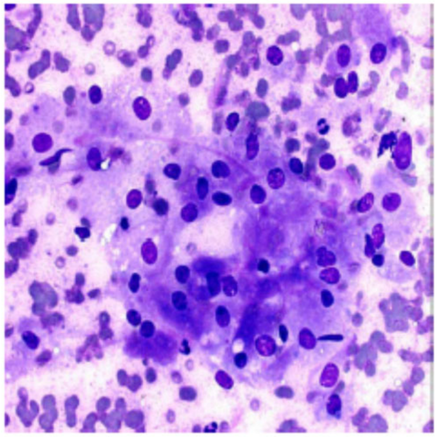
Let's work with real cancer data!

- Data from Wolberg, Street, and Mangasarian (1994)
- Fine-needle aspirates = biopsy for breast cancer
- Black dots – cell nuclei. Irregular shapes/sizes may mean cancer
- Statistics of all cells in the image
- 212 cancer patients and 357 healthy individuals (column 1)
- 30 other properties (see table)



Variable	Mean	S.Error	Extreme
Radius (average distance from the center)	Col 2	Col 12	Col 22
Texture (standard deviation of gray-scale values)	Col 3	Col 13	Col 23
Perimeter	Col 4	Col 14	Col 24
Area	Col 5	Col 15	Col 25
Smoothness (local variation in radius lengths)	Col 6	Col 16	Col 26
Compactness (perimeter^2 / area - 1.0)	Col 7	Col 17	Col 27
Concavity (severity of concave portions of the contour)	Col 8	Col 18	Col 28
Concave points (number of concave portions of the contour)	Col 9	Col 19	Col 29
Symmetry	Col 10	Col 20	Col 30
Fractal dimension ("coastline approximation" - 1)	Col 11	Col 21	Col 31

Matlab exercise #2

- Download cancer data in cancer_wdbc.mat
- Data in the table cancerwdbc (569x30). First 357 patients are healthy. The remaining 569-357=212 patients have cancer.
- Make scatter plots of area vs perimeter and texture vs radius.
- Calculate Pearson and Spearman correlations
- Calculate the correlation matrix of all-against-all variables: there are 30*29/2=435 correlations. Hint: corr_mat=corr(cancerwdbc);
- Plot the histogram of these 435 correlation coefficients. Hint: use [i,j,v]=find(corr_mat); then find all i>j and analyze v evaluated on this subset of 435 matrix elements

Midterm will be held
here in this classroom
on Tuesday 10/29
during our regular class hours
2pm-3:50pm

Midterm Info

- Closed book exam; no books, notes, laptops, phones…
- Calculators (not on smartphones) can be used
- You can prepare one 2-sided cheat sheet
- I want you to submit your cheat sheet alongside the exam. **Do not copy answers**!
- The following two printouts will be provided

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Name	Probability Distribution	Mean	Variance	Section in Book
Discrete				
Uniform	$\frac{1}{n}, a \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a+1)^2-1}{12}$	3-5
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \ldots, n, 0 \le p \le 1$	np	$np(1-p)$	3-6
Geometric	$(1-p)^{x-1}p$, $x = 1, 2, \ldots, 0 \le p \le 1$	$1/p$	$(1-p)/p^2$	3-7.1
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r}p^r$ $x = r, r+1, r+2, \ldots, 0 \le p \le 1$	r/p	$r(1-p)/p^2$	3-7.2
Hypergeometric	$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ $x = \max(0, n-N+K), 1, \ldots$ $\min(K, n), K \le N, n \le N$	np, where $p = \frac{K}{N}$	$np(1-p)\left(\frac{N-n}{N-1}\right)$	3-8
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots, 0 < \lambda$	λ	λ	3-9
Continuous				
Uniform	$\frac{1}{b-a}, a \le x \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a)^2}{12}$	4-5
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty, -\infty < \mu < \infty, 0 < \sigma$	μ	σ^2	4-6
Exponential	$\lambda e^{-\lambda x}, 0 \le x, 0 < \lambda$	$1/\lambda$	$1/\lambda^2$	4-8
Erlang	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}, 0 < x, r = 1, 2, \ldots$	r/λ	r/λ^2	4-9.1
Gamma	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda$	r/λ	r/λ^2	4-9.2

What is included in the midterm?

