Let
$$\lambda = np = E(x)$$
, so $p = \frac{\lambda}{n}$

$$P(X = x) = \binom{n}{x} p^{x} (1 - p)^{n - x}$$

$$= \frac{n(n - 1) \dots (n - x + 1)}{x!} \left(\frac{\lambda}{n}\right)^{x} \left(1 - \frac{\lambda}{n}\right)^{n - x} \sim \frac{n^{x}}{x!} \left(\frac{\lambda}{n}\right)^{x} = \frac{\lambda^{x}}{x!};$$

$$\sum_{x} \frac{\lambda^{x}}{x!} = e^{\lambda}.$$

Normalization requires $\sum_{x} P(X = x) = 1$.

Thus
$$P(X = x) = \frac{\lambda^x}{x!}e^{-\lambda}$$

Poisson Mean & Variance

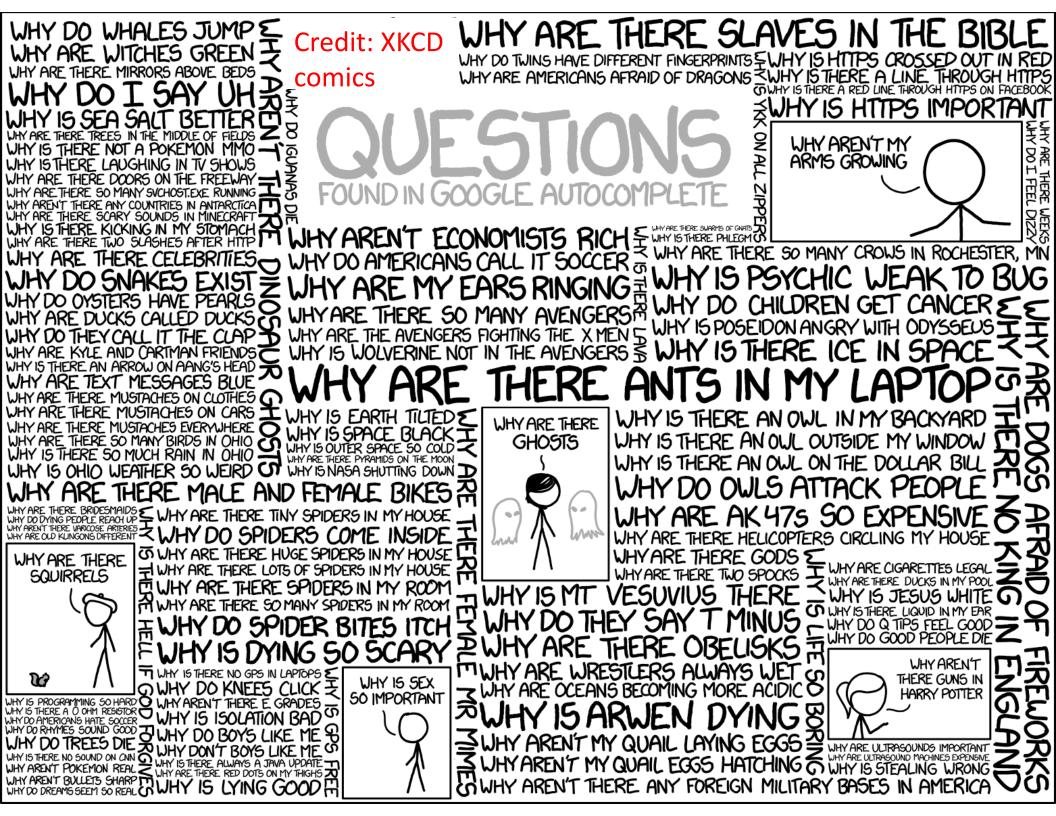
If X is a Poisson random variable, then:

- Mean: $\mu = E(X) = \lambda = \emptyset$
- Variance: $\sigma^2 = V(X) = \lambda n \cdot \rho$ (if was Standard deviation: $\sigma = \lambda^{1/2}$

