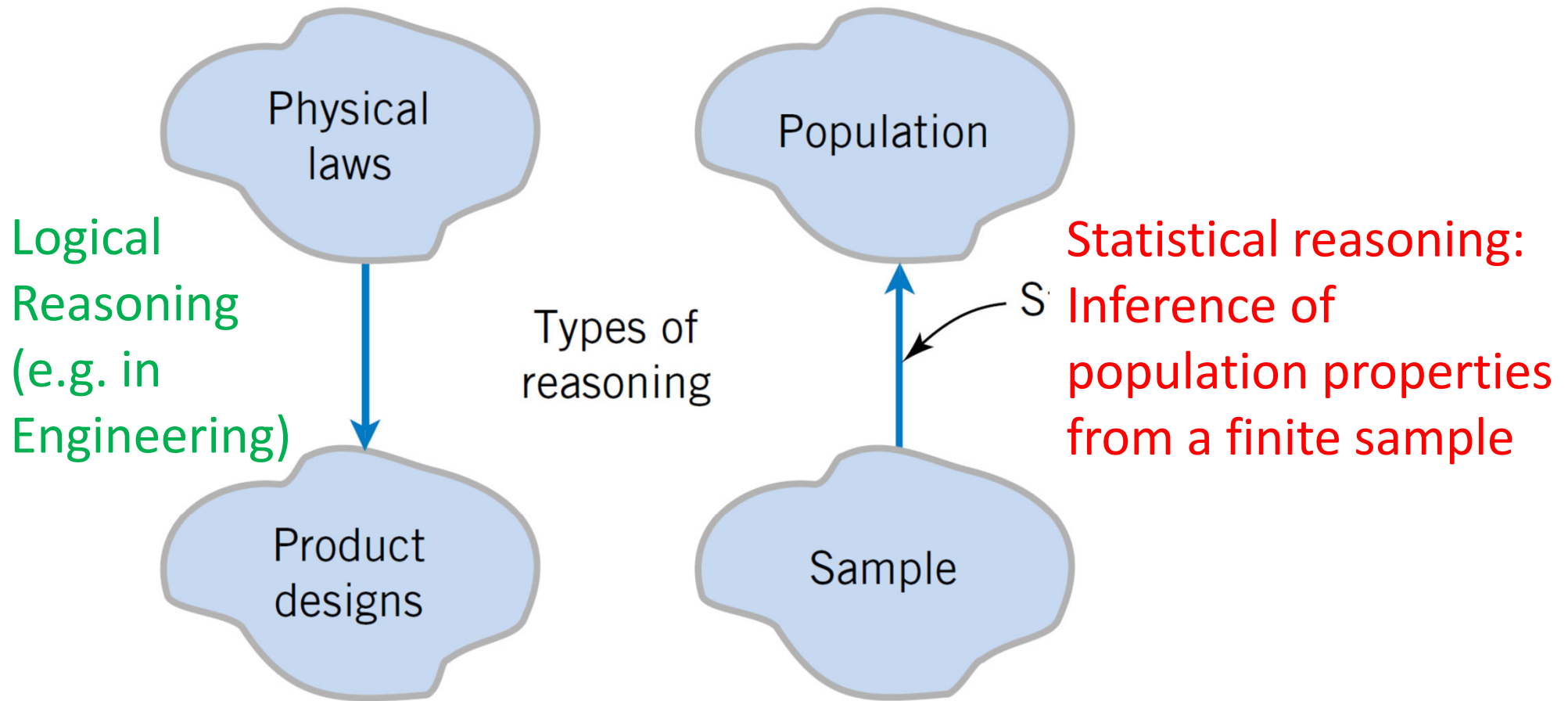


Descriptive statistics:
Populations, Samples
Histograms, Quartiles
Sample mean and
variance

Two types of reasoning



Numerical Summaries of Data

- Data are the **numerical observations** of a **phenomenon of interest**.
- The totality of all observations is a **population**.
 - **Population can be infinite** (e.g. abstract random variables)
 - **It can be very large** (e.g. 7 billion humans or all patients who have cancer of a given type)
- A (usually small) portion of the population collected for analysis is a random **sample**.
- We want to **use sample** to **infer facts about populations**
- The **inference** is not perfect but **gets better and better as sample size increases**.

