Pre-lecture brain teaser

Show that NP is closed under the kleene-star operation.

CS/ECE-374: Lecture 28 - Final Exam review

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Pre-lecture brain teaser

Show that NP is closed under the kleene-star operation.

Final Topics

Topics for the final exam include:

- Regular expressions
- DFAs, NFAs,
- Fooling Sets and Closure properties
- Turing Machines and Decidability
- Recursion and Dynamic Programming
- DFS/BFS
- Djikstra, Bellman-Ford (Path finding)
- Reductions/ NP-Completeness

Final Topics

In today's lecture let's focus on a few that you guys had trouble on in the midterms (and the most recent stuff whih you'll be tested on).

- Regular expressions
- DFAs, NFAs,
- **Fooling Sets and Closure properties**
- **Turing Machines and Decidability**
- Recursion and Dynamic Programming
- DFS/BFS
- Djikstra, Bellman-Ford (Path finding)
- **Reductions/ NP-Completeness**

Practice: Asymtotic bounds

Given an asymptotically tight bound for:

$$\sum_{i=1}^{n} i^3 \tag{1}$$

Practice: Regular expressions

Find the regular expression for the language:

$$\{w \in \{0,1\}^* | w \text{does not contain 00 as a substring}\} \qquad (2)$$

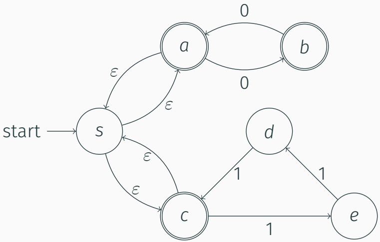
Practice: Fooling Sets

Is the following language regular?

$$L = \{w | w \text{ has an equal number of } 0\text{'s and } 1\text{'s }\}$$

Practice: NFAs and DFAs

Let M be the following NFA:



Which of the following statements about M are true?

Practice: NFAs and DFAs

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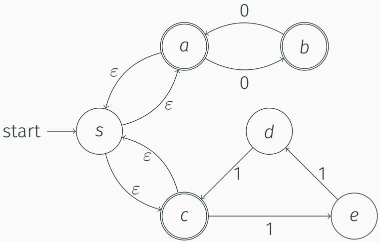

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1. M accepts the empty string ε -

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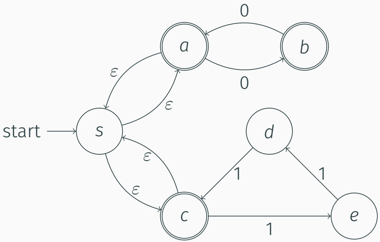

Which of the following statements about M are true?

1. M accepts the empty string ε -
2. $\delta(s, 010) = \{s, a, c\}$ -

Practice: NFAs and DFAs

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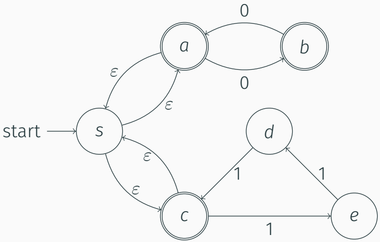

1. M accepts the empty string ε -
2. $\delta(s, 010) = \{s, a, c\}$ -
3. $\varepsilon - \mathrm{reach}(a) = \{s, a, c\}$ -

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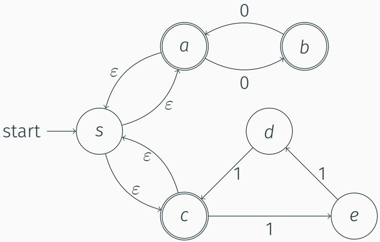

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4. M rejects the string 11100111000 -

Practice: NFAs and DFAs

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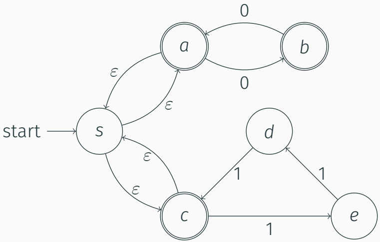

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3. $\varepsilon - \text{reach}(a) = \{s, a, c\}$ -
4. M rejects the string 11100111000 -
5. $L(M) = (00)^* + (111)^*$ -

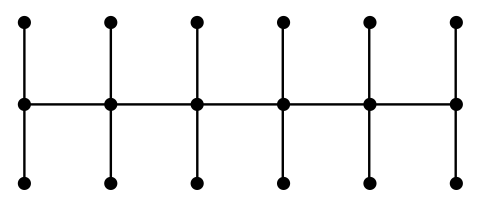
Practice: Closure

Which of the following is true for **every** language $L \subseteq \{0,1\}^*$

1. L^* is non-empty -
2. L^* is regular -
3. If L is NP-Hard, then L is not regular -
4. If L is not regular, then L is undecidable -

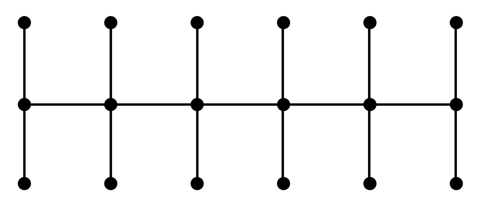
Practice: NP-Complete Reduction

A *centipede* is an undirected graph formed by a path of length k with two edges (legs) attached to each node on the path as shown in the below figure. Hence, the centipede graph has $3k$ vertices. The **CENTIPEDE** problem is the following: given an undirected graph $G = (V, E)$ and an integer k, does G contain a *centipede* of $3k$ vertices as a subgraph? Prove that **CENTIPEDE** is **NP-Complete**.