Note: Variance = Mean

Note: Standard deviation/Mean = $\lambda^{-1/2}$ decreases with λ

Sec 2-



Poisson Distribution in Genome Assembly

Poisson Example: Genome Assembly

- Goal: DNA sequence of the entire genome of an organism
- Problem: Sequencers generate short reads of random portions of a genome
- Solution: assemble genome from short reads using computers
- Whole Genome Shogun Assembly pioneered by Craig Venter in 1990s
- The human genome was jointly announced in 2001 by the Human Genome Project (public) and Celera Genomics (Craig Venter's company)

Short Reads assemble into Contigs

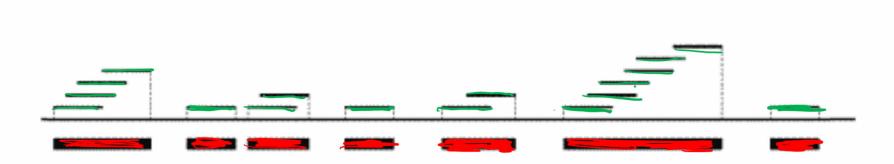


Figure 5.1.



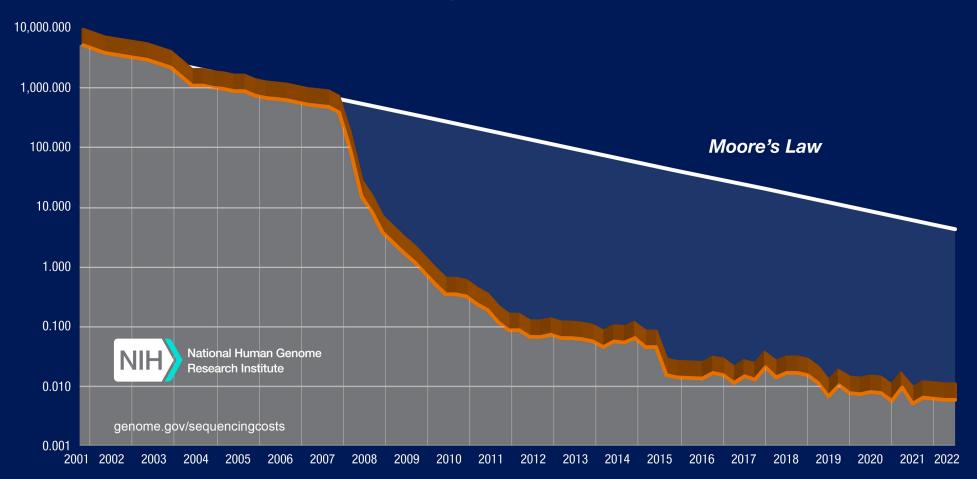
Promise of Genomics



Drew Sheneman, New Jersey -- The Newark Star Ledger, E-mail Drew.

I think I found the corner piece!

