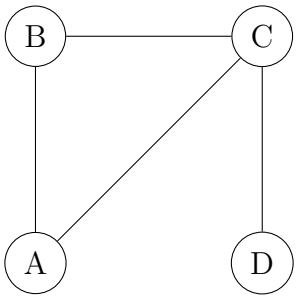
Collections of Sets - Additional Tutorial Problems

Partitions

a) Let $A = \{2, 5, 7, 8, 13, 21\}$, and define $p : A \to \mathbb{P}(A)$ by $p(n) = \{s \in A \mid \gcd(s, n) \neq 1\}$. Let $M = \{p(s) \mid s \in A\}$.

- What are the elements of M?
- Is M a partition of A? Explain why or why not.

b) Consider this graph on vertex set $X = \{A, B, C, D\}$:



Define $D : N \to \mathbb{P}(X)$ by $D(n) = \{v \in X | \text{degree of node } v \text{ is } n\}$. Let $S = \{D(n) \mid n \in \mathbb{N}\} = \{D(0), D(1), D(2), ...\}$.

- What are the elements of S?
- Is S a partition of X? Explain why or why not.

Set-valued functions

Define $f : \mathbb{P}(\mathbb{Z}) \to \mathbb{P}(\mathbb{Z})$ by $f(S) = \{n/2 \mid n \in S \text{ and } n \text{ is even}\}$.

a) Is f one-to-one?

b) Is f onto?

Counting

Let n, k be integers with $n \leq k$. Compute the number of positive integer solutions to the equation $\sum_{i=1}^{n} x_i = k$. *Hint: this is similar to problem 17.5a.*