- Probability of events (set operations), Multiplication rules. Combinatorics
- Bayes Theorem
- Discrete Random Variables
- Continuous Random Variables
- Other topics covered
  (see HW1-HW2 for inspiration)
- No joint probabilities, correlation and covariation
- No Matlab exercises (since no computers)

Probability Multiplication Rules
Combinatorics

Mr. Jones has 6 different books that he is going to put on his bookshelf. Of these, 3 are chemistry books, 2 are physics books, and 1 is a mathematics book. Jones wants to arrange his books so that two conditions are met:

(1) all the books dealing with the same subject are together on the shelf

AND

(2) all chemistry books are on the leftmost side.

How many such different arrangements are possible?

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(1) all the books dealing with the same subject are together on the shelf

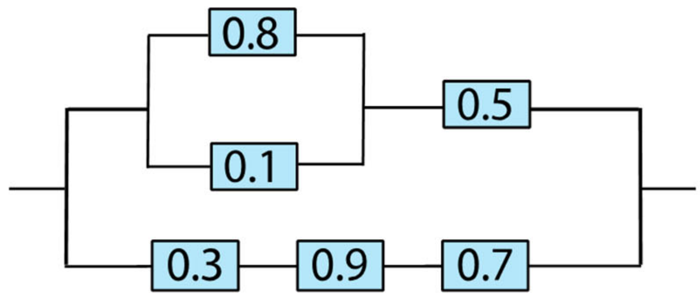
AND

(2) all chemistry books are on the leftmost side.

How many such different arrangements are possible?

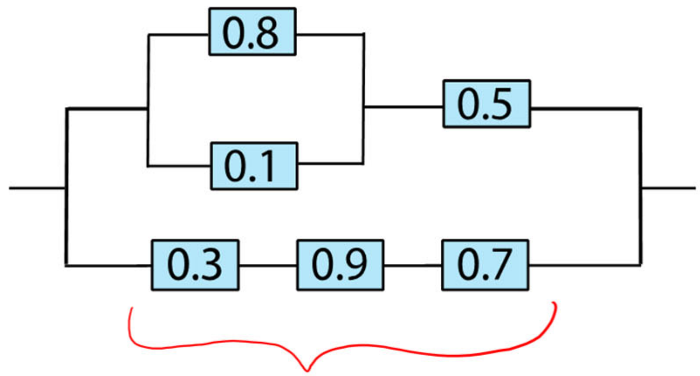
Answer: (3!*2!*1!)*2!=24

4. (4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



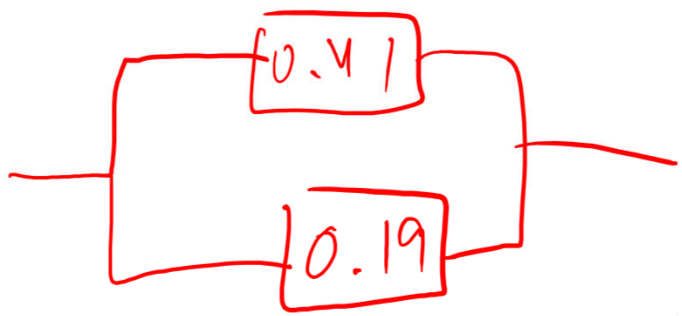
4. (4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?

$1-(1-0.8)\cdot(1-0.1)=0.82$



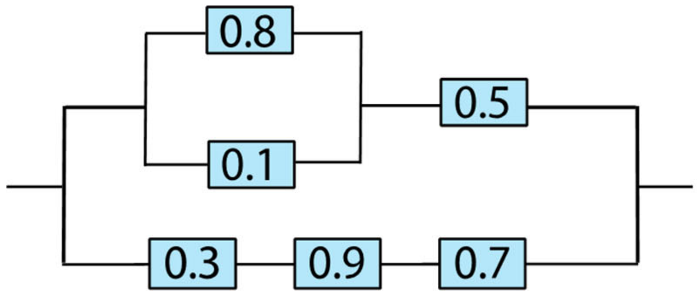
$0.3\cdot 0.9\cdot 0.7=0.19$

$0.82\times 0.5=0.41$



$1-(1-0.41)\cdot(1-0.19)=0.52$

4. (4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Answer: P(Operate) = 1-(1-0.3*0.9*0.7)*(1-0.5*(1-(1-0.8)*(1-0.1))) = 0.52

