Review for the Final Exam

The final exam for BIOE 505 will be held in this room

0018 Campus Instructional Facility

12/17/2024, 7pm-9pm

Rules are the same as for midterm

- Closed book exam; no books, notes, laptops, smartphones, etc.
- However, calculators (not on a smartphone) can be used.
- You can prepare one cheat sheet (letter size, two-sided if needed)
- Printouts provided:
  - Distributions means/variances/pdfs
  - Standard normal distribution CDF table

Name	Probability Distribution	Mean	Variance	Section in Book
Discrete				
Uniform	$\frac{1}{n}, a \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a+1)^2-1}{12}$	3-5
Binomial	$\binom{n}{x} p^x(1-p)^{n-x}$, $x = 0, 1, \ldots, n, 0 \le p \le 1$	np	$np(1-p)$	3-6
Geometric	$(1-p)^{x-1}p$, $x = 1, 2, \ldots, 0 \le p \le 1$	$1/p$	$(1-p)/p^2$	3-7.1
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r}p^r$ $x = r, r+1, r+2, \ldots, 0 \le p \le 1$	r/p	$r(1-p)/p^2$	3-7.2
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots, 0 < \lambda$	λ	λ	3-9
Continuous				
Uniform	$\frac{1}{b-a}, a \le x \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a)^2}{12}$	4-5
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty, -\infty < \mu < \infty, 0 < \sigma$	μ	σ^2	4-6
Exponential	$\lambda e^{-\lambda x}, 0 \le x, 0 < \lambda$	$1/\lambda$	$1/\lambda^2$	4-8
Erlang	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}, 0 < x, r = 1, 2, \ldots$	r/λ	r/λ^2	4-9.1
Gamma	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda$	r/λ	r/λ^2	4-9.2

This will be provided

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

What may be on the final exam?

- Probability Multiplication, Combinatorics
- Bayes Theorem
- Discrete & Continuous Random Variables
- Joint Probability Distributions, Covariation/Correlations
- Sampling distributions and parameter point estimation
- Confidence Intervals
- Hypothesis testing for one and two samples
- Other topics
- Look at Homework 1-4 for examples of problems

One-sample hypothesis testing

3. **(8 points)** The college bookstore tells prospective students that the average cost of its textbooks is \$52 with a standard deviation of \$4.50. A group of statistics students think that the average cost is **actually higher**. In order to test bookstore's claim against this alternative hypothesis, the students bought a random sample of 100 books. The mean price of this sample was \$52.80. Perform the hypothesis test at the 5% level of significance and state your decision.

What type of hypothesis should I apply?

A. Two-sided: $\mu_1 \neq \mu_0$
B. One-sided: $\mu_1 > \mu_0$
C. One-sided: $\mu_1 < \mu_0$
D. Three-sided
E. I have no idea

Get your i-clickers

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The standard deviation of $\bar{x}$ in this sample is:

A. \$4.50
B. \$45
C. \$0.45
D. I have no idea

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Answer: Hypothesis: $\begin{Bmatrix} H_0 : \mu = 52 \\ H_1 : \mu > 52 \end{Bmatrix}$. The critical z-value can be obtained from $z^* = \dfrac{52.8 - 52}{4.5/10} = 1.78$ Since $z^* > z_\alpha = 1.65$, this test statistic lies in the rejection region for H_0. Thus null hypothesis H_0 will be rejected and alternative hypothesis H_1 is accepted.

Two-sample hypothesis

Mating Calls. In a study of mating calls in the gray treefrogs *Hyla hrysoscelis* and *Hyla versicolor*, Gerhart (1994) reports that in a location in Lousiana the following data on the length of male advertisement calls have been collected:

	Sample size	Average duration	SD of duration	Duration range
Hyla chrysoscelis	43	0.65	0.18	0.36–1.27
Hyla versicolor	12	0.54	0.14	0.36–0.75

The two species cannot be distinguished by external morphology, but *H. chrysoscelis* are diploids while *H. versicolor* are tetraploids. The triploid

crosses exhibit high mortality in larval stages, and if they attain sexual maturity, they are sterile. Females responding to the mating calls try to avoid mismatches.

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha = 0.05$.