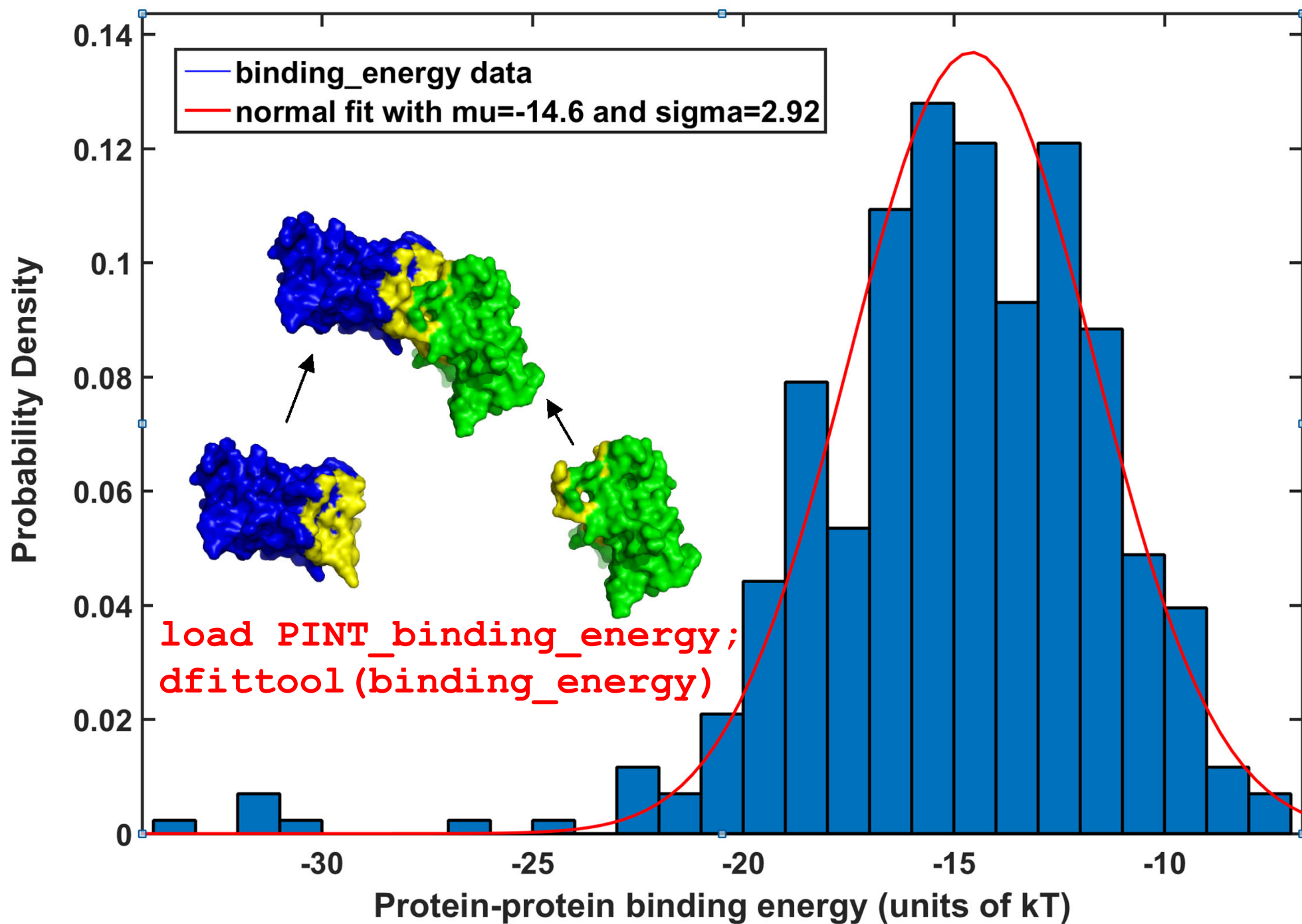
Some Definitions

- The random variables $X_1, X_2, \dots, X_n$ are a **random sample** of **size n** if:
 - a) The X_i are **independent** random variables.
 - b) Every X_i has **the same probability distribution**.
- Such $X_1, X_2, \dots, X_n$ are also called **independent and identically distributed** (or **i. i. d.**) random variables