Bayes theorem

In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let 1/3 be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

A. 1/4
B. 1/3
C. 2/3
D. 1/5
E. I don’t know

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In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let 1/3 be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

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D. 1/5
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Answer: P(K)=1/3, P(K')=2/3, P(C|K)=1, P(C|K')=1/4.
P(K|C)=P(C|K)*P(K)/P(C)=1*(1/3)/(1*1/3+(1/4)*(2/3))=2/3=0.666…

(10 points) Suppose that a bag contains ten coins, three of which are fair, while the remaining seven are biased: they have probability of 0.6 of heads when flipped. A coin was taken at random from the bag and flipped five times. All five flips gave heads. What's the probability that this coin is fair?

(10 points) Suppose that a bag contains ten coins, three of which are fair, the remaining seven having probability 0.6 of giving heads when flipped. A coin is taken at random from the bag and flipped five times. All five flips give heads. What's the probability that a coin is fair given the five coin flips?

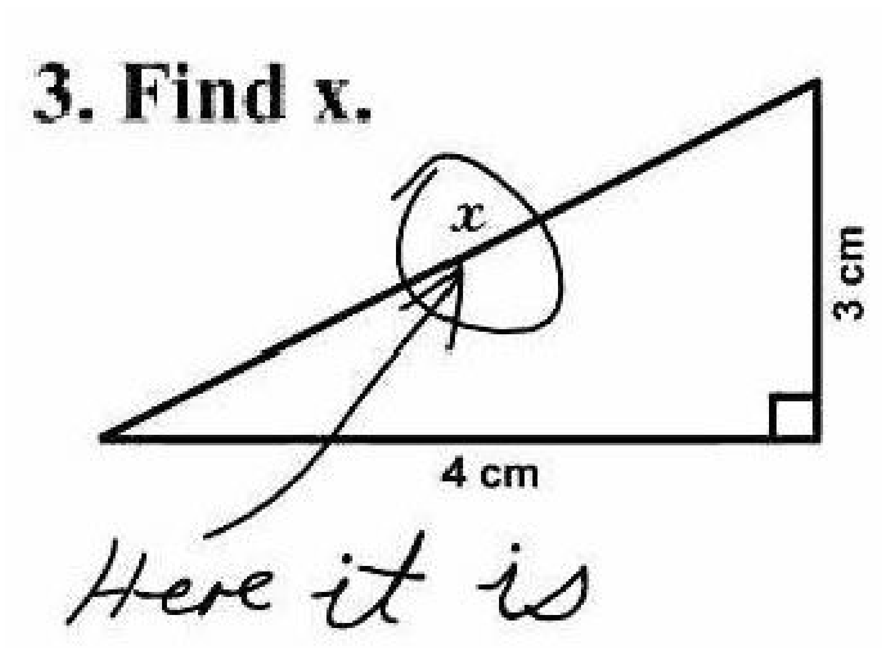
Answer: Let us denote H_1 as the hypothesis that a coin is fair and H_2 that a coin is biased. The data that all five flips were heads is denoted as D. Therefore,

$P(H_1|D) = P(D|H_1)P(H_1)/P(D) = 0.5^5 * 0.3/(0.5^5 * 0.3 + 0.6^5 * 0.7) = 0.147$

Discrete Probability Distributions

What is X in this problem?

