11.4.6

Epilogue: On selection in linear time

## Summary: Selection in linear time

## Theorem

The algorithm select $(\boldsymbol{A}[\mathbf{1} \ldots \boldsymbol{n}], \boldsymbol{k})$ computes in $\boldsymbol{O}(\boldsymbol{n})$ deterministic time the $\boldsymbol{k}$ th smallest element in $\mathbf{A}$.

On the other hand, we have:

## Lemma

The algorithm QuickSelect( $\boldsymbol{A}[\mathbf{1} \ldots \boldsymbol{n}], \boldsymbol{k})$ computes the $\boldsymbol{k}$ th smallest element in $\boldsymbol{A}$. The running time of QuickSelect is $\Theta\left(\boldsymbol{n}^{2}\right)$ in the worst case.

## Questions to ponder

(1) Why did we choose lists of size 5 ? Will lists of size 3 work?
(2) Write a recurrence to analyze the algorithm's running time if we choose a list of size $\boldsymbol{k}$.

## Median of Medians Algorithm

Due to:<br>M. Blum, R. Floyd, D. Knuth, V. Pratt, R. Rivest, and R. Tarjan. "Time bounds for selection". Journal of Computer System Sciences (JCSS), 1973.

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```


## Takeaway Points

(1) Recursion tree method and guess and verify are the most reliable methods to analyze recursions in algorithms.
(2) Recursive algorithms naturally lead to recurrences.

- Some times one can look for certain type of recursive algorithms (reverse engineering) by understanding recurrences and their behavior.