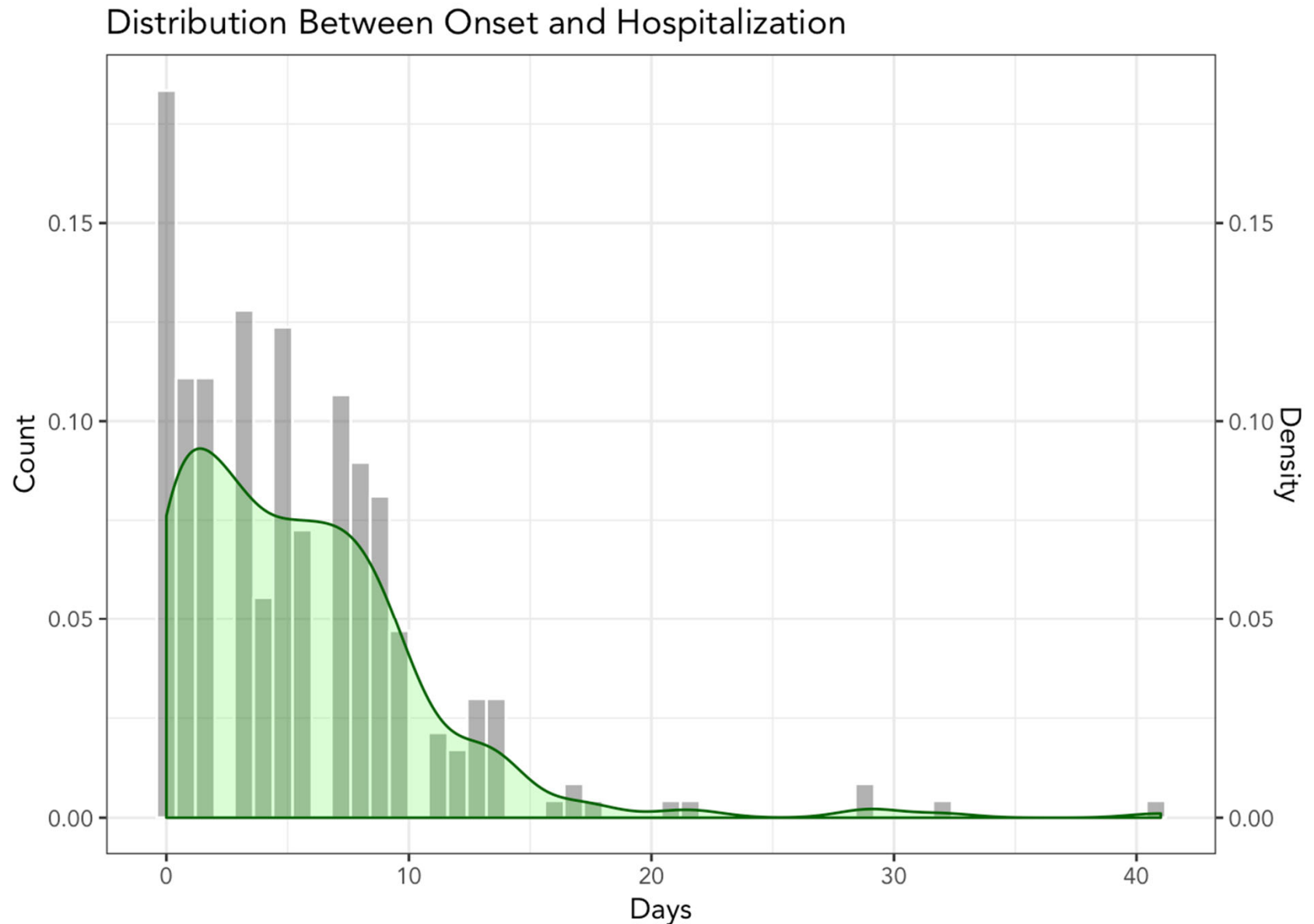
Ways to describe a sample:

