

Marginal Probability Distributions (discrete)

For a **discrete** joint PDF, there are **marginal distributions** for **each random variable**, formed by summing the joint PMF over the other variable.

$$f_X(x) = \sum_y f_{XY}(x, y)$$

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Called **marginal** because they are **written in the margins**

y = number of times city name is stated	x = number of bars of signal strength			$f_Y(y) =$
	1	2	3	
1	0.01	0.02	0.25	0.28
2	0.02	0.03	0.20	0.25
3	0.02	0.10	0.05	0.17
4	0.15	0.10	0.05	0.30
$f_X(x) =$	0.20	0.25	0.55	1.00

Figure 5-6 From the prior example, the joint PMF is shown in green while the two marginal PMFs are shown in purple.

Conditional Probability Distributions

Recall that $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$P(Y=y | X=x) = P(X=x, Y=y) / P(X=x) = f(x, y) / f_X(x)$$

From Example 5-1

$$P(Y=1 | X=3) = 0.25/0.55 = 0.455$$

$$P(Y=2 | X=3) = 0.20/0.55 = 0.364$$

$$P(Y=3 | X=3) = 0.05/0.55 = 0.091$$

$$P(Y=4 | X=3) = 0.05/0.55 = 0.091$$

$$\text{Sum} = 1.00$$

y = number of times city name is stated	x = number of bars of signal strength			$f_Y(y) =$
	1	2	3	
1	0.01	0.02	0.25	0.28
2	0.02	0.03	0.20	0.25
3	0.02	0.10	0.05	0.17
4	0.15	0.10	0.05	0.30
$f_X(x) =$	0.20	0.25	0.55	1.00

Note that there are 12 probabilities conditional on X , and 12 more probabilities conditional upon Y .

X and Y are Bernoulli variables

	Y=0	Y=1
X=0	2/6	1/6
X=1	2/6	1/6

What is the marginal $P_Y(Y=0)$?

A. 1/6

B. 2/6

C. 3/6

D. 4/6

E. I don't know

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X and Y are Bernoulli variables

	Y=0	Y=1
X=0	2/6	1/6
X=1	2/6	1/6

What is the conditional $P(X=0 | Y=1)$?

A. 2/6

B. 1/2

C. 1/6

D. 4/6

E. I don't know

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Reminder

Statistically independent events

Always true: $P(A \cap B) = P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$

■ Two events

Two events are **independent** if **any one** of the following equivalent statements is true:

- (1) $P(A|B) = P(A)$
- (2) $P(B|A) = P(B)$
- (3) $P(A \cap B) = P(A)P(B)$

■ Multiple events

The events E_1, E_2, \dots, E_n are independent if and only if for any subset of these events $E_{i_1}, E_{i_2}, \dots, E_{i_k}$,

$$P(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times \dots \times P(E_{i_k})$$

Independence of Random Variables X and Y

- **Random variable independence**
means that knowledge of **any** of the values of X **does not change** probabilities of **any** of the values of Y
- Opposite: **Dependence** implies that **some** values of X influence the probability of **some** values of Y

Independence for Discrete Random Variables

- Remember independence of events (slide 13 lecture 4) : Events are independent if **any one** of the three conditions are met:
 - 1) $P(A|B) = P(A \cap B)/P(B) = P(A)$ or
 - 2) $P(B|A) = P(A \cap B)/P(A) = P(B)$ or
 - 3) $P(A \cap B) = P(A) \cdot P(B)$
- Random variables independent if **all events** A that $Y=y$ and B that $X=x$ are independent if any one of these conditions is met:
 - 1) $P(Y=y|X=x) = P(Y=y)$ for any x or
 - 2) $P(X=x|Y=y) = P(X=x)$ for any y or
 - 3) $P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$**for every pair x and y**

X and Y are Bernoulli variables

	Y=0	Y=1
X=0	2/6	1/6
X=1	2/6	1/6

Are they independent?

A. yes

B. no

C. I don't know

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X and Y are Bernoulli variables

	Y=0	Y=1
X=0	1/2	0
X=1	0	1/2

Are they independent?

