CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2022

**Midterm 1** **Monday, September 26, 2022: 7-9pm**

**Location: Foellinger Auditorium**

| Name | | ⇐ Please PRINT |
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- Please print your name and NetID. Anonymous submissions would not be graded.
- There are five questions – you should answer all of them.
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original, by yourself) $8\frac{1}{2}$" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. We will not return or scan the cheat sheets, so photocopy them before the exam if you want a copy.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- Please read all the questions before starting to answer them. Please ask for clarification if any question is unclear.
- **This exam lasts 120 minutes.** The clock started when you got the questions.
- If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
- Give complete solutions, not examples. Declare all your variables. If you don't know the answer admit it and move to the next question. We will happily give 0 points for nonsense answers.
- **Style counts.** Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper.
- ***Good luck!***

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**1** (20 PTS.) For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth two points; each incorrect answer is worth nothing.

**1.A.** If $L_1 \subseteq L_2 \subseteq L_3 \subseteq \cdots$ are all finite languages, then the language $\bigcup_i^\infty L_i$ is context-free. **False:** ☐ **True:** ☐

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**1.B.** Fooling sets for regular languages are always finite. **False:** ☐ **True:** ☐

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**1.C.** Let $\Sigma = \{0, 1\}$. For a string $w$ and a character $c$, let $\#_c(w)$ be the number of appearances of $c$ in $w$. The strings 0101 and 1010 are distinguishable for the language **False:** ☐ **True:** ☐

$$L = \left\{ x \in \Sigma^* \;\middle|\; \forall p \text{ prefix of } x \text{ we have } |\#_0(p) - \#_1(p)| \le 1 \right\}.$$

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**1.D.** For all languages $L$, if $L$ is context-free, then the complement language $\overline{L}$ is also context-free. **False:** ☐ **True:** ☐

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**1.E.** For all context-free languages $L$ and $L'$, the language $L \setminus L'$ is also context-free. **False:** ☐ **True:** ☐

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**1.F.** For all languages $L_1, L_2, L_3 \subseteq \Sigma^*$, if $L_1, L_2$ and $L_3$ are recognized by DFAs $M_1$, $M_2$, and $M_3$, respectively, then the language **False:** ☐ **True:** ☐

$$L' = \{w \in \Sigma^* \mid w \text{ is in exactly one of the languages } L_1, L_2 \text{ or } L_3\}$$

can be represented by a regular expression.

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**1.G.** Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then it must be that $L(M) \neq L(M')$. **False:** ☐ **True:** ☐

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**1.H.** The language $\left\{0^i 1^j 0^k 1^\ell \;\middle|\; (i + j) \bmod 7 = (k + \ell) \bmod 7\right\}$ is regular. **False:** ☐ **True:** ☐

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**1.I.** Let $L$ be a regular language over alphabet $\Sigma$, and consider the language **False:** ☐ **True:** ☐

$$L' = \{xy \mid x, y \in \Sigma^*, \alpha \in \Sigma, \text{ and } \alpha x \alpha y \alpha \in L\}.$$

The language $L'$ is regular.

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**1.J.** If a language $L \subseteq \{0\}^*$ contains at least four distinct strings, then the language $L^*$ is regular. **False:** ☐ **True:** ☐

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**2** (20 PTS.) For each of the following languages, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.*** Here, $\#_a(x)$ denotes the number of occurrences of the symbol $a$ in the string $x$.

**2.A.** (10 PTS.) $L = \left\{ x \in \{0,1\}^* \;\middle|\; \text{the value in base 2 of } x \text{ is divisible by } 5 \right\}$.
Thus $L = \{0, 101, 1010, 1111, \cdots\}$ (since, for example, $1010_2$ is ten, which is divisible by 5).

**2.B.** (10 PTS.) $L = \left\{ 0^x 1 0^y \;\middle|\; x, y \geq 0, \text{ and } x \text{ divides } y \right\}$

**3** (20 PTS.) In the following, provide short justifications of your answer (no need for a proof).

**3.A.** (10 PTS.) For $\Sigma = \{0, 1\}$, and any string $w \in \Sigma^*$, let $\#_0(w)$ and $\#_1(w)$ be the number of 0s and 1s in $w$, respectively. Provide a DFA for the following language $L$. (A drawing of the DFA in this case is not useful.)

$$L = \left\{ w \in \Sigma^* \;\middle|\; (\#_0(w) \bmod 3) = (\#_1(w) \bmod 374) \right\}$$

**3.B.** (10 PTS.) Provide a regular expression for the following language: The set of all strings $w$ in $\{0, 1, 2\}^*$, such that $w$ contains all the symbols in the alphabet $\Sigma = \{0, 1, 2\}$. For example, 02110, 2102, 0122, and 012 are in the language whereas 0101 is not.

## 4 (20 PTS.) CONTEXT FREE LANGUAGES.

In the following, provide a short explanations for your answers (proof is not required).

**4.A.** (10 PTS.) Describe a context-free grammar (CFG) for the following language:

$$L_1 = \left\{ xyz \;\middle|\; |x| = |y| = |z|,\ x \in \{0\}^*,\ \text{and } y, z \in \{0,1\}^* \right\}.$$

**4.B.** (10 PTS.) Describe a CFG for the following language:

$$L_2 = \left\{ xy \;\middle|\; |x| = |y|, x, y \in \{0,1\}^*,\ \text{and } \big(\#_0(x) \bmod 3\big) = 0 \right\}$$

(In other words, $L_2$ consists of all even-length strings whose first half has a number of 0 divisible by three.)

**5** (20 PTS.) Let $\Sigma = \{0, 1\}$. Let $\overline{0} = 1$ and $\overline{1} = 0$. For any string $w = w_0 w_1 \dots w_n \in \Sigma^*$, define

$$\text{flip}_k(w) = \overline{w_0} w_1 w_2 \cdots w_{k-1} \overline{w_k} w_{k+1} \dots w_{2k-1} \overline{w_{2k}} w_{2k+1} \dots$$

be the string resulting from flipping the $k$th bits of $w$. For example, $\text{flip}_3(0001111) = 1000110$.

For any language $L \subseteq \Sigma^*$, let $L' = \{\text{flip}_3(w) \mid w \in L\}$.

**Prove** that for any regular language $L$, the language $L'$ is also regular.

(scratch paper)

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