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1. Use two-sided hypothesis
2. z_{alpha/2}=1.96
3. Z=(0.65-0.54)/sqrt(0.18.^2/43+0.14.^2/12)=2.2516
4. Since Z> z_{alpha/2} null hypothesis can be rejected

Confidence intervals

2. **(6 points)** The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes. After observing a sample of 100 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a **90% confidence interval** for the population mean of the assembly time.

two-sided

What Z should I look up in the table?

A. $\Phi(Z)=0.9$
B. $\Phi(Z)=0.05$
C. $\Phi(Z)=0.95$
D. $\Phi(Z)=0.1$
E. I have no idea

Get your i-clickers

2. **(6 points)** The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes. After observing a sample of 100 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a **90% confidence interval** for the population mean of the assembly time.

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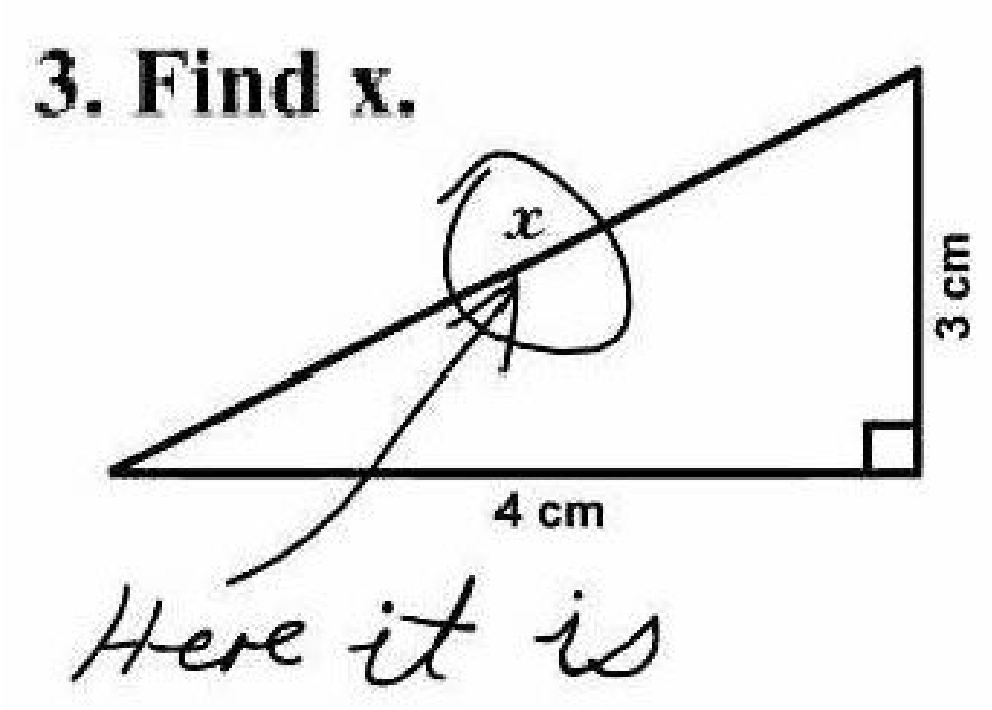
2. **(6 points)** The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes. After observing a sample of 100 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a **90% confidence interval** for the population mean of the assembly time.

Answer: Let μ denote the mean assembly time (in minutes). We want a 90% confidence interval for μ based on the following information: $n = 100,\ \bar{X} = 16.2,\ \alpha=0.1,\ \sigma=3.6$. Since σ is known, we can use normal distribution to calculate confidence interval:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 16.2 \pm (1.65)\frac{3.6}{10} = [15.61, 16.79]$$

What is X in this problem?

- What is X? Look for keywords:
  - Find the probability that….
  - What is the mean (or variance) of…
- What are the parameters? Look for keywords:
  - Given that…
  - Assuming that…
- **Is X discrete or continuous?**



Discrete Probability Distributions

(8 points) You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.
(A) (2 points) What is the probability that you first succeed on tenth experiment?

(B) (2 points) What is the probability that it requires more than five experiments for you to succeed?

(C) (2 points) What is the mean number of experiments needed to succeed once?

(D) (2 points) What is the probability that the second experiment that worked is the tenth one since you started?

2. **(8 points, 2 points each)** You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.

(a) What is the probability that you first succeed on tenth experiment?

$P(X=10) = (1-0.02)^9 *0.02 = 0.0167$

(b) What is the probability that it requires more than five experiments for you to succeed?