A. yes

☒ B. no

C. I don't know

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Joint Probability Density Function Defined

The **joint probability density function** for the continuous random variables X and Y , denoted as $f_{XY}(x,y)$, satisfies the following properties:

(1) $f_{XY}(x,y) \geq 0$ for all x, y

(2)
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y) dx dy = 1$$

(3)
$$P((X,Y) \subset R) = \iint_R f_{XY}(x,y) dx dy \quad (5-2)$$

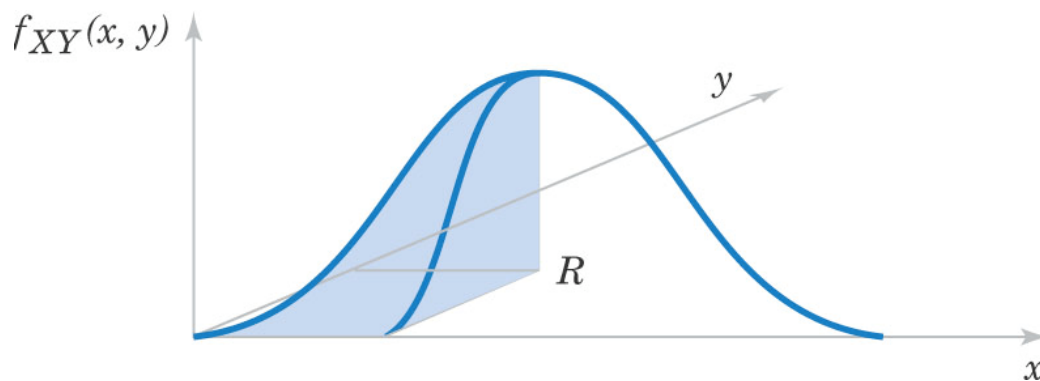


Figure 5-2 Joint probability density function for the random variables X and Y . Probability that (X, Y) is in the region R is determined by the **volume** of $f_{XY}(x,y)$ over the region R .