Cost per Raw Megabase of DNA Sequence



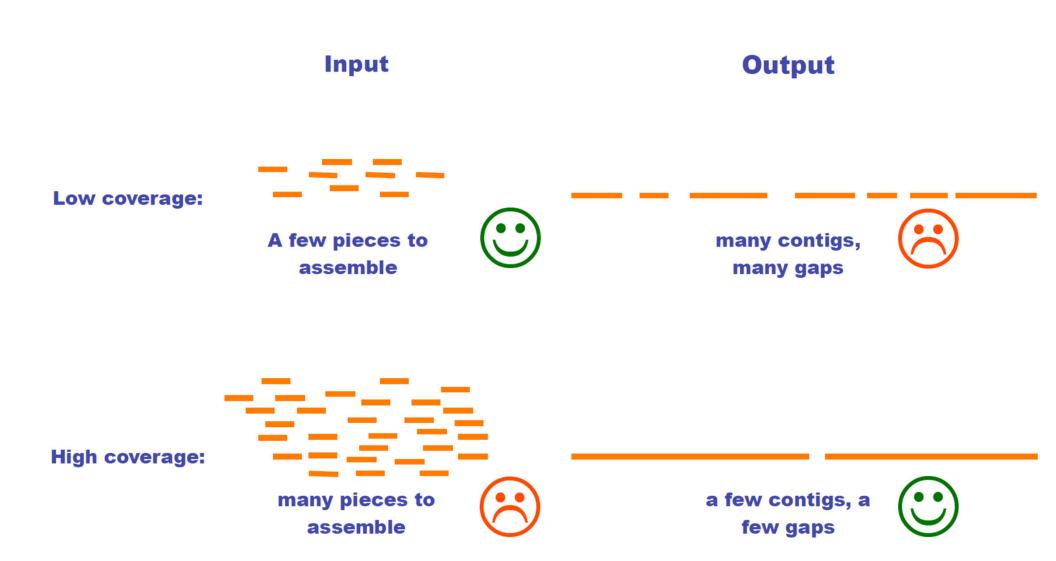
Current sequencing technologies

Technology	Read Length	Error Rate	Cost per Gbase		
Illumina NovaSeq	75-500 bp	~0.1%	\$5-\$150		
BGI DNBSEQ	•		\$5-\$120		
Ion Torrent			\$70-\$1000		
PacBio PacBio	10,000-25,000 bp	<mark>13%</mark>	<mark>\$7-\$40</mark>		
Oxford Nanopore	10.000-100.000+ bp		<mark>\$30-\$60</mark>		



MinION, a palm-sized gene sequencer made by UK-based Oxford Nanopore Technologies

How many short reads do we need?



Genome Assembly

Whole-genome "shotgun" sequencing starts by copying and fragmenting the DNA

("Shotgun" refers to the random fragmentation of the whole genome; like it was fired from a shotgun)

Input: GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

35bp

Copy GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

by GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

PCR: GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

Fragment: GGCGTCTA TATCTCGG CTCTAGGCCCTC ATTTTTT
GGC GTCTATAT CTCGGCTCTAGGCCCTCA TTTTTT

GGCGTC TATATCT CGGCTCTAGGCCCT CATTTTTT

GGCGTCTAT ATCTCGGCTCTAG GCCCTCA TTTTTT

Courtesy of **Ben Langmead**. Used with permission.

Assembly

Assume sequencing produces such a large # fragments that almost all genome positions are *covered* by manyfragments...

...but we don't know what came from where

CTAGGCCCTCAATTTTT

GGCGTCTATATCT

CTCTAGGCCCTCAATTTTT

CTCTAGGCCCTCAATTTTT

TCTATATCTCGGCTCTAGG

this

GGCTCTAGGCCCTCATTTTT

TATCTCGACTCTAGGCCCTCA

GGCGTCGATATCT

TATCTCGACTCTAGGCC

GGCGTCTATATCTCG

GGCGTCTATATCTCG

Courtesy of **Ben Langmead**. Used with permission.

Assembly

Overlaps between short reads help to put them together

CTAGGCCCTCAATTTTT
CTCTAGGCCCTCAATTTTT

GGCTCTAGGCCCTCATTTTTT

CTCGGCTCTAGCCCCTCATTTT

TATCTCGACTCTAGGCCCTCA

TATCTCGACTCTAGGCC

TCTATATCTCGGCTCTAGG

GGCGTCTATATCTCG

GGCGTCGATATCT

GGCGTCTATATCT

GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT

177 nucleotides

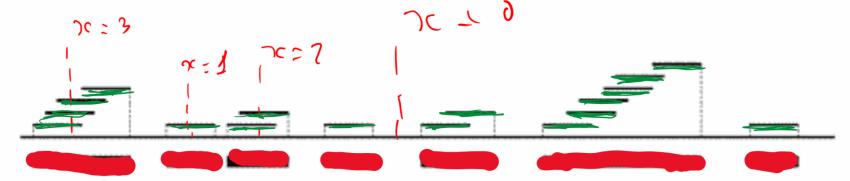
35 nucleotides

Where is the Poisson?

- G genome length (in bp)
- L short read average length
- N number of short read sequenced
- λ sequencing coverage redundancy = LN/G
- x- number of short reads covering a given site on the genome

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Poisson as a limit of Binomial: For a given site on the genome for each short read Prob(site covered): p=L/G is very small. Number of attempts (short reads): N is very large. Their product (sequencing redundancy): $\lambda = NL/G$ is O(1).



What fraction of the genome is missing?

What fraction of genome is covered?

