#### Figure 2. US Trends in Diabetes Prevalence per 100 Adults Aged 20 Years or Older by BMI Category



Menke A, Casagrande S, Geiss L, Cowie CC JAMA. 2015;314: 1021–1029

## Large sample confidence estimate of population proportion

- Want to know the fraction p of the population that belongs to a class, e.g., the class "people with cancer"
- Each variable is a Bernoulli trial with one parameter p. We can use moments or MLE estimator to estimate p
- Both give the same estimate: sample fraction  $\hat{p}$ =(# of *people with cancer in the sample)/(sample size n)*
- How to put confidence bounds on *p* based on
- • Each participants in the sample is a Bernoulli trial: "success" = sampled participant has diabetes : -(
- Standard deviation of Bernoulli trial  $\sqrt{p(1-p)}$
- Standard error of the fraction of successes is  $\frac{\sqrt{p(1-p)}}{p}$  $\pmb{n}$

### **8-5 A Large-Sample Confidence Interval For a Population Proportion** (Eq. 8-23)

If  $\hat{p}$  is the proportion of observations in a random sample of size *n* that belongs to a class of interest, an approximate  $100(1 - \alpha)$ % confidence interval on the proportion  $p$  of the population that belongs to this class is

$$
\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
$$
(8-23)

where  $z_{\alpha/2}$  is the upper  $\alpha/2$  percentage point of the standard normal distribution.

This interval is known as the Wald interval (Wald and Wolfowitz, 1939).

Hypothesis testing: one sample

Is P53 gene expressed at a lower level in cancer patients than in healthy people?

- • We are interested if a P53 gene expression is **lowered** in population of cancer patients compared to the healthy population.
- •We know that mean gene expression in the healthy population is  $\mu_h$ =50 mRNAs/cell We are interested in deciding whether or not the mean expression in cancer population is **lower than** in healthy population. Let's call hypothesis H<sub>1</sub>. Here H<sub>1</sub> is **one-sided**
- If we asked: cancer is <u>not equal</u> to healthy H<sub>1</sub> would be a *two***sided hypothesis**
- •Assume we have a sample of 100 cancer patients with sample mean  $\bar{x}$  =48 mRNAs/cell and standard deviation σ=10 mRNA/cell
- • Can we use our sample to reject the "business as usual" or **null hypothesis H0:** cancer = healthy and select **one-sided hypothesis H1:** cancer **<sup>&</sup>lt;** healthy

### Two types of errors



 $\alpha = P$ (type I error) = P(reject H<sub>0</sub> when H<sub>0</sub> is true)

Sometimes the **type I error probability α** is called the significance level, or the  $\alpha$ -error

 ${\sf Instructions:}$  get  $\alpha$  from your boss or PI (e.g., 5% or 1%)

Prob(H<sub>0</sub> is true given the sample data) <  $\alpha$  $\rightarrow$ reject H $_{\rm 0}$  and accept H $_{\rm 1}$ 

Prob(H<sub>0</sub> is true given the sample data) >  $\alpha$  $\rightarrow$  accept H<sub>0</sub> and reject H<sub>1</sub>

Type II error is much harder to estimate. Will deal with it later

## **P-Values of Hypothesis Tests**

- P-value: what is the probability to get the observed value of sample mean of  $\bar{x}$  =48 mRNAs/cell (or even smaller) and  $\sigma$ =10 mRNAs/cell in a healthy population with  $\mu_{\sf h}^{\sf}$ =50 mRNAs/cell
- If P-value is small the null hypothesis is likely wrong and thus, the probability of making a type I error (incorrectly rejecting the null hypothesis) is small
- P-value answers the question: if I reject the null hypothesis H<sub>0</sub> based on the sample, what is the probability that I am making a type I error?

# **P-Value vs**  $\alpha$  **in Hypothesis Testing**

- Problem with using a predefined  $\alpha$ : you don't know by how much you exceeded it
- Another approach is to calculate  $\mathsf{Prob}(\mathsf{H}_{0})$  is true given the sample data) referred to as P-value. It the smallest  $\alpha$  that would lead to rejection of null hypothesis
- You give your boss the P-value and let him/her decide if it is good enough
- Routinely with big datasets in genomics and systems biology P-values can be 10<sup>-large number~10-100</sup>. This number is used to judge the quality of the hypothesis

 $M=100$ ,  $X=48$ ,  $S=10$  $\mu_{\mathbf{k}}=5$ One-sided  $H_1$ :  $M_c < M_R$  $H_0$ :  $\mu_c = \mu_h$  $2\overline{x} = \frac{S}{\sqrt{n}} = \frac{76}{\sqrt{100}} = 1$  $x = 48$  $P-\text{Value} = Pr1D(X_h<48)W0J$  $2.5\%$ 





