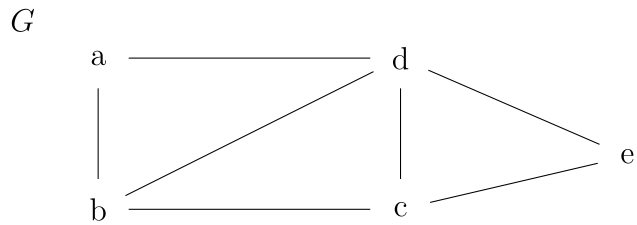
Paths

List all the paths from b to e in graph G below.



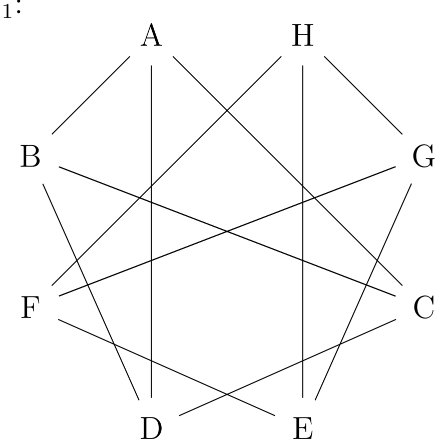
Solution:

Using the text definition of path we get the following paths from b to e.

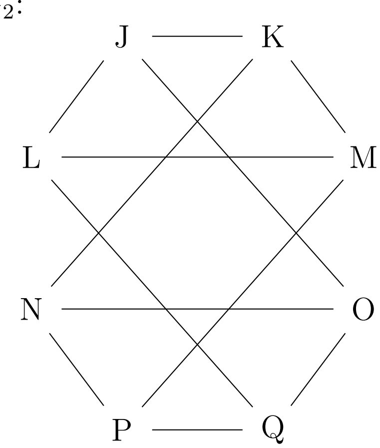
(b, c, e)
(b, d, e)
(b, a, d, e)
(b, d, c, e)
(b, c, d, e)
(b, a, d, c, e)

Component Is each of these graphs connected? If not, list the nodes in each connected component.

G_1:



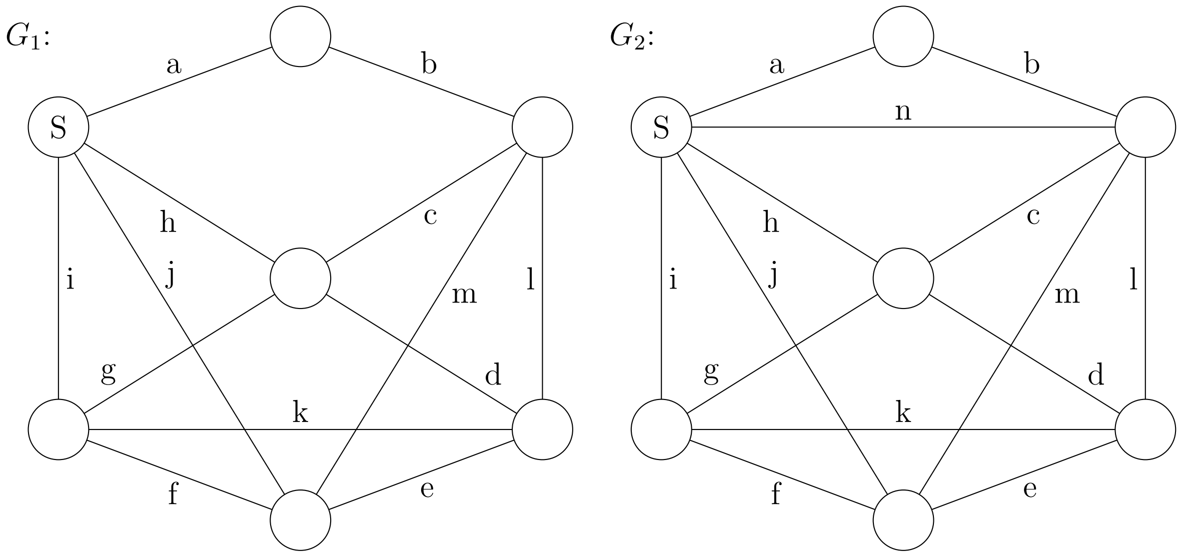
G_2:



Solution:

In graph G_1 there are two connected components $\{A, B, C, D\}$ and $\{E, F, G, H\}$.
In graph G_2 there is one connected component.

Euler Circuits Find an Euler circuit in each graph beginning at S, or explain why this isn't possible.



Solution:

There is a Euler Circuit in G_1 using the following $a, b, c, d, e, f, g, h, i, k, l, m, j$.
There is no Euler Circuit in G_2 since there is a vertex with odd degree.

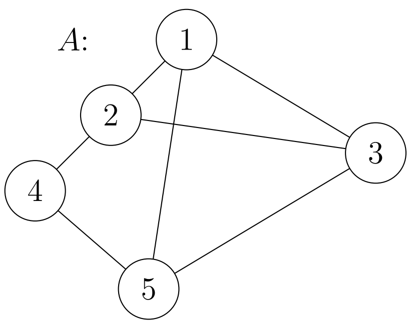
Chromatic Number

Recall that the justification that a particular chromatic number is valid requires bounding the number from above *and* below. Therefore you must give an *explicit* coloring to produce an upper bound *and* produce a valid argument that no smaller number of colors will work to produce a lower bound.

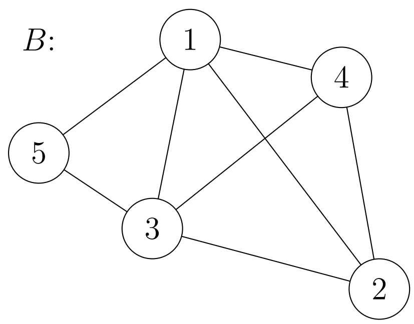
The argument justifying the lower bound often involves finding a copy of K_n (where n is the chromatic number you are attempting to validate) as a subgraph. Sometimes, however, you have to work through the space of possible $n-1$ colorings by hand and show that none of them work.

Find and justify the chromatic numbers for each of the following graphs.

A:



B:



Solution:

In Graph A You can tell that the 3 chromatic number is 3. Since you get an upper bound of 3 by coloring vertices as follows Red for 1 and 4, Blue for 2 and 5, Green for 3. You get a lower bound of 3 since vertices 1,2,3 make K_3 which takes a minimum of 3 colors.

The Graph B has a chromatic number of 4. You can find an upper bound of 4 by coloring the graph as follows Red for 1, Blue for 2 and 5, Green for 3, Yellow for 5. You can find a lower bound of 4 since the vertices $1, 2, 3, 4$ is the subgraph that is K_4 which has a minimum of 4 colors.