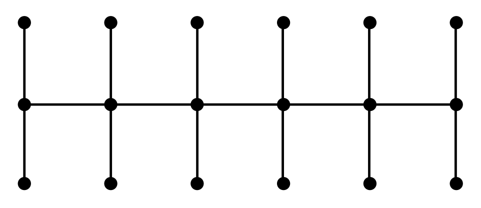
Practice: NP-Complete Reduction

What do we need to do to prove Centipede is NP-Complete?



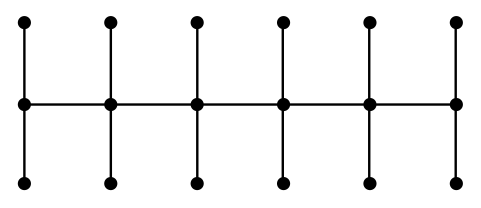
Practice: NP-Complete Reduction

Prove Centipede is in NP:



Practice: NP-Complete Reduction

Prove Centipede is in **NP-hard**:



Practice: Decidability

Prove (via reduction) that the following language is undecidable.

$$\text{AcceptOrBust} = \{\langle M\rangle | M \text{does not reject any input}\}$$

Your reduction must involve the **SelfHalts** problem whihc is known to be undecidable:

$$\text{SelfHalts} = \{\langle M\rangle | M \text{ halts on input } \langle M\rangle\}$$

Practice: Decidability

$$\text{AcceptOrBust} = \{\langle M\rangle | M\text{does not reject any input}\}$$

$$\text{SelfHalts} = \{\langle M\rangle | M \text{ halts on input } \langle M\rangle\}$$

Reduction: 3SAT to Clique

Consider the two problems:

Problem: 3SAT

Instance: Given a CNF formula φ with n variables, and k clauses
Question: Is there a truth assignment to the variables such that φ evaluates to true

Problem: Clique

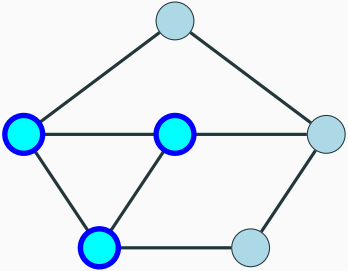
Instance: A graph G and an integer k.
Question: Does G has a clique of size $\geq k$?

Reduce **3SAT** to **CLIQUE**

Reduction: 3SAT to Clique

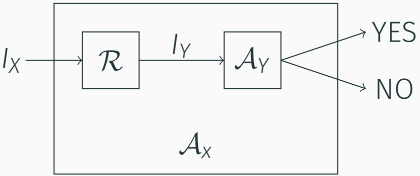
Given a graph G, a set of vertices V' is:

clique: *every* pair of vertices in V' is connected by an edge of G.



Reduction: 3SAT to Clique

Bust out the reduction diagram:



Reduction: 3SAT to Clique

Some thoughts:

- Clique is a fully connected graph and very similar to the independent set problem
- We want to have a clique with all the satisfying literals
  - Can't have literal and its negation in same clique
  - Only need one satisfying literal per clique

Reduction: 3SAT to Clique

Hence the reduction creates a undirected graph G:

- Nodes in G are organized in k groups of nodes. Each triple corresponds to one clause.
- The edges of G connect all but:
    - nodes in the same triple
    - nodes with contradictory labels (x_1 and $\overline{x_1}$)

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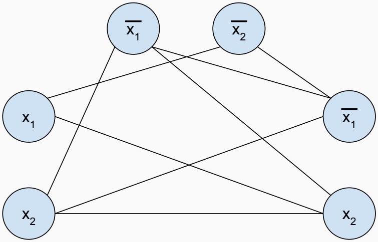
$\varphi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$



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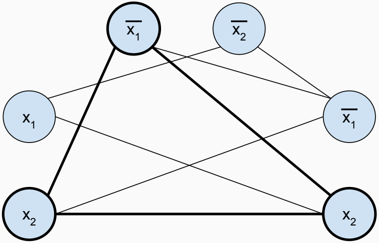

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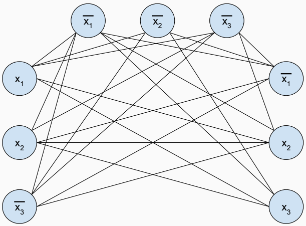

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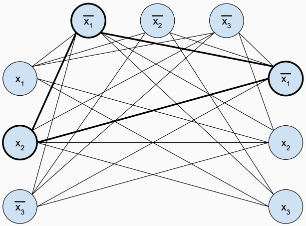

3SAT to Independent Set Reduction

Very similar to 3SAT to independent set reduction:

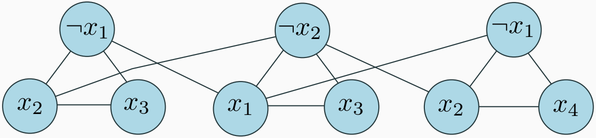

Figure 1: Graph for $\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$