Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states $Q$, the start state $s$, the accept states $A$, and the transition function $\delta$ are all clear. Try to keep the number of states small.

1. All strings in which the number of $0$s is even and the number of $1$s is not divisible by 3.

2. All strings in which the number of $0$s is even or the number of $1$s is not divisible by 3.

3. Given DFAs $M_1$ and $M_2$, all strings in $L(M_1) \oplus L(M_2)$.
   
   Recall that for two sets $A$ and $B$, their symmetric distance $A \oplus B$ is the set of elements in either $A$ or $B$, but not both.

Work on these later:

4. All strings that are both the binary representation of an integer divisible by 3 and the ternary (base-3) representation of an integer divisible by 4.

   For example, the string $1100$ is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

5. All strings in which the subsequence $0101$ appears an even number of times.

6. All strings $w$ such that $\left( \left\lfloor \frac{|w|}{2} \right\rfloor \right) \mod 6 = 4$. [Hint: Maintain both $\left( \left\lfloor \frac{|w|}{2} \right\rfloor \right) \mod 6$ and $|w| \mod 6$.]

7. All strings $w$ such that $F_{\#(10,w)} \mod 10 = 4$, where $\#(10,w)$ denotes the number of times $10$ appears as a substring of $w$, and $F_n$ is the $n$th Fibonacci number:

   $$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$