Joint Probability Density Function Graph

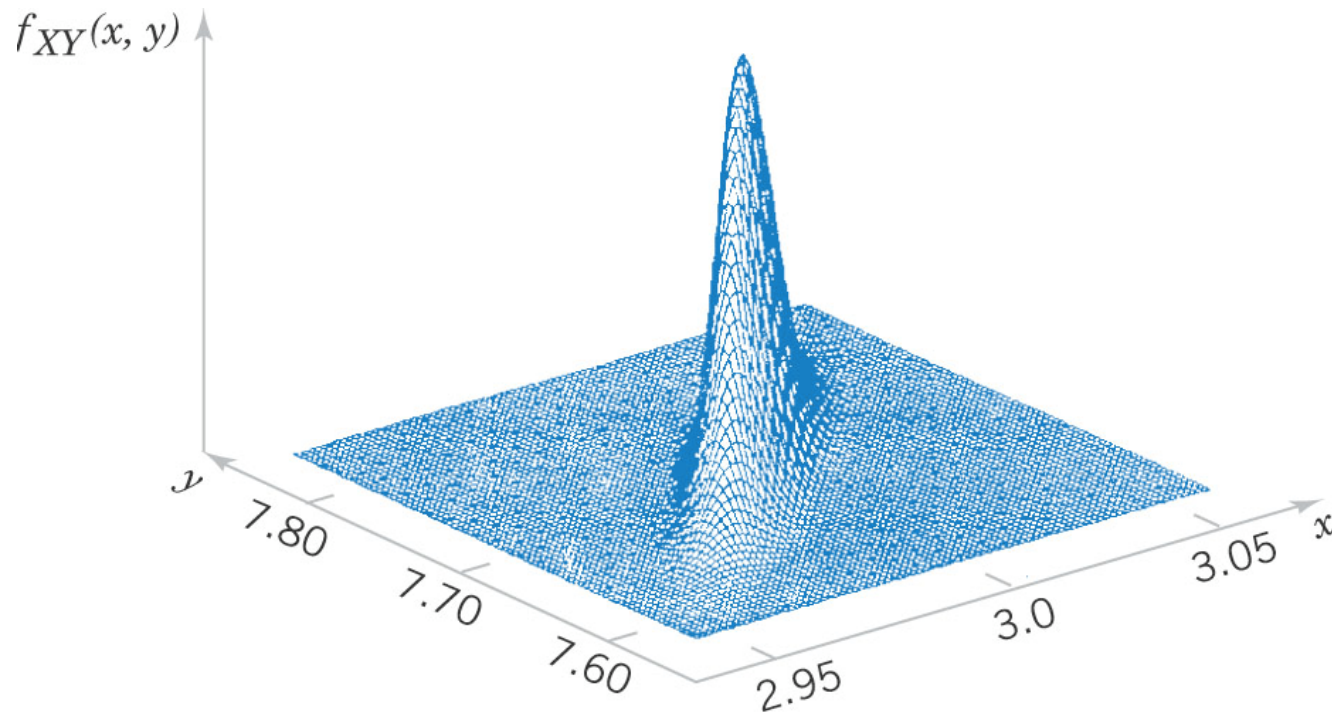


Figure 5-3 Joint probability density function for the continuous random variables X and Y of expression levels of two different genes. Note the asymmetric, narrow ridge shape of the PDF – indicating that small values in the X dimension are more likely to occur when small values in the Y dimension occur.

Marginal Probability Distributions (continuous)

- Rather than summing a discrete joint PMF, we integrate a continuous joint PDF.
- The marginal PDFs are used to make probability statements about one variable.
- If the joint probability density function of random variables X and Y is $f_{XY}(x,y)$, the marginal probability density functions of X and Y are:

$$f_X(x) = \int_y f_{XY}(x, y) dy$$

$$f_Y(y) = \int_x f_{XY}(x, y) dx \quad (5-3)$$

$$f_X(x) = \sum_y f_{XY}(x, y)$$

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Conditional Probability Density Function Defined

Given continuous random variables X and Y with joint probability density function $f_{XY}(x, y)$, the conditional probability density function of Y given $X=x$ is

$$f_{Y|x}(y) = \frac{f_{XY}(x, y)}{f_X(x)} = \frac{f_{XY}(x, y)}{\int_y f_{XY}(x, y) dy} \text{ if } f_X(x) > 0 \quad (5-4)$$

which satisfies the following properties:

(1) $f_{Y|x}(y) \geq 0$

(2) $\int f_{Y|x}(y) dy = 1$

(3) $P(Y \in B | X = x) = \int_B f_{Y|x}(y) dy$ for any set B in the range of Y

Compare to discrete: $P(Y=y | X=x) = f_{XY}(x, y) / f_X(x)$

Conditional Probability Distributions

- Conditional probability distributions can be developed for multiple random variables by extension of the ideas used for two random variables.
- Suppose $p = 5$ and we wish to find the distribution of X_1, X_2 and X_3 conditional on $X_4=x_4$ and $X_5=x_5$.

$$f_{X_1X_2X_3|x_4x_5}(x_1, x_2, x_3) = \frac{f_{X_1X_2X_3X_4X_5}(x_1, x_2, x_3, x_4, x_5)}{f_{X_4X_5}(x_4, x_5)}$$

for $f_{X_4X_5}(x_4, x_5) > 0$.

Independence for Continuous Random Variables

For random variables X and Y , if any one of the following properties is true, the others are also true. Then X and Y are **independent**.

$$(1) f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$$

$$(2) f_{Y|x}(y) = f_Y(y) \text{ for all } x \text{ and } y \text{ with } f_X(x) > 0$$

$$(3) f_{X|y}(x) = f_X(x) \text{ for all } x \text{ and } y \text{ with } f_Y(y) > 0$$

$$(4) P(X \in A, Y \in B) = P(X \in A) \cdot P(Y \in B) \text{ for any sets } A \text{ and } B \text{ in the range of } X \text{ and } Y, \text{ respectively.} \quad (5-7)$$

$$P(Y=y|X=x)=P(Y=y) \text{ for any } x \text{ or}$$

$$P(X=x|Y=y)=P(X=x) \text{ for any } y \text{ or}$$

$$P(X=x, Y=y)=P(X=x) \cdot P(Y=y) \text{ for any } x \text{ and } y$$

Covariation, Correlations

Quick and dirty check for
linear (in)dependence
between variables

Covariance - 1 number to
measure dependance
between random variables

$\text{Cov}(X, Y)$ or σ_{xy}

$$\sigma_{xy} = E[(X - \mu_x) \cdot (Y - \mu_y)] = \\ = E(X \cdot Y) - \mu_x \cdot \mu_y$$

- $\text{Var}(X) = \text{Cov}(X, X)$
- If X & Y are independent

$$\text{Cov}(X, Y) = E[X - \mu_x] \cdot E[Y - \mu_y] = 0$$

- $-\infty$ < $\text{Cov}(X, Y)$ < $+\infty$ Can be negative!