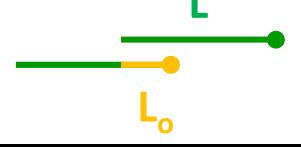
Coverage: λ=NL/G,
 X – random variable equal to the number of times a given site is covered by short reads.
 Poisson: P(X=x)= λ*exp(- λ)/x!
 P(X=0)=exp(- λ), P(X>0)=1- exp(- λ)

• Total length covered: $G^*[1-exp(-\lambda)]$

λ	2	4	6	6 8 10		12	
Mean proportion of genome covered	.864665	.981684	.997521	.999665	.999955	.999994	

Table 5.1. The mean proportion of the genome covered for different values of λ

How long should be the length L_{ov} of the overlap to connect two short reads into a contig?



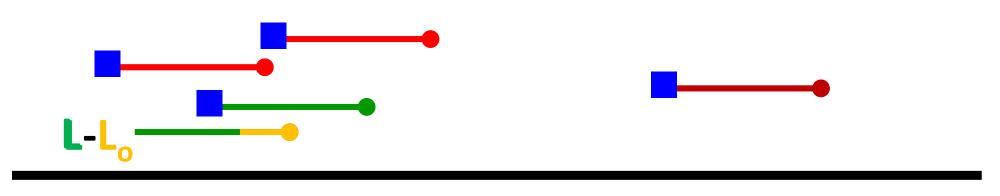
G

If DNA was a random chain with $p_A = p_C = p_G = p_T = 1/4$ L_{ov} ~16-20 would be enough

$$2 \cdot G \cdot 4^{-Lov} = 2 \cdot 3x10^{9} \cdot 4^{-16} = 1.4$$

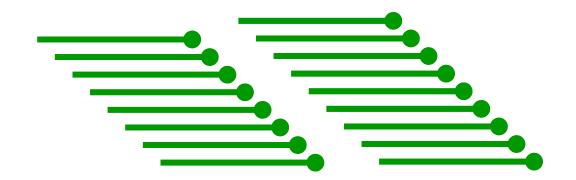
 $2 \cdot 3x10^{9} \cdot 4^{-20} = 0.0055 < < 1$

How many contigs?



G

P(short read can be extended by another short read)= $\frac{L-L_o}{G}$ =p P(short read cannot be extended by any short reads)= $e^{-pN}\approx Ne^{-\lambda}$ number of contigs= $Ne^{-pN}\approx Ne^{-\lambda}$



How many contigs?

- A given short read is the right end of a contig if and only if no left ends of other short reads fall within it.
- The left end of another short read has the probability p=(L-1)/G to fall within a given read. There are N-1 other reads. Hence the expected number of left ends inside a given shot read is $p \cdot (N-1) = (N-1) \cdot (L-1)/G \approx \lambda$
- If significant overlap required to merge two short reads is L_{ov} , modified λ is given by $(N-1) \cdot (L-L_{ov})/G$
- Probability that no left ends fall inside a short read is $exp(-\lambda)$. Thus the Number of contigs is $N_{contigs} = Ne^{-\lambda}$:

λ	0.5	0.75	1	1.5	2	3	4	5	6	7
Mean number of contigs	60.7	70.8	73.6	66.9	54.1	29.9	14.7	6.7	3.0	1.3

Table 5.2. The mean number of contigs for different levels of coverage, with $G=100{,}000$ and L=500.

Average length of a contig?