Histogram

approximates PDF
(or PMF)



PDF of time between COVID-19 symptoms onset and hospitalization in IL, April 2020

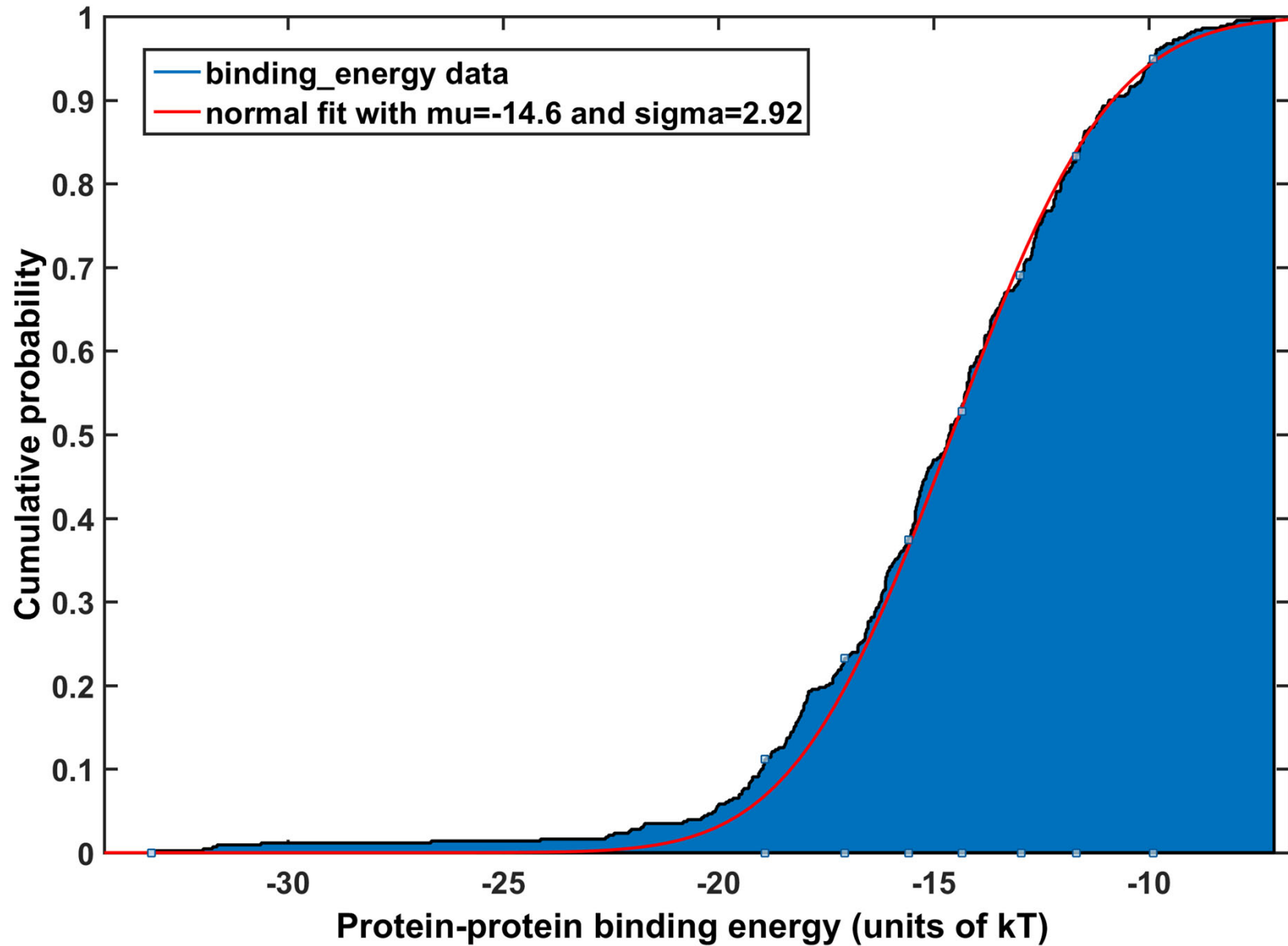


Histograms with Unequal Bin Widths

- If the data is tightly clustered in some regions and scattered in others, it is visually helpful to use **narrow bin widths** in the **clustered region** and **wide bin widths** in the **scattered areas**.
- To approximate the PDF, the **rectangle area**, not the height, must be proportional to the **bin relative frequency**.

