CS/ECE 374: Algorithms & Models of Computation, Spring 2019

Midterm 1 **Monday, February 18, 7-9pm**

Which exam room to go to based on your discussion section.

ECEB 1002	SC 1404	DCL 1320	ECEB 1013	ECEB 1015
AYA 9am Yipu AYB 10am Xilin AYC 11am Xilin AYD noon Mitch AYE 1pm Ravi AYJ 1pm Shant	AYF 2pm Konstantinos AYG 3pm Robert BYE 3pm Jiaming	AYH 4pm Robert AYK 2pm Shant BYA 9am Zhongyi BYC 1pm Shu	BYB 10am Zhongyi BYF 4pm Jiaming	BYD 2pm Shu

Name:	
NetID:	

⇐ Please PRINT

- ***Don't panic!***
- Please print your name, print your NetID, and circle your discussion section in the boxes above.
- There are five questions – you should answer all of them.
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original) 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. We will not return or scan the cheat sheets, so photocopy them before the exam if you want a copy.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- Please read all the questions before starting to answer them. Please ask for clarification if any question is unclear.
- **This exam lasts 120 minutes.** The clock started when you got the questions.
- If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
- As usual, answering any (sub)problem with I don't know (and nothing else) is worth 25% partial credit. Correct, complete, but sub-optimal solutions are *always* worth more than 25%. A blank answer is not the same as I don't know.
- Total IDK points for the whole exam would not exceed 10.
- Give complete solutions, not examples. Declare all your variables. If you don't know the answer admit it and use IDK. Write short concise answers.
- **Style counts.** Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper.
- ***Good luck!***

1 (20 PTS.) For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth two points; each incorrect answer is worth nothing – checking "I don't know" is worth ½ a point.

1.A. If $L_1, L_2, \ldots$ are all regular languages, then the language $\bigcap_i^{\infty} \overline{L_i}$ is context free. **False**: ☐ **True**: ☐ **IDK**: ☐

1.B. Consider the logical statement "If the earth is a disc lying on top a giant turtle, then pigs can fly."
This statement is: **False**: ☐ **True**: ☐ **IDK**: ☐

1.C. The strings 010 and 101 are distinguishable for the language

$$L = \left\{ x \in \Sigma^* \;\middle|\; |\#_0(x) - \#_1(x)| \le 1 \right\}.$$

False: ☐ **True**: ☐ **IDK**: ☐

1.D. For all languages L, if L is regular, then L has a finite fooling set. **False**: ☐ **True**: ☐ **IDK**: ☐

1.E. For all context-free languages L and L', the language $L \cap L'$ is also context-free. **False**: ☐ **True**: ☐ **IDK**: ☐

1.F. For all languages $L, L' \subset \Sigma^*$, if L and L' are recognized by DFAs M and M', respectively, then

$$L' \oplus L = (L' \setminus L) \cup (L \setminus L')$$

can be represented by a regular expression. **False**: ☐ **True**: ☐ **IDK**: ☐

1.G. Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states.
Then $\overline{L(M)} = L(M')$. **False**: ☐ **True**: ☐ **IDK**: ☐

1.H. $\left\{ 0^i 1^j 0^i 1^\ell \;\middle|\; j \le i \le 10 \text{ and } \ell \ge 0 \right\}$ is regular. **False**: ☐ **True**: ☐ **IDK**: ☐

1.I. Let L be a regular language over alphabet Σ, and consider the language

$$L' = \{ x\alpha y \mid x, y \in \Sigma^*, \alpha \in \Sigma, \text{ and } xy \in L \}.$$

The language L' is regular. **False**: ☐ **True**: ☐ **IDK**: ☐

1.J. If a language $L \subseteq \{0\}^*$, that is not regular, contains a string of length two, then the language L^* is regular. **False**: ☐ **True**: ☐ **IDK**: ☐

---

2 (20 PTS.) For each of the following languages, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.*** Here, $\#_a(x)$ denotes the number of occurrences of the symbol a in the string x.

2.A. (10 PTS.) $L = \left\{ x \in \{0,1\}^* \;\middle|\; \min\bigl\{\#_0(x), \#_1(x)\bigr\} \geq 4 \right\}.$

2.B. (10 PTS.) $L = \left\{ x \in \{0,1\}^* \;\middle|\; \min\{\#_0(x), \#_1(x)\} \text{ is divisible by } 5 \right\}.$

3 In the following, provide short justifications of your answer (no need for a proof).

3.A. (8 PTS.) For $\Sigma = \{0, 1\}$, and any string $w \in \Sigma^*$, let $\#_0(w)$ and $\#_1(w)$ be the number of 0s and 1s in w, respectively. Provide a DFA for the following language L. (You might find it easier to describe the DFA than to draw it.)

$$L = \left\{ w \in \Sigma^* \;\middle|\; \big(\#_0(w) \cdot \#_1(w)\big) = 1 \mod 3 \right\}.$$

3.B. (4 PTS.) Provide a DFA for the following language: The set of all strings in $\{0, 1, 2\}^*$ that do not contain the substring 012.

3.C. (8 PTS.) Provide a regular expression for the following language: The set of all strings in $\{0, 1, 2\}^*$ that do not contain at least one of the symbols in the alphabet. For example, 00110, 2112, 0022, and 00 are in the language whereas 0121 is not.

4 (20 PTS.) For any string $w \in \Sigma^*$, define $skip(w)$ as a subsequence of w containing only the odd symbols of w. For example, skip($CS374$) = $C34$, skip($UIUC$) = UU, skip($(01)^5$) = 0^5, skip(ϵ) = ϵ, and skip($MIDTERM$) = $MDEM$.

For any language L, let $L' = \{skip(w) \mid w \in L\}$.

Prove that for any regular language L, the language L' is also regular.

5 (20 PTS.) CONTEXT FREE LANGUAGES.

In the following, provide a short explanations for your answers (proof is not required).

5.A. (8 PTS.) Describe a context-free grammar (CFG) for the following language:

$$L_1 = \left\{ xy \;\middle|\; |x| = |y|,\ x \in \{0\}^*,\ \text{ and } y \in \{0,1\}^* \right\}.$$

(In other words, L_1 consists of all even-length strings whose first half consists of only 0's.)

5.B. (8 PTS.) Describe a CFG for the following language:

$$L_2 = \left\{ xy \;\middle|\; |x| = |y|, x \in \{0,1\}^* \text{ has an odd number of 0's, and } y \in \{0,1\}^* \right\}$$

(In other words, L_2 consists of all even-length strings whose first half has an odd number of 0's and any number of 1's.)

5.C. (4 PTS.) (Harder.) We now generalize the previous two parts: for a language L over the alphabet $\Sigma = \{0,1\}$, define a new language

$$\text{first-half-in}(L) \;=\; \{xy \mid |x| = |y|,\ x \in L,\ \text{and } y \in \{0,1\}^*\}.$$

Given a DFA $M = (Q, \Sigma, \delta, s, A)$ that accepts L, describe formally how to construct a CFG $G = (V, T, P, S)$ that generates first-half-in(L). (As a consequence, this would show that if L is regular, then first-half-in(L) is context-free.) You do ***not*** need to give a formal proof of correctness.

(scratch paper)

(scratch paper)