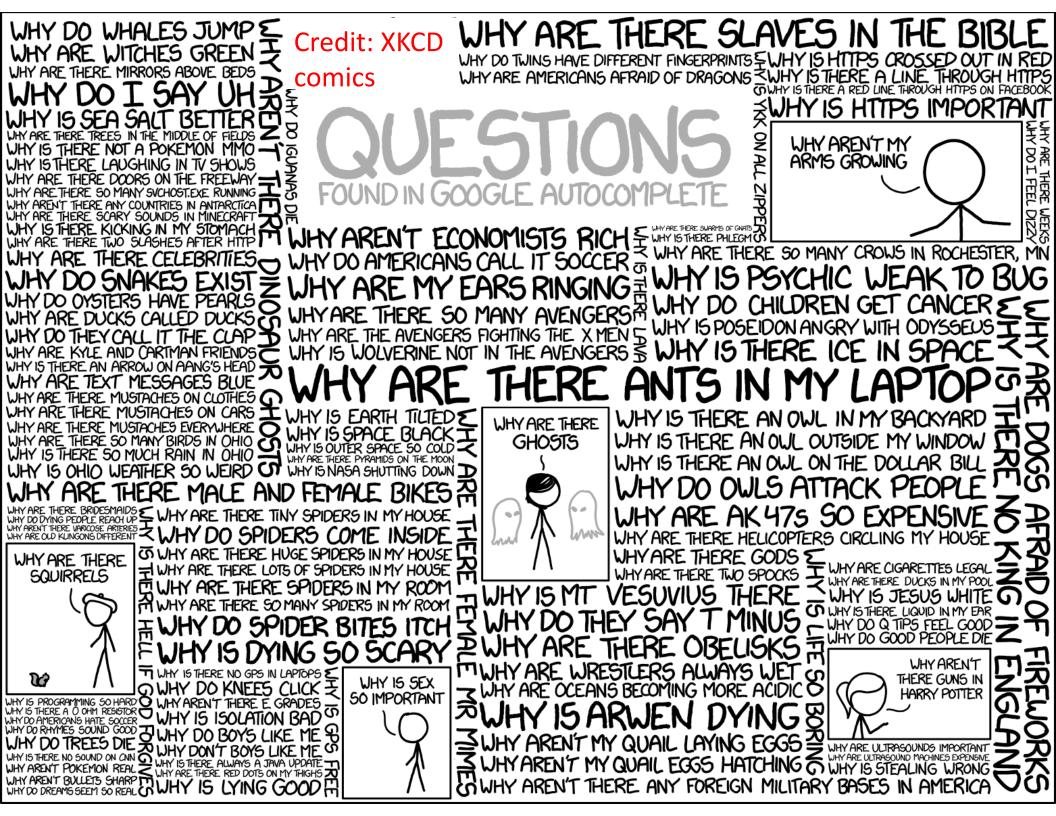
• Length of a genome covered: $G_{covered} = G \cdot P(X>0) = G \cdot (1 - exp(-\lambda))$

- Number of contigs $N_{contigs} = N \cdot e^{-\lambda}$
- Average length of a contig =

$$< L> = \sum_{i} L_{i}/N_{contigs} = G_{covered}/N_{contigs} =$$

$$G \cdot (1 - \exp(-\lambda))/N \cdot e^{-\lambda} = L \cdot (1 - \exp(-\lambda))/\lambda \cdot e^{-\lambda}$$

Table 5.3. The mean contig size for different values of a for the case L = 500.



Matlab exercise: Poisson distribution

- Generate a sample of size 100,000 for Poissondistributed random variable X with $\lambda = 2$, 6, 20
- Plot the <u>approximation</u> to the Probability Mass Functions based on <u>these samples</u>. Combine them in the same figure.
- Calculate the mean and variance of <u>this</u> <u>sample</u> and compare it to theoretical calculations:

$$E[X] = \lambda$$
 and $V[X] = \lambda$

Matlab exercise: Poisson distribution

```
    Stats=100000; lambda=2;

r2=random('Poisson',lambda,Stats,1);
mu p=sum(r2)./Stats;
disp(mu p);

    var_p=sum((r2-mu_p).^2)./Stats;

disp(var_p);
std_p=sqrt(var_p)
[a,b]=hist(r2, 0:max(r2));
p p=a./sum(a);
figure; stem(b,p_p);

    figure; semilogy(b,p p,'ko-');
```

Estimate

- Human genome is 3x10⁹ bp long
- Chromosome 1 is about G=0.25x10⁹ bp
- Illumina generates short reads L=100 bp long
- What number of reads N are needed to completely assemble the 1st chromosome?
- The formula to use is: $1=N_{contigs}=Ne^{-\lambda}=Ne^{-NL/G}$
- Answer: N=4.4x10⁷ short (100bp) reads
 Test: 4.4e7*exp(-4.4e7*100/0.25e9)=0.9997
- What coverage redundancy λ will it be? Answer: $\lambda = NL/G = 17.6$ coverage redundancy

How much would it cost to assemble human genome now?