$$\text{Rectangle height} = \frac{\text{bin relative frequency}}{\text{bin width}}$$

Cumulative Frequency Plot



Median, Quartiles, Percentiles

- The **median** q_2 divides the sample into two equal parts: 50% ($n/2$) of sample points below q_2 and 50% ($n/2$) points above q_2
- The **three quartiles** partition the data into four equally sized counts or segments.
 - 25% of the data is less than q_1 .
 - 50% of the data is less than q_2 , the median.
 - 75% of the data is less than q_3 .
- There are **100 percentiles**. n -th percentile p_n is defined so that $n\%$ of the data is less than p_n

Box-and-Whisker Plot

- A box plot is a graphical display showing **S**pread, **O**utliers, **C**enter, and **S**hape (**SOCS**).
- It displays the **5-number summary**: *min*, q_1 , *median*, q_3 , and *max*.

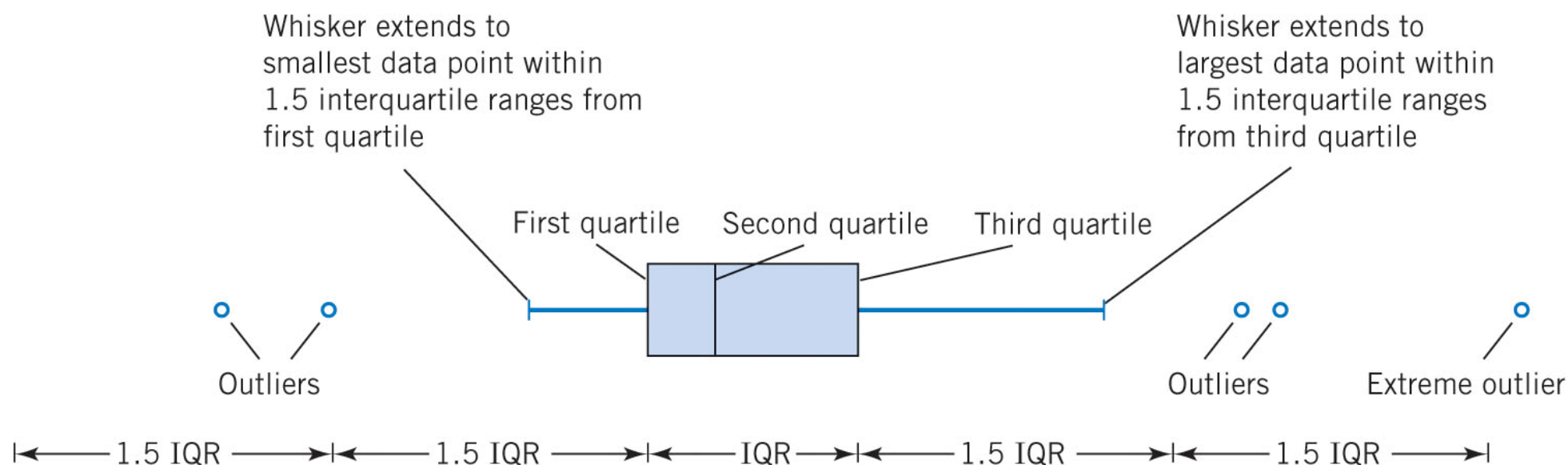


Figure 6-13 Description of a box plot.

