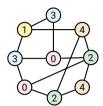
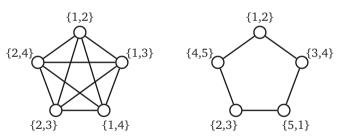
1. Recall that a 5-coloring of a graph *G* is a function that assigns each vertex of *G* a "color" from the set {0, 1, 2, 3, 4}, such that for any edge *uv*, vertices *u* and *v* are assigned different "colors". A 5-coloring is *careful* if the colors assigned to adjacent vertices are not only distinct, but differ by more than 1 (mod 5). Prove that deciding whether a given graph has a careful 5-coloring is NP-hard. [Hint: Reduce from the standard 5COLOR problem.]



A careful 5-coloring.

- 2. Prove that the following problem is NP-hard: Given an undirected graph G, find any integer k > 374 such that G has a proper coloring with k colors but G does not have a proper coloring with k 374 colors.
- 3. A *bicoloring* of an undirected graph assigns each vertex a set of *two* colors. There are two types of bicoloring: In a *weak* bicoloring, the endpoints of each edge must use *different* sets of colors; however, these two sets may share one color. In a *strong* bicoloring, the endpoints of each edge must use *distinct* sets of colors; that is, they must use four colors altogether. Every strong bicoloring is also a weak bicoloring.
 - (a) Prove that finding the minimum number of colors in a weak bicoloring of a given graph is NP-hard.
 - (b) Prove that finding the minimum number of colors in a strong bicoloring of a given graph is NP-hard.



Left: A weak bicoloring of a 5-clique with four colors. Right A strong bicoloring of a 5-cycle with five colors.