$P(X > 5) = 1 - P(X=1) – P(X=2) – P(X=3) – P(X=4) – P(X=5)$

$=1 – 0.98^0 *0.02 – 0.98^1 *0.02 – 0.98^2 *0.02 – 0.98^3 *0.02 – 0.98^4 *0.02 = 0.9039$

Easier solution: $P(X>5)=0.98^5=0.9039$

(c) What is the mean number of experiments needed to succeed once?

Since X follows geometric distribution, the mean value of X is 1/0.02 = 50.

(d) What is the probability that the second experiment that worked is the tenth one since you started

Probability = $9*0.02*0.98^8 *0.02 = 0.0031$

Continuous Probability Distributions

(12 points) Time interval separating subsequent bus arrivals at a stop is an exponential random variable with mean 20 minutes. Steve and Andrew work at the same place and each will be late to work unless they board a bus on or before 8:40am. Steve comes to the bus stop exactly at 8am. Andrew also comes to the same bus stop but at a random time, uniformly distributed between 8am and 8:30am. Both of them take the first bus that arrives.

(a) (4 points) What is the probability that Steve will be late for work tomorrow?

(b) **(4 points)** What is the probability that Andrew will be late for work tomorrow?

(c) **(4 points)** What is the probability that Steve and Andrew will ride the same bus

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(a) **(4 points)** What is the probability that Steve will be late for work tomorrow?

Answers: $P(Steve \text{ late}) = 1 - P(T < 40) = 1 - \frac{1}{20}\int_0^{40} e^{-t/20} dt = e^{-2} = 0.1353$

(b) **(4 points)** What is the probability that Andrew will be late for work tomorrow?

Answers:

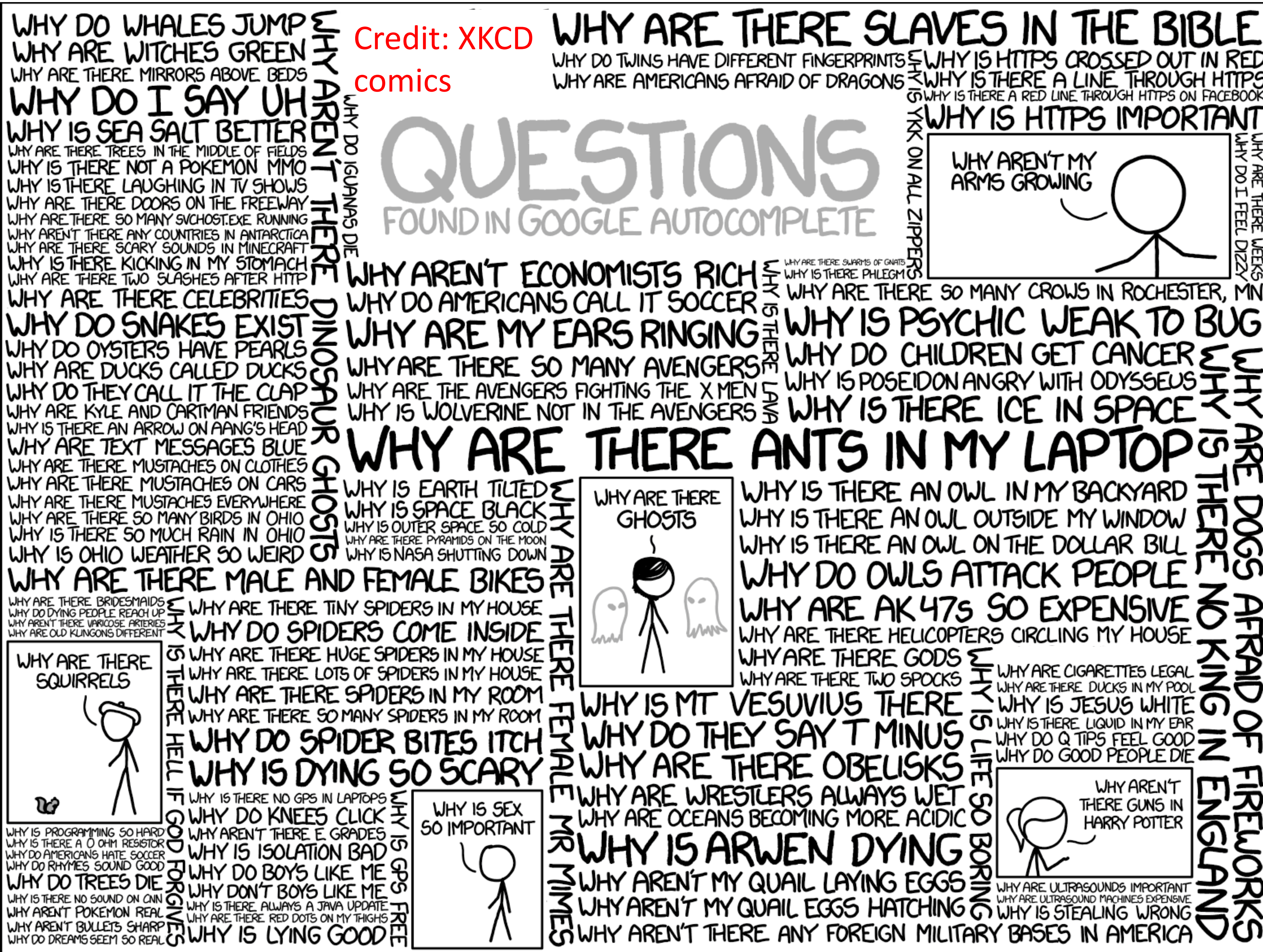
$$P(\text{Andrew late}) = \int_0^{30} \frac{dx}{30} P(T >= 40 \,|\, \text{T} > \text{x}) = \int_0^{30} \frac{dx}{30} e^{-(40-x)/20} = \frac{e^{-2}}{30}\int_0^{30} e^{x/20} dx = \frac{20e^{-2}}{30}\left(e^{30/20} - 1\right) = 0.3141$$