Credit: XKCD
comics

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WHY IS THERE AN OWL OUTSIDE MY WINDOW

WHY IS THERE AN OWL ON THE DOLLAR BILL

WHY DO OWLS ATTACK PEOPLE

WHY ARE AK 47s SO EXPENSIVE

WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE

WHY ARE THERE GODS

WHY ARE THERE TWO SPOCKS

WHY IS LIFE SO BORING

WHY ARE CIGARETTES LEGAL

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Matlab exercise #1

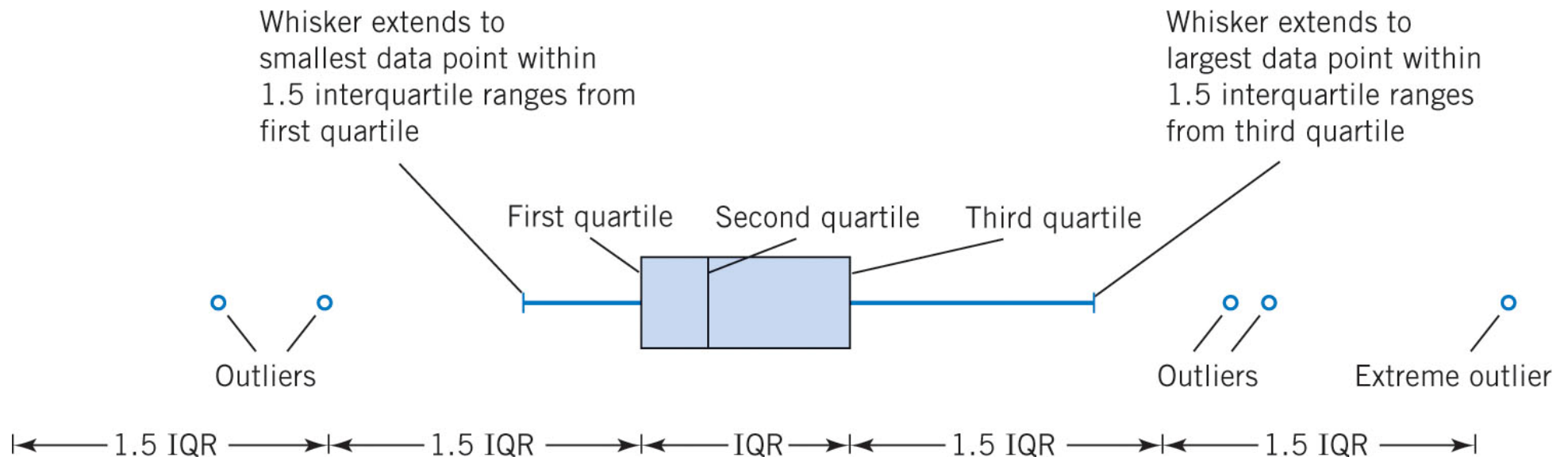
- Find the median and lower & upper quartiles of a $n=1000$ sample drawn from a standard normal distribution
- Do not use built-in Matlab functions for this exercise!
- Hint: use `[a,b]=sort(r1);` to rank order your sample. The variable `a` returns `r1` sorted in the increasing order.
- How to find the median and both quartiles from `a`?

How to find the median & quartiles

- % Example: find median and lower quartile of
- % a sample with $n=100$ drawn from uniform
- `r1=randn(1000,1);`
- `[a,b]=sort(r1);`
- `med=(a(500)+a(501))./2`
- `sum(r1<med) % verify`
- `q1=(a(250)+a(251))./2`
- `sum(r1<q1) % verify`
- `q3=(a(750)+a(751))./2`
- `sum(r1<q3) % verify`

Matlab exercise #2:

- Generate a sample with $n = 1000$ following **standard normal distribution**
- Calculate **median, first, and third quartiles**
- Calculate **IQR** and find ranges shown below
- Find and count **left** and **right outliers**
- **Do not use built-in Matlab functions for this!**
- Make box and whisker plot: use **boxplot**



How many right outliers one expects in a sample of $n=1000$ following normal distribution?

