

1. Consider the following  $k$ COLOR problem: Given an undirected graph  $G$ , can its vertices be colored with  $k$  colors, so that every edge touches vertices with two different colors?
  - (a) Describe a direct polynomial-time reduction from 3COLOR to 4COLOR.
  - (b) Prove that  $k$ COLOR problem is NP-hard for any  $k \geq 3$ .
2. Prove that each of the following problems is NP-hard.
  - (a) Given an undirected graph  $G$ , does  $G$  contain a simple path that visits all but 374 vertices?
  - (b) Given an undirected graph  $G$ , does  $G$  have a spanning tree in which every node has degree at most 374?
  - (c) Given an undirected graph  $G$ , does  $G$  have a spanning tree with at most 374 leaves?