Covariance Defined

Covariance is a number quantifying the average *linear* dependence between two random variables.

The covariance between the random variables X and Y , denoted as $\text{cov}(X, Y)$ or σ_{XY} is

$$\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X\mu_Y$$

Montgomery, Runger 5th edition Eq. (5–14)

The units of σ_{XY} are the units of X times the units of Y .

Unlike the range of the variance, covariance can be negative: $-\infty < \sigma_{XY} < \infty$.

Covariance and PMF tables

y = number of times city name is stated	x = number of bars of signal strength		
	1	2	3
1	0.01	0.02	0.25
2	0.02	0.03	0.20
3	0.02	0.10	0.05
4	0.15	0.10	0.05

The probability distribution of Example 5-1 is shown.

By inspection, note that the **larger probabilities** occur as X and Y move in opposite directions. This indicates a **negative covariance**.

Covariance and Scatter Patterns

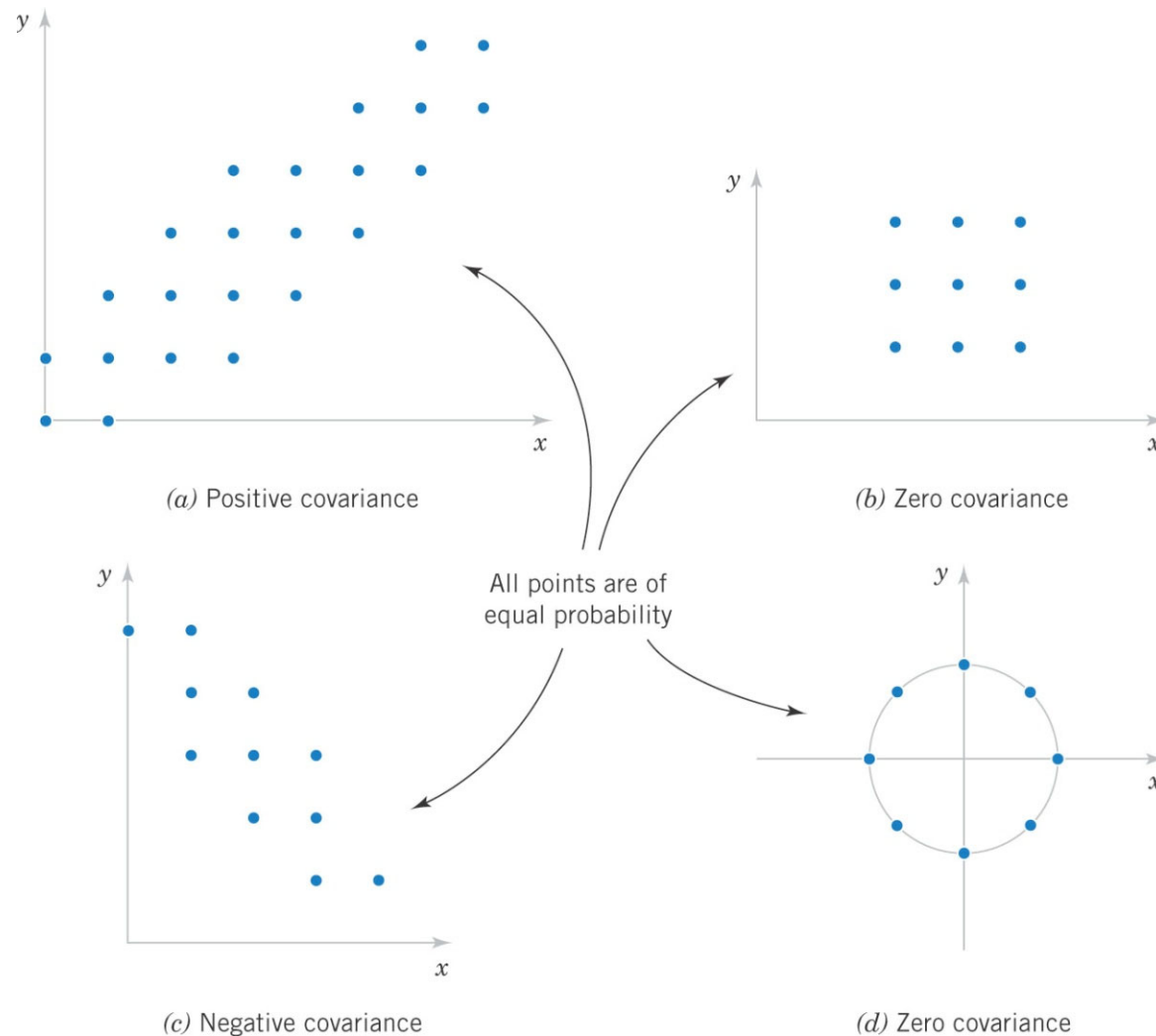


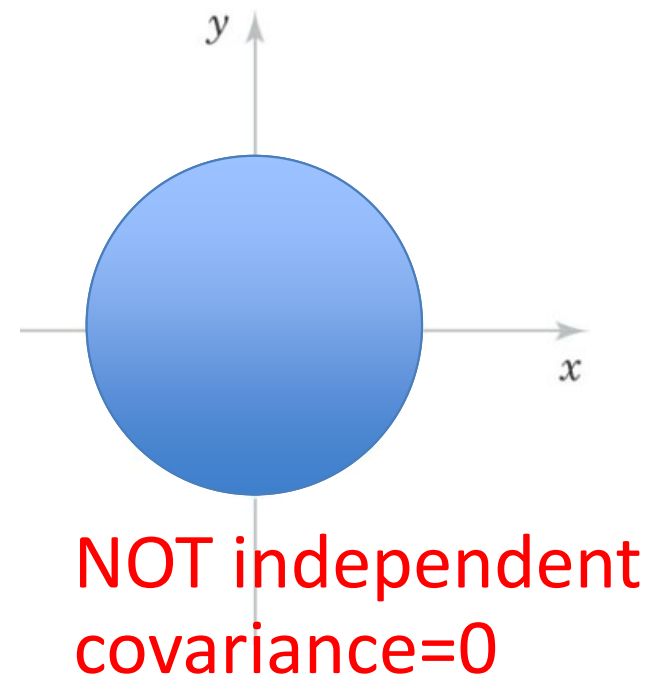
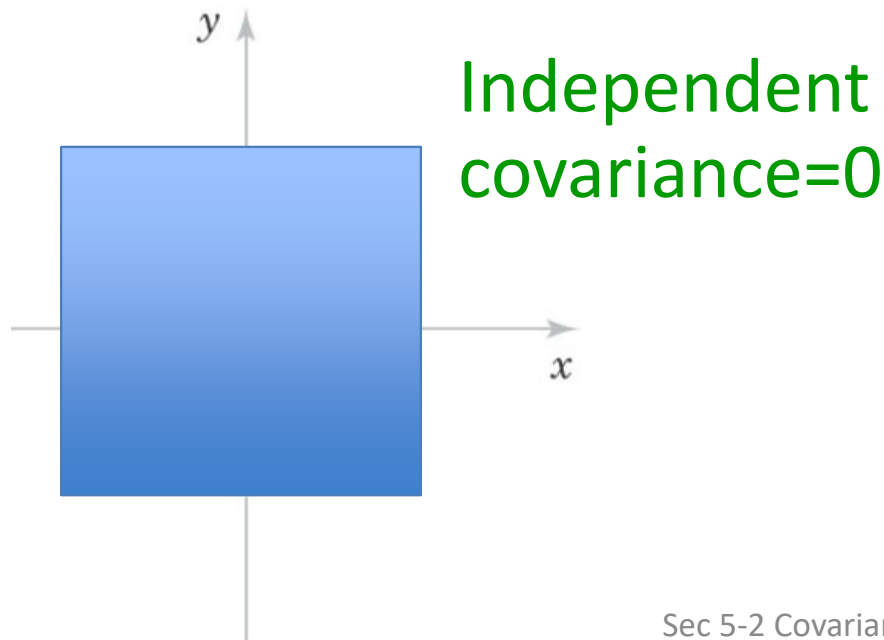
Figure 5-13 Joint probability distributions and the sign of $\text{cov}(X, Y)$. Note that covariance is a measure of linear relationship. Variables with non-zero covariance are **correlated**.