(c) **(4 points)** What is the probability that Steve and Andrew will ride the same bus?

Probability that Steve will not leave by the time x when Andrew comes is exp(-x/20).

It needs to be integrated over Int_0^30 dx/30 exp(-x/20)=

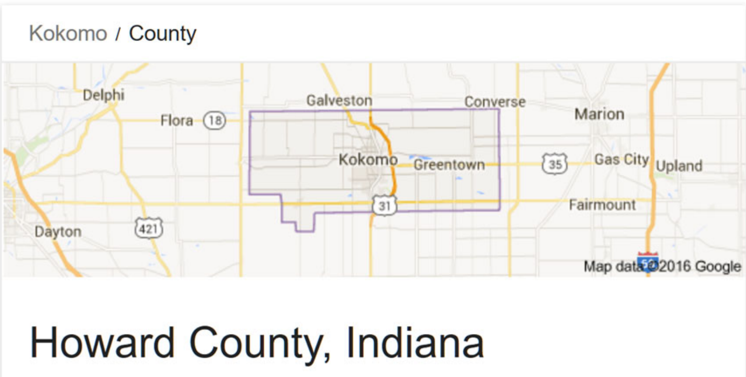
Answers: $P(\text{Steve and Andrew meet}) = \int_0^{30} \frac{dx}{30} e^{-x/20} = \frac{20}{30}(1 - e^{-30/20}) = 0.5179$