## Generalizations

- What if  $H_1$  is a two-sided hypothesis?
- A: P-value is *2(1-Φ(*|*Z*|*))*, where *Z=( -<sup>μ</sup>0)/[S/* Compare it to: For one sized  $\mu_1 > \mu_0$  it is  $1-\Phi(Z)$ For one sized  $\mu_1 \leq \mu_0$  it is  $\Phi(Z)$
- If α is given, use  $\mu_0$  +/-z<sub>α/2</sub> \*S as thresholds to reject the null hypothesis

- What if the sample size n is small (say n<10):
- A: Use t-distribution with n-1 degrees of freedom for 2-sided *P-value=2(1-CDF\_Tdist*(|*T*|)) where  $T=(\bar{X}-\mu_0)/[S/\sqrt{n}]$ .
- For a given <sup>α</sup> use *μ0 +/- <sup>t</sup>α/2,n-1 T* to reject the null hypothesis

### **Type II Error and Choice of Sample Size**

Assume you know the minimum  $\delta = |\mu_1 - \mu_0|$  that you care about. What is the minimal sample you should use to separate H0 and H1 hypotheses if your tolerance to type I and type II errors is  $\alpha$  and  $\beta$  ?



# Standard notation to indicate P-value with

\* \*\* \*\*\*

Table 11.1: A commonly adopted convention for reporting  $p$  values: in many places it is conventional to report one of four different things (e.g.,  $p < .05$ ) as shown below. I've included the "significance stars" notation (i.e., a  $*$  indicates  $p < .05$ ) because you sometimes see this notation produced by statistical software. It's also worth noting that some people will write *n.s.* (not significant) rather than  $p > .05$ .







Credit: Trust me, I'm a "Biologist" Facebook community



Credit: XKCD comics

### A peculiar prevalence of  $p$  values just below .05

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#### **MASICAMPO AND LALANDE**





Hypothesis testing: two samples



### **Figure 10-1** Two independent populations.

### **Assumptions**

- 1.  $X_{11}, X_{12}, \ldots, X_{1n_1}$  is a random sample from population 1.
- 2.  $X_{21}, X_{22}, \ldots, X_{2n}$  is a random sample from population 2.
- The two populations represented by  $X_1$  and  $X_2$  are independent.  $3.$
- Both populations are normal. 4.

$$
E(\overline{X}_1 - \overline{X}_2) = E(\overline{X}_1) - E(\overline{X}_2) = \mu_1 - \mu_2
$$
  

$$
V(\overline{X}_1 - \overline{X}_2) = V(\overline{X}_1) + V(\overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}
$$

The quantity  
\n
$$
Z = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}
$$
\n(10-1)  
\nhas a *N*(0, 1) distribution.



**10-2.1 Hypotheses Tests on the Difference in Means, Variances Unknown**

Case 2: 
$$
\sigma_1^2 \neq \sigma_2^2
$$

If  $H_0: \mu_1 - \mu_2 = \Delta_0$  is true, the statistic

$$
T_0^* = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}
$$
 (10-15)

is distributed as t-distribution with degrees of freedom given by

$$
v = n_1 + n_2 - 2,
$$
  
or more generally

Sec 10-2 Hypotheses Tests on the Difference in Means, Variances Unknown

## Manhattan plot for Genome-Wide Association Study (GWAS)



Fig. 1 Genome-wide association results comparing 2,702 cases and 5,726 controls: a Manhattan and b quantile-quantile plots of  $-\log 10$ tranformed  $P$  values of 285,984 SNPS genotyped

Li J,, et al. A combined analysis of genome-wide association studies in breast cancer. Breast Cancer Res Treat. 2011;126: 717–727

## Multiple null hypotheses: Bonferroni correction

- What if you have **m** independent null hypotheses? Say you have **m=25,000 genes** in a genome?
- What is the probability that at least one of the nullhypotheses will be shown to be false at significance threshold  $\boldsymbol{\alpha}_1$ ?
- Answer: Family-Wise Error Rate or FWER=1-(1-  $\alpha_{\scriptscriptstyle 1}$ )<sup>m</sup>  $\approx$ m $\alpha_{\scriptscriptstyle 1}$
- If m=20 and  $\alpha_1$ =0.05, FWER= 0.6415

**Carlo Emilio Bonferroni** (1892 –1960) Italian mathematicianwho worked onprobability theory.



• If you want to get FWER< α, use  $α<sub>1</sub> = α/m$ 

#### Example 10-7

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Is there ey plasma ant

### Chocolate and Cardiovascular Health An article in Nature (2003, Vol. 48, p. 1013) described an Plasma antioxidants from chocolate

Dark chocolate may offer its consumers health benefits the milk variety cannot match.