Independence Implies $\sigma = \rho = 0$ but not vice versa

- If X and Y are independent random variables,

$$\sigma_{XY} = \rho_{XY} = 0 \quad (5-17)$$

- $\rho_{XY} = 0$ is necessary, but **not a sufficient** condition for independence.



Correlation is “normalized covariance”

- Also called:
Pearson correlation
coefficient

$\rho_{XY} = \sigma_{XY} / \sigma_X \sigma_Y$
is the covariance
normalized to
be $-1 \leq \rho_{XY} \leq 1$



Karl Pearson (1852– 1936)

English mathematician and biostatistician

Prove that ρ_{xy} is in $[-1, 1]$

$$Z_x = \frac{X - \mu_x}{\sigma_x} ; \quad Z_y = \frac{Y - \mu_y}{\sigma_y}$$

$$\begin{aligned} 0 \leq E((Z_x - Z_y)^2) &= E(Z_x^2) + E(Z_y^2) - \\ &- 2E(Z_x \cdot Z_y) = 2 - 2 \frac{1}{\sigma_x \sigma_y} E((X - \mu_x)(Y - \mu_y)) = \end{aligned}$$

$$2 - 2\rho_{xy} \implies \boxed{\rho_{xy} \leq 1}$$

$$\begin{aligned} 0 \leq E((Z_x + Z_y)^2) &= E(Z_x^2) + E(Z_y^2) + \\ &+ 2E(Z_x \cdot Z_y) = 2 + 2\rho_{xy} \implies \end{aligned}$$

$$\implies \boxed{\rho_{xy} \geq -1}$$

Spearman rank correlation

- **Pearson correlation** tests for **linear relationship** between X and Y
- **Unlikely for** variables with **broad distributions** → non-linear effects dominate
- **Spearman correlation** tests for any **monotonic relationship** between X and Y
- **Calculate ranks** (1 to n), $r_X(i)$ and $r_Y(i)$ of variables in both samples. Calculate Pearson correlation between ranks:
 $\text{Spearman}(X,Y) = \text{Pearson}(r_X, r_Y)$
- **Ties:** convert to fractions, e.g. tie for 6s and 7s place both get 6.5. This can lead to artefacts.
- If lots of ties: use **Kendall rank correlation** (Kendall tau)