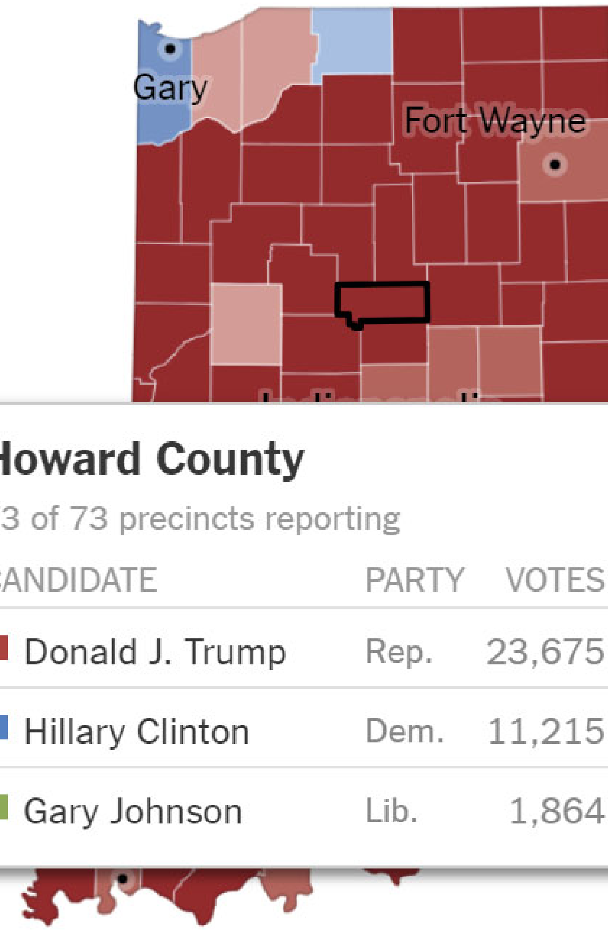
Credit: XKCD comics

Bayes theorem

Kokomo, Indiana. In Kokomo, IN, 65% of the people are conservative, 20% are liberal, and 15% are independent. Records show that in a particular election, 82% of conservatives voted, 65% of liberals voted, and 50% of independents voted. If a person from the city is selected at random and it is learned that she did not vote, what is the probability that the person is liberal?



As of the 2010 census, the population was 82,752. The county seat is Kokomo, IN.



Howard County

73 of 73 precincts reporting

CANDIDATE	PARTY	VOTES	PCT.
Donald J. Trump	Rep.	23,675	**63.4%**
Hillary Clinton	Dem.	11,215	**30.0**
Gary Johnson	Lib.	1,864	**5.0**

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P(L|NV)=P(NV|L)*P(L)/P(NV)=0.35*0.2/(0.18*0.65+0.35*0.2+0.5*0.15)=0.2672

Joint Probability Distributions

1. **(20 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by $f_{XY}(x, y) = c*(x + y)$. Determine the following:
   a) **(2 points)** Find c
   b) **(2 points)** Find probability of the event, where $X = 1$ and $Y < 3$
   c) **(2 points)** Find marginal probability $P_Y(Y = 2)$
   d) **(2 points)** Find marginal probability distribution of the random variable X
   e) **(2 points)** Find E(X), E(Y), V(X), and V(Y)
   f) **(2 points)** Find conditional probability distribution of Y given that X = 1
   g) **(2 points)** Conditional probability distribution of X given that Y = 2
   h) **(2 points)** Are X and Y independent?
   i) **(2 points)** What is the covariance for X and Y?
   j) **(2 points)** What is the correlation for X and Y?

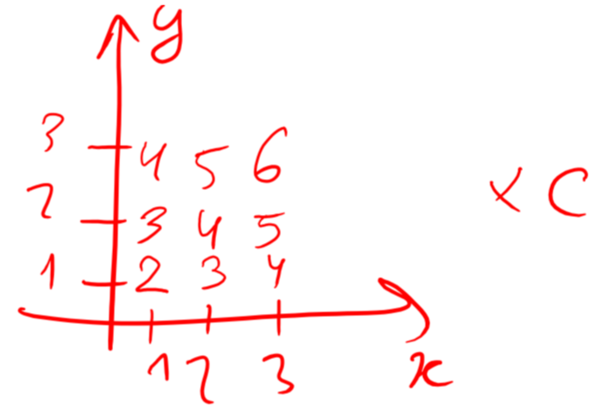
1. **(20 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by $f_{XY}(x, y) = c*(x + y)$. Determine the following:

a) **(2 points)** Find c

b) **(2 points)** Find probability of the event, where X = 1 and Y < 3

c) **(2 points)** Find marginal probability $P_Y(Y = 2)$

f) **(2 points)** Find conditional probability distribution of Y given that X = 1



(a) $1 = c\cdot(2+3+4+3+4+5+4+5+6)$

$c = 1/36$

(b) $P(X=1, Y<3) = \frac{2+3}{36} = \frac{5}{\ }$

(c) $P_Y(Y=2) = \frac{3+4+5}{36} = \frac{12}{36} = \frac{1}{3}$

(f) $P(Y=2 \mid X=1) = \frac{P(Y=2, X=1)}{P_X(X=1)} = \frac{3/36}{(2+3+4)/36} = \frac{1}{3}$

