one adjacent pair that are either both even or both odd. Describe and analyze an efficient algorithm to find the unique index i such that $A[i]+A[i+1]$ is even.

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A *zigzag walk* in in a directed graph G is a sequence of vertices connected by edges in G, but the edges alternately point forward and backward along the sequence. Specifically, the first edge points forward, the second edge points backward, and so on. The *length* of a zigzag walk is the sum of the weights of its edges, both forward and backward.

Suppose you are given a directed graph G with non-negatively weighted edges, along with two vertices s and t. Describe and analyze an algorithm to find the shortest zigzag walk from s to t in G.

(scratch paper)

(scratch paper)

(scratch paper)

(scratch paper)

(scratch paper)