

## Homework #2

Please present 4 significant figures in your final answers for probabilities. Also, make sure to explain your thought process as if the reader is one of your classmates.

1. (11 points) Assume  $X$  is normally distributed with a mean of 4 and a standard deviation of 2.

(a) Determine  $P(X > 2)$

$$\text{Answer: } P(X > 2) = 1 - P(X \leq 2) = 1 - 0.15866 = 0.84134$$

(b) Determine  $P(0 < X < 7)$

$$\text{Answer: } P(2 < X < 7) = P(X < 7) - P(X < 2) = 0.93319 - 0.15866 = 0.77353$$

(c) If  $P(x < X < 7) = 0.2$ , what is  $x$ ?

$$\text{Answer: If } P(x < X < 7) = 0.2, \text{ then } P(X < x) = 0.73319. \text{ By looking at the table, we know that } x = 5.24498.$$

2. (7 points) The height of people is often assumed to be normally distributed. Let the mean height of men in the US is  $\mu = 1.77\text{m}$ , and standard deviation is  $\sigma = 0.08$ .

(a) What is the probability that a randomly selected man is taller than 1.83m?

$$\text{Answer: The probability that a randomly selected man is taller than 1.83m is } P(X > 1.83) = 1 - P(X \leq 1.83) = 1 - 0.77337 = 0.22663.$$

(b) What is the probability that if 10 men are randomly selected, at least 3 of them are taller than 1.83m?

$$\text{Let } Y \sim \text{Binom}(10, 0.22663)$$

$$\begin{aligned} P(Y \geq 3) &= 1 - (P(Y=0) + P(Y=1) + P(Y=2)) \\ &= 1 - (0.59659) \\ &= 0.40341 \end{aligned}$$

3. (7 points) Measurement error that is normally distributed with a mean of zero and a standard deviation of 0.5 grams is added to the true weight of a sample. Then the measurement is rounded to the nearest gram. Suppose that the true weight of a sample is 156.5 grams.

(a) What is the probability that the rounded result is exactly 157 grams?

$$\text{Answer: Let us denote } X \text{ as measurement and } \epsilon \text{ as error. Then, we have } X = 156.5 + \epsilon. \text{ The probability that the rounded result is exactly 157 grams is } P(156.5 \leq X < 157.5) = P(X < 157.5) - P(X < 156.5) = P(\epsilon < 1) - P(\epsilon < 0) = 0.97725 - 0.5 = 0.47725$$

(b) What is the probability that the rounded result is 155 grams or lesser?

$$\text{Answer: } P(X < 155.5) = P(\epsilon < -1) = 1 - P(\epsilon < 1) = 0.02275$$