1. (20 points) The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by a formula $f_{XY}(x, y) = c*(x + y)$. Determine the following:

a) **(2 points)** Find c

Answer: $\sum_R f(x,y) = c*(2+3+4+3+4+5+4+5+6) = 1$, c*36 = 1. Thus, c = 1/36

b) **(2 points)** Find probability of the event where X = 1 and Y < 3

Answer: $P(X=1, Y<3) = f_{XY}(1,1) + f_{XY}(1,2) = \frac{1}{36}(2+3) = 5/36$

c) **(2 points)** Find marginal probability $P_Y(Y = 2)$

Answers: $P(Y=2) = f_{XY}(1,2) + f_{XY}(2,2) + f_{XY}(3,2) = \frac{1}{36}(3+4+5) = 1/3$

f) **(2 points)** Find conditional probability distribution of Y given that X = 1

Answers: $f_{Y|X}(y) = \dfrac{f_{XY}(1,y)}{f_X(1)}$

y	$f_{Y\|X}(y)$
1	(2/36)/(1/4)=2/9
2	(3/36)/(1/4)=1/3
3	(4/36)/(1/4)=4/9

Discrete Probability Distributions

Which distribution is this?

$$\binom{n}{x} p^x (1-p)^{n-x}$$

A. Uniform
B. Binomial
C. Geometric
D. Negative Binomial
E. Poisson

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Which distribution is this?

$$\binom{x-1}{r-1}(1-p)^{x-r}p^r$$

A. Uniform
B. Binomial
C. Geometric
D. Negative Binomial
E. Poisson

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Which distribution is this?

$$\frac{e^{-\lambda}\lambda^{x}}{x!}$$

A. Uniform
B. Binomial
C. Geometric
D. Negative Binomial
E. Poisson

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Guide to probability distributions

- Binomial: # of samples, n, is fixed, # of successes, x, is variable

$$P(X=x)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

- Geometric: # of samples, x is variable. # of successes 1 is fixed. Success comes in the end

$$P(X=x)=(1-p)^{x-1}\cdot p$$

- Negative binomial: # of samples, x is variable. # of successes, r, is fixed. rth success in ~~the~~ end

$$P(X=x)=\frac{(x-1)!}{(r-1)!(x-r)!}p^r(1-p)^{x-r}$$

Which distribution applies to this problem?

Deighton's Novel. In his World War II historical novel *Bomber* Len Dieghton argues that a pilot is "mathematically certain" to be shot down in 50 missions if the probability of being shot down on each mission is 0.02.
(a) Assuming independence of outcomes in each mission, is Deighton's reasoning correct?
(b) Find the probability of surviving all 50 missions without being shot down?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots, 0 < \lambda$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \ldots, n, 0 \le p \le 1$
Geometric	$(1-p)^{x-1} p$ $x = 1, 2, \ldots, 0 \le p \le 1$
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r} p^r$ $x = r, r+1, r+2, \ldots, 0 \le p \le 1$

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Deighton's Novel. In his World War II historical novel *Bomber* Len Dieghton argues that a pilot is "mathematically certain" to be shot down in 50 missions if the probability of being shot down on each mission is 0.02.
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$0.98^{50} = 0.3642$

(8 points) You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.

(A) (2 points) What is the probability that you first succeed on tenth experiment?

(B) (2 points) What is the probability that it requires more than five experiments for you to succeed?

(C) (2 points) What is the mean number of experiments needed to succeed once?

(D) (2 points) What is the probability that the second experiment that worked is the tenth one since you started?

2. **(8 points, 2 points each)** You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.

(a) What is the probability that you first succeed on tenth experiment?

$P(X=10) = (1-0.02)^9 * 0.02 = 0.0167$

(b) What is the probability that it requires more than five experiments for you to succeed?

$P(X > 5) = 1 - P(X=1) – P(X=2) – P(X=3) – P(X=4) – P(X=5)$
$=1 – 0.98^0 *0.02 – 0.98^1 *0.02 – 0.98^2 *0.02 – 0.98^3 *0.02 – 0.98^4 *0.02 = 0.9039$
Easier solution: $P(X>5)=0.98^5=0.9039$

(c) What is the mean number of experiments needed to succeed once?

Since X follows geometric distribution, the mean value of X is 1/0.02 = 50.

(d) What is the probability that the <u>second</u> experiment that worked is the tenth one since you started
Probability = $9*0.02*0.98^8 *0.02 = 0.0031$