- Human Genome Project: \$2.7 billion in 1991 dollars.
- Now a de novo full assembly of the whole human genome would now cost $3 \times 10^9 \times 17.6 / 10^9 \times 10^9 / GBase = 530
- 2nd genome (and after) would be even cheaper as we would already have a reference genome to which we can map short reads. (Puzzle: picture on the box)
- But this is a naïve estimate. In reality, there are complications. See the next slides:

What spoils these estimates?

>gi|224514922|ref|NT_024477.14| Homo sapiens chromosome 12 genomic contig, GRCh37.p13 Primary Assembly (displaying 3' end)
CGGGAAATCAAAAGCCCCTCTGAATCCTGCGCACCGAGATTCTCCCCAGCCAAGGTGAGGCGGCAGCAGT
GGGAGATCCACACCGTAGCATTGGAACACAAATGCAGCATTACAAATGCAGACATGACACCCGAAAATATA
ACACACCCCATTGCTCATGTAACAAGCACCTGTAATGCTAATGCACTGCCTCAAAACAAAATATTAATAT
AAGATCGGCAATCCGCACACTGCCGTGCAGTGCTAAGACAGCAATGAAAATAGTCAACATAATAACCCTA
ATAGTGTTAGGGTTAGGGTCAGGGTCCCGGTCCGGGTCCGGGTCCGGGTCCGGGTCAGGGTGA
GGGTTAGG

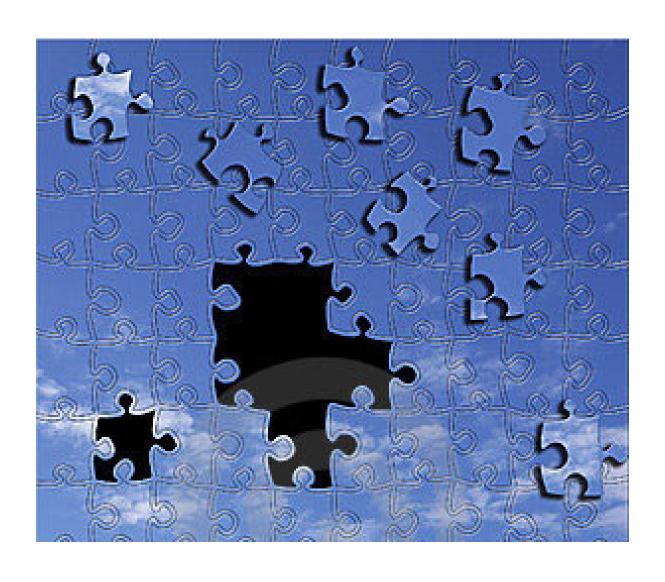
FIGURE 8.11 A BLASTN search of the human genome (all assemblies) database was performed at the NCBI website using TTAGGGTTAGGGTTAGGG as query (i.e., three TTAGGG repeats). There were matches to hundreds of genomic scaffolds. This figure shows an example (NT_024477.14) assigned to the telomere of chromosome 12q having many dozens of TTAGGG repeats. These occurred at the 3' end of the genomic contig sequence.

There were 100s of matches while one expects << 1 match:

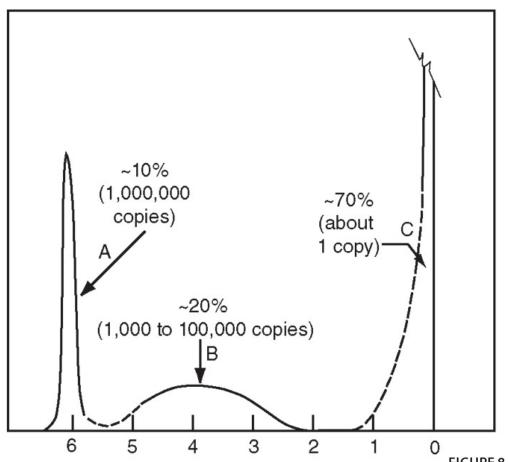
 $2 \cdot 3x10^9 \cdot 4^{-18} = 0.08 < < 1$

DNA repeats make assembly difficult

Repeats are like sky puzzle pieces



How many repeats are in eukaryotic genomes?