- **% find the third quartile of a standard distribution**
- **norminv(0.75) %ans = 0.6745**
- **% Calculate IQR - Interquartile Range**
- **IQR=2.*norminv(0.75) % 1.3490**
- **% Calculate $0.5 \cdot \text{IQR} + 1.5 \cdot \text{IQR}$ - the right whisker position**
- **whisker=0.5.*IQR+1.5*IQR %ans = 2.6980**
- **% Find the probability to be above the right whisker**
- **1-normcdf(whisker) %ans = 0.00349**
- **% Find number of right outliers in a sample of 1000 points**
- **1000.*(1-normcdf(whisker)) %ans = 3.49**

Credit: XKCD
comics

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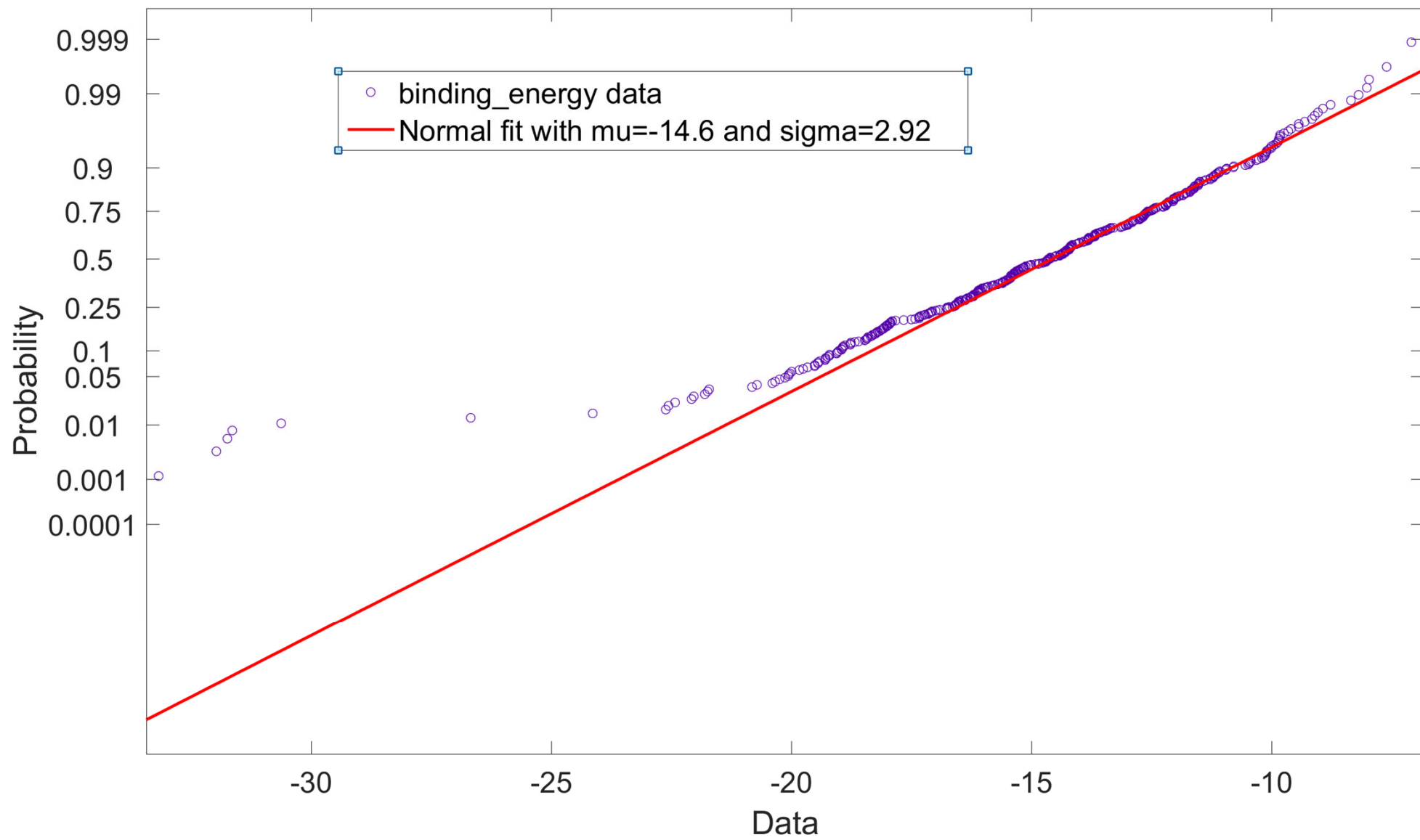
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Probability Plots

- How do we know if a particular probability distribution is a reasonable model for a data set?
- A histogram of a large data set reveals the shape of a distribution. The histogram of a small data set does not provide a clear picture.
- A probability plot is helpful for all data set size. How good is the model based on a particular probability distribution can be verified using a subjective visual examination.



