Big-O Tutorial Questions

Exponential vs Factorial

a) Prove that 2^n is $O(n!)$. *(Apply the definition of big-O; do not just appeal to our $\ll$ hierarchy, though you may adapt the related proof from the textbook if you need help.)*

b) Prove or disprove the following: If $f(n)$ is $O(2^n)$ and $g(n)$ is $O(n!)$ then $f(n)$ is $O(g(n))$

Transitivity of big-O

Prove that if $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n)$ is $O(h(n))$. *(Apply the definition of big-O; do not appeal to general arguments about which functions must grow faster than which others.)*