There is some speculation that dietary flavonoids from chocolate, in particular (-)epicatechin, may promote cardiovascular health as a result of direct antioxidant effects or through antithrombotic mechanisms<sup>1-3</sup>. Here we show that consumption of plain, dark chocolate (Fig. 1) results in an increase in both the total antioxidant capacity and the  $(-)$ epicatechin content of blood plasma, but that these effects are markedly reduced when the chocolate is consumed with milk or if milk is incorporated as milk chocolate. Our findings indicate that milk may interfere with the absorption of antioxidants from chocolate in vivo and may therefore negate the potential health benefits that can be derived from eating moderate amounts of dark chocolate.

To determine the antioxidant content of different chocolate varieties, we took dark chocolate and milk chocolate prepared from the same batch of cocoa beans and defatted them twice with  $n$ -hexane before extracting them with a mixture of water, acetone and acetic acid  $(70.0.29.8.0.2)$  by volume). We measured their in vitro total antioxidant capacities using the ferric-reducing antioxidant potential (FRAP) assay<sup>4</sup>; FRAP

reduced iron per 100 g for dark and milk chocolate, respectively. Volunteers must therefore consume twice as much milk chocolate as dark chocolate to receive a similar intake of antioxidants.

We recruited 12 healthy volunteers (7) women and 5 men with an average age of  $32.2 \pm 1.0$  years (range, 25-35 years). Subjects were non-smokers, had normal blood lipid levels, were taking no drugs or vitamin supplements, and had an average weight of  $65.8 \pm 3.1$  kg (range, 46.0-86.0 kg) and body-mass index of  $21.9 \pm 0.4$  kg m<sup>-2</sup> (range,  $18.6 - 23.6$  kg m<sup> $-2$ </sup>). On different days, following a crossover experimental design, subjects consumed 100 g dark chocolate, 100 g dark chocolate with 200 ml full-fat milk, or 200 g milk chocolate (containing the equivalent of up to 40 ml milk).

One hour after subjects had ingested the chocolate, or chocolate and milk, we measured the total antioxidant capacity of their plasma by FRAP assay. Plasma antioxidant levels increased significantly after consumption of dark chocolate alone, from  $100 \pm 3.5\%$  to  $118.4 \pm 3.5\%$  (*t*-test.  $P < 0.001$ ), returning to baseline values  $(95.4 \pm 3.6\%)$  after 4 h (Fig. 2a). There was



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Figure 1 Stack of benefits? Unlike its milky counterpart, dark chocolate may provide more than just a treat for the tastebuds.

could be due to the formation of secondary bonds between chocolate flavonoids and milk proteins<sup>6,7</sup>, which would reduce the biological accessibility of the flavonoids and therefore the chocolate's potential antioxidant properties in vivo.

Our findings highlight the possibility

 $V$  o  $\frac{1}{2}$   $424$ 

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## Sweet matlab exercise #1

- **Download dark\_vs\_milk\_chocolate\_analysis\_template.m at the course website. Correct all ?? In the file**
- **dark=[118.8 122.6 115.6 113.6 119.5 115.9 115.8 115.1 116.9 115.4 115.6 107.9];**
- **milk=[102.1 105.8 99.6 102.7 98.8 100.9 102.8 98.7 94.7 97.8 99.7 98.6]**
- **Use Z-statistics to calculate P-value of the null hypothesis H 0 that milk = dark against H 1 that**  dark > milk. P value  $z=2*[1-normcdf(|Z|)]$
- **Repeat using T-statistics. # of degrees of freedom is dof=2\*(n-1) P\_value\_t=2\*tcdf(|T|, dof)**

## Sweet matlab exercise #1

- • **dark=[118.8 122.6 115.6 113.6 119.5 115.9 115.8 115.1 116.9 115.4 115.6 107.9];**
- **milk=[102.1 105.8 99.6 102.7 98.8 100.9 102.8 98.7 94.7 97.8 99.7 98.6]**
- **x\_dark=mean(dark) % sample mean dark chocolate**
- **x\_milk=mean(milk) % sample mean milk chocolate**
- **s\_dark=std(dark) % sample std dark chocolate**
- **s\_milk=std(milk) % sample std milk chocolate**
- **n=12 % sample size of both dark and milk**
- $\bullet$ **std\_xdiff=sqrt(s\_dark.^2./2+s\_milk.^2./n) % std diff x**
- **z\_stat=(x\_dark-x\_milk)./std\_xdiff % z-statistic**
- **P\_value\_z=erfc(z\_stat./sqrt(2))./2 % P-value of null true**
- **% P\_value\_z=9.9629e-34**
- $\bullet$ **dof=(n-1)+(n-1) % # of degrees of freedom**
- **P\_value\_t=tcdf(z\_stat,dof,'upper') % P-value of null true**
- **%P\_value\_t= 1.8417e-11**