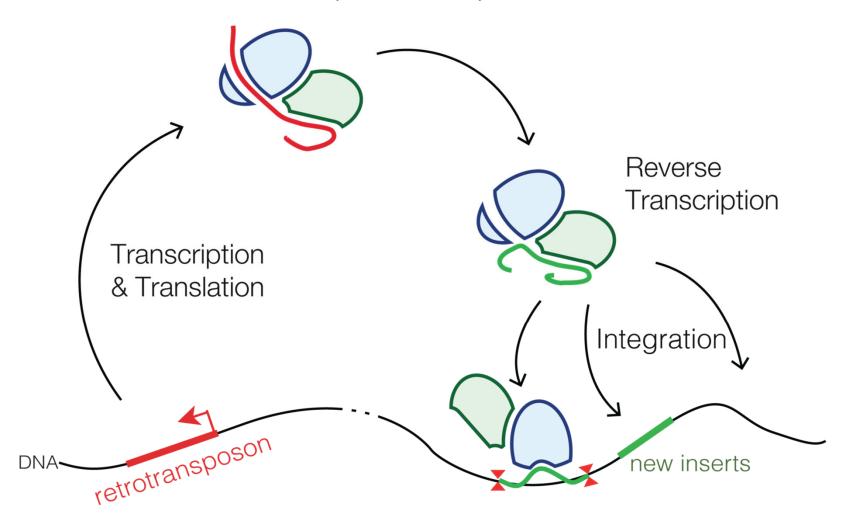


Log₁₀ repetition frequency

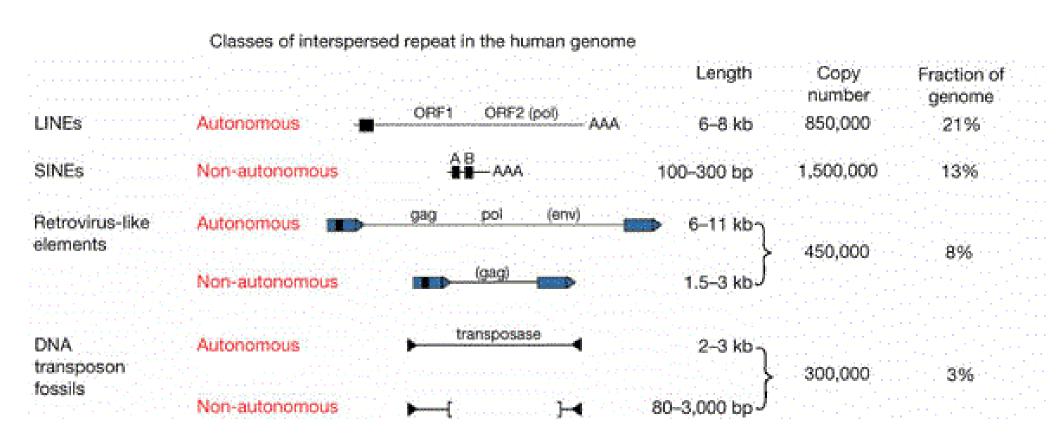
Data for mouse genome obtained in 1961 (sic!) using DNA denaturation and renaturation curves

FIGURE 8.6 The complexity of genomic DNA can be estimated by denaturing then renaturing DNA. This figure (redrawn from Britten and Kohne, 1968) depicts the relative quantity of mouse genomic DNA (y axis) versus the logarithm of the frequency with which the DNA is repeated. The data are derived from a C_0 $t_{1/2}$ curve, which describes the percent of genomic DNA that reassociates at particular times and DNA concentrations. A large C_0 $t_{1/2}$ value implies a slower reassociation reaction. Three classes are apparent. The fast component accounts for 10% of mouse genomic DNA (arrow A), and represents highly repetitive satellite DNA. An intermediate component accounts for about 20% of mouse genomic DNA and contains repeats having from 1000 to 100,000 copies. The slowly reassociating component, comprising 70% of the mouse genome, corresponds to unique, single-copy DNA. Britten and Kohne (1968) obtained similar profiles from other eukaryotes, although distinct differences were evident between species. Used with permission.

Formation of Ribonucleoprotein complexes



Almost all transposable elements in mammals fall into one of four classes



Slide by Ross Hardison, Penn State U.