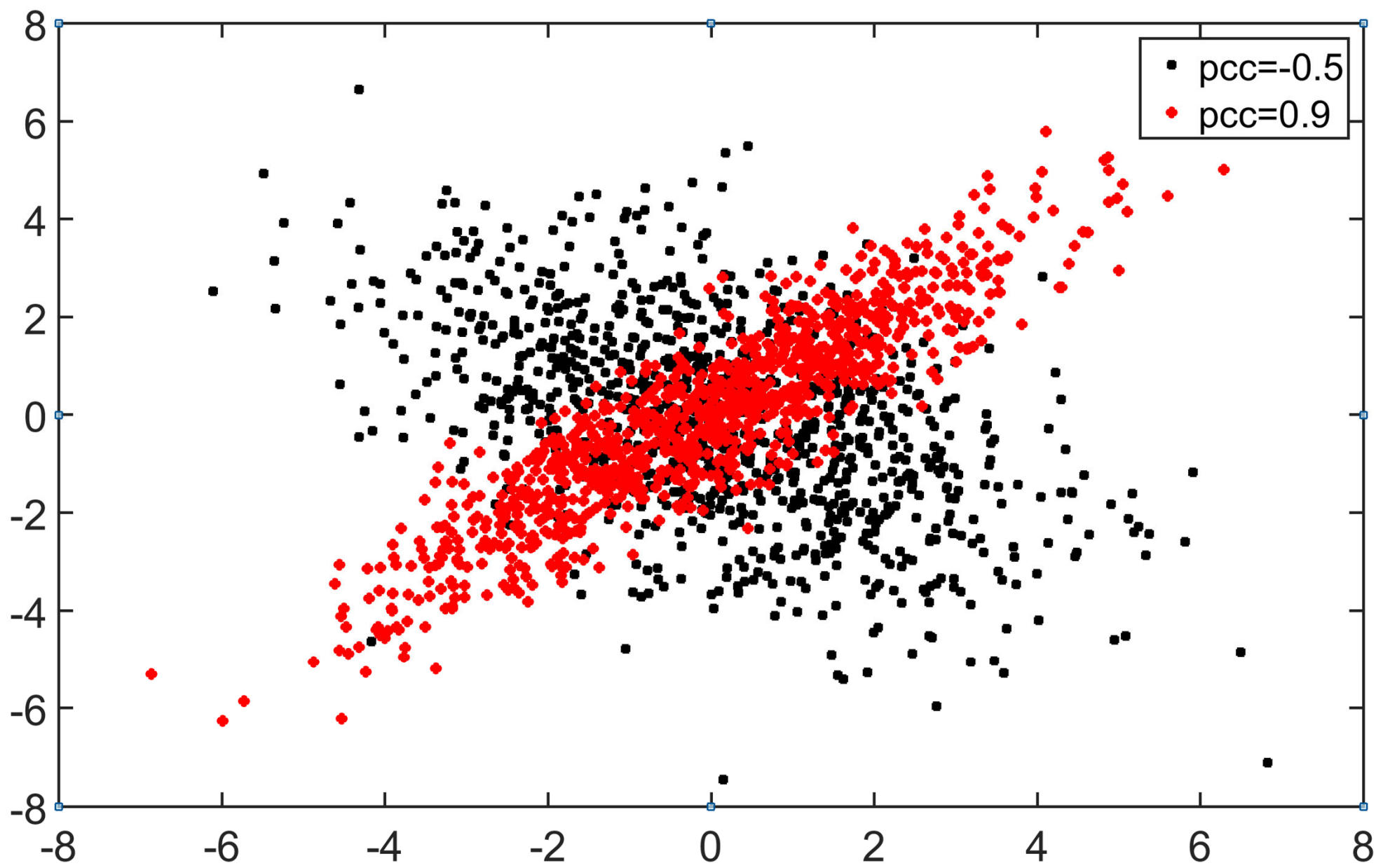
WHY AREN'T BOWTIES SHARP? WHY IS LYIN

Matlab exercise: Correlation/Covariation

- Generate a sample with **Stats=100,000** of two Gaussian random variables **r1** and **r2** which have **mean 0** and **standard deviation 2** and are:
 - **Uncorrelated**
 - Correlated with **correlation coefficient 0.9**
 - Correlated with **correlation coefficient -0.5**
 - Trick: first make **uncorrelated r1** and **r2**. Then make a new variable: **$r1_{mix} = mix \cdot r2 + (1 - mix^2)^{0.5} \cdot r1$** ; where **mix= corr. coeff.**
- For each value of **mix** calculate covariance and **correlation coefficient** between **r1mix** and **r2**
- In each case make a scatter plot: **`plot(r1mix,r2,'k.')`**;

Matlab exercise: Correlation/Covariation

1. Stats=100000;
2. r1=2.*randn(Stats,1);
3. r2=2.*randn(Stats,1);
4. disp('Covariance matrix='); disp(cov(r1,r2));
5. disp('Correlation=');disp(corr(r1,r2));
6. figure; plot(r1,r2,'k.');
7. mix=0.9; %Mixes r2 to r1 but keeps same variance
8. r1mix=mix.*r2+sqrt(1-mix.^2).*r1;
9. disp('Covariance matrix='); disp(cov(r1mix,r2));
10. disp('Correlation=');disp(corr(r1mix,r2));
11. figure; plot(r1mix,r2,'k.');
12. mix=-0.5; %REDO LINES 8-11



Credit: XKCD
comics

WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS

WHY IS HTTPS CROSSED OUT IN RED
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WHY IS WOLVERINE NOT IN THE AVENGERS

WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN

WHY IS PSYCHIC WEAK TO BUG

WHY DO CHILDREN GET CANCER

WHY IS POSEIDON ANGRY WITH ODYSSEUS

WHY IS THERE ICE IN SPACE

WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED

WHY IS SPACE BLACK

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WHY IS DYING SO SCARY

WHY IS THERE NO GPS IN LAPTOPS

WHY DO KNEES CLICK

WHY AREN'T THERE E GRADES

WHY IS ISOLATION BAD

WHY DO BOYS LIKE ME

WHY DON'T BOYS LIKE ME

WHY IS THERE ALWAYS A JAVA UPDATE

WHY ARE THERE RED DOTS ON MY THIGHS

WHY IS LYING GOOD

WHY ARE THERE
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WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE

WHY AREN'T
THERE GUNS IN
HARRY POTTER



WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG

WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER

WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP

WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD

WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE ARTERIES
WHY ARE OLD KINGDOMS DIFFERENT

WHY ARE THERE SQUIRRELS

WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
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WHY IS LYING GOOD

WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL

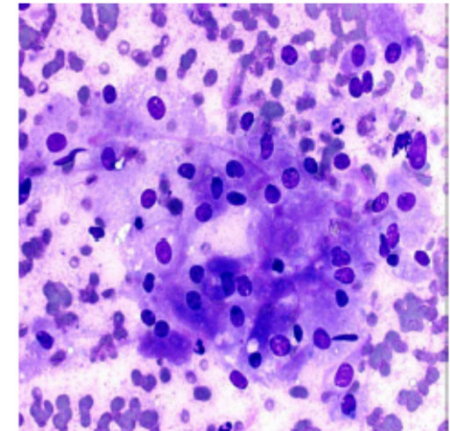
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY

WHY IS SEX
SO IMPORTANT



Let's work with real cancer data!

- Data from Wolberg, Street, and Mangasarian (1994)
- Fine-needle aspirates = biopsy for breast cancer
- Black dots – cell nuclei. Irregular shapes/sizes may mean cancer
- Statistics of all cells in the image
- 212 cancer patients and 357 healthy individuals (column 1)
- 30 other properties (see table)



Variable	Mean	S.Error	Extreme
Radius (average distance from the center)	Col 2	Col 12	Col 22
Texture (standard deviation of gray-scale values)	Col 3	Col 13	Col 23
Perimeter	Col 4	Col 14	Col 24
Area	Col 5	Col 15	Col 25
Smoothness (local variation in radius lengths)	Col 6	Col 16	Col 26
Compactness ($\text{perimeter}^2 / \text{area} - 1.0$)	Col 7	Col 17	Col 27
Concavity (severity of concave portions of the contour)	Col 8	Col 18	Col 28
Concave points (number of concave portions of the contour)	Col 9	Col 19	Col 29
Symmetry	Col 10	Col 20	Col 30
Fractal dimension ("coastline approximation" - 1)	Col 11	Col 21	Col 31

Matlab exercise #2

- Download cancer data in cancer_wdbc.mat
- Data in the file cancerwdbc.mat (569x30). First 357 patients are healthy. The remaining 569-357=212 patients have cancer.
- Make scatter plots of radius vs perimeter and texture vs radius.
- Calculate Pearson and Spearman correlations in both cases
- Calculate the correlation matrix of all-against-all variables: there are $30 \times 29 / 2 = 435$ correlations.
Hint: `corr_mat=corr(cancerwdbc);`
- Plot the histogram of these 435 correlation coefficients.
Hint: use `[i,j,v]=find(corr_mat);` then find all $i > j$ and analyze `v` evaluated on this subset of 435 matrix elements

Credit: XKCD
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WHY DO GOOD PEOPLE DIE

WHY AREN'T
THERE GUNS IN
HARRY POTTER



WHY ARE ULTRASOUNDS IMPORTANT

WHY ARE ULTRASOUND MACHINES EXPENSIVE

WHY IS STEALING WRONG

WHY ARE THERE DOGS AFRAID OF FIREWORKS

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WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARIOUSE ARTERIES
WHY ARE OLD KUNGONS DIFFERENT
WHY ARE THERE SQUIRRELS
WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL

WHY AREN'T THERE DINOSAUR GHOSTS

WHY ARE THERE FEMALE MR NIMES

WHY IS SEX
SO IMPORTANT



WHY IS GPS FREE