How To Build a Probability Plot

- Sort the data observations in ascending order:
 $X_{(1)}, X_{(2)}, \dots, X_{(n)}.$
- Empirically determined cumulative frequency
 $\text{Prob}(x \leq x_{(j)}) = j/n$. To correct for discreteness of $x_{(j)}$ better use $\text{Prob}(x \leq x_{(j)}) = (j-0.5)/n$
- If you believe that $\text{CDF}(x)$ describes your random variable $(j-0.5)/n$ should be close to $\text{CDF}(x_{(j)})$
- Probability plot is $x_{(j)} \cdot [(j-0.5)/n] / \text{CDF}(x_{(j)})$ plotted versus the observed value $x_{(j)}$.
- If the fit is good - one gets a straight line
- Deviations can be seen especially at tails.

Probability Plot Variations

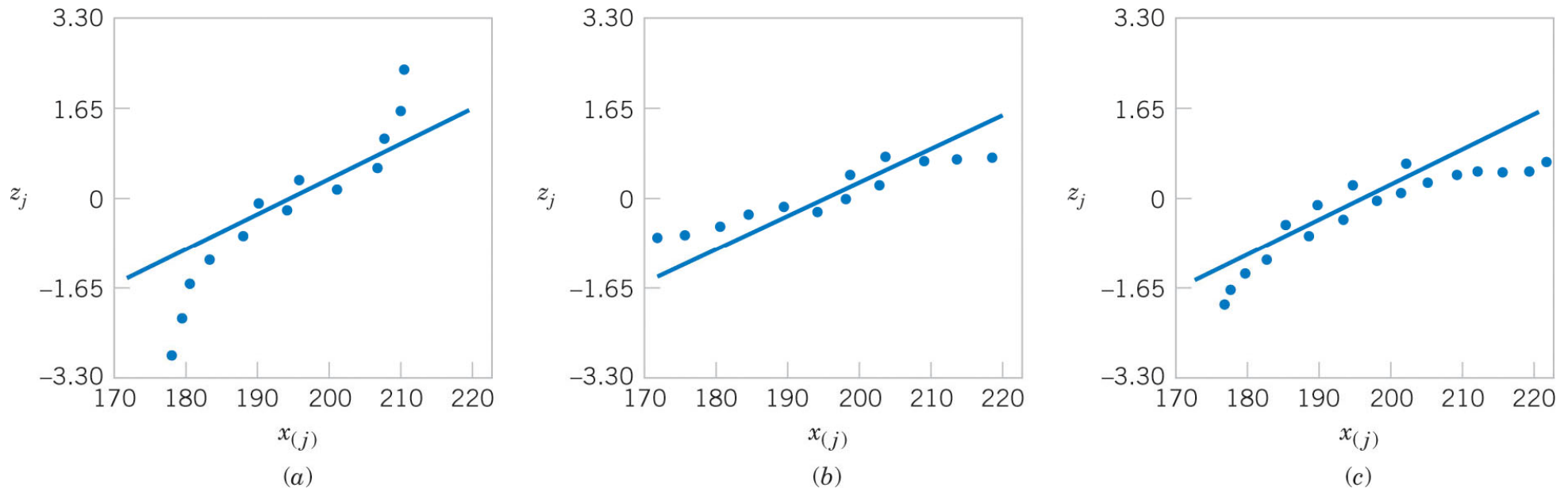


Figure 6-21 Normal probability plots indicating a non-normal distribution.

- (a) Light tailed distribution (squeezed together)
- (b) Heavy tailed distribution (stretched out)
- (c) Right skewed distribution (left end squeezed, right end stretched)

Credit: XKCD
comics

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