- What is the random variable: Look for keywords:
  - Find the probability that….
  - What is the mean (or variance) of…
- What are parameters? Look for keywords:
  - Given that…
  - Assuming that…



Guide to probability distributions

- Binomial: # of samples, n, is fixed, # of successes, x, is variable

$$P(X=x)=\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}$$

- Geometric: # of samples, x is variable. # of successes 1 is fixed. Success comes in the end

$$P(X=x)=(1-p)^{x-1}\cdot p$$

- Negative binomial: # of samples, x is variable. # of successes, r, is fixed rth success in ~~the~~ end

$$P(X=x)=\frac{(x-1)!}{(r-1)!(x-r)!}p^{r}(1-p)^{x-r}$$

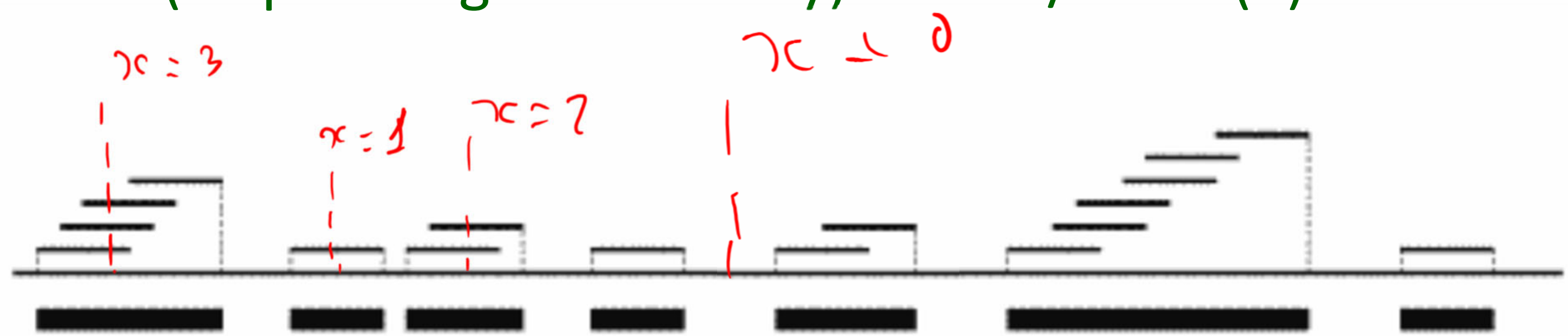
Poisson distribution in genomics

- *G - genome length (in bp)*
- *L - short read average length*
- *N – number of short read sequenced*
- *λ – sequencing redundancy = LN/G*
- *x- number of short reads covering a given site on the genome*

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Ewens, Grant, Chapter 5.1

Poisson as a limit of Binomial. For a given site on the genome for each short read Prob(site covered): p=L/G is very small. Number of attempts (short reads): N is very large. Their product (sequencing redundancy): λ = NL/G is O(1).



Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

Which discrete probability distribution describes the number of attempts?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0, 1, 2, \ldots, 0 < \lambda$
Binomial	$\binom{n}{x} p^{x}(1-p)^{n-x}$
	$x = 0, 1, \ldots, n, 0 \le p \le 1$
Geometric	$(1-p)^{x-1} p$
	$x = 1, 2, \ldots, 0 \le p \le 1$
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r} p^{r}$
	$x = r, r+1, r+2, \ldots, 0 \le p \le 1$

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Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

What are the values of p, r?

A. p=0.5, r=5
B. p=0.1, r=0.5
C. p=0.1, r=5
D. p=0.5, r=0.1
E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots, 0 < \lambda$
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E. I have no idea

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	$x = r, r+1, r+2, \ldots, 0 \le p \le 1$

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Cancer happens when the gene p53 mutates.
Probability of p53 to mutate per year is 5%.

How many years before a patient gets disease?

Which discrete probability distribution would you use to answer?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^{x}}{x!},\ x = 0, 1, 2, \ldots, 0 < \lambda$
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	$x = r, r+1, r+2, \ldots, 0 \le p \le 1$

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Continuous Probability Distributions

2. (8 points) The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.

(A) (4 points) What is the probability of a length of stay greater than 10 hours?

Answer: (10-4.6)/2.9=1.86 Using table one finds Prob=1-0.9687=0.0313

(B) (4 points) How long does one have to stay in this emergency room to know that approximately 25% of all visits last even longer?

Answer: Using table one finds P(Z<0.67)=0.75 meaning it is 4.6+2.9*0.67=6.543

1. **(8 points)** The expression level of a *TP53* tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

   (A)(4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

   (B)(4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such "close to average" *TP53* expression?

1. **(8 points)** The expression level of a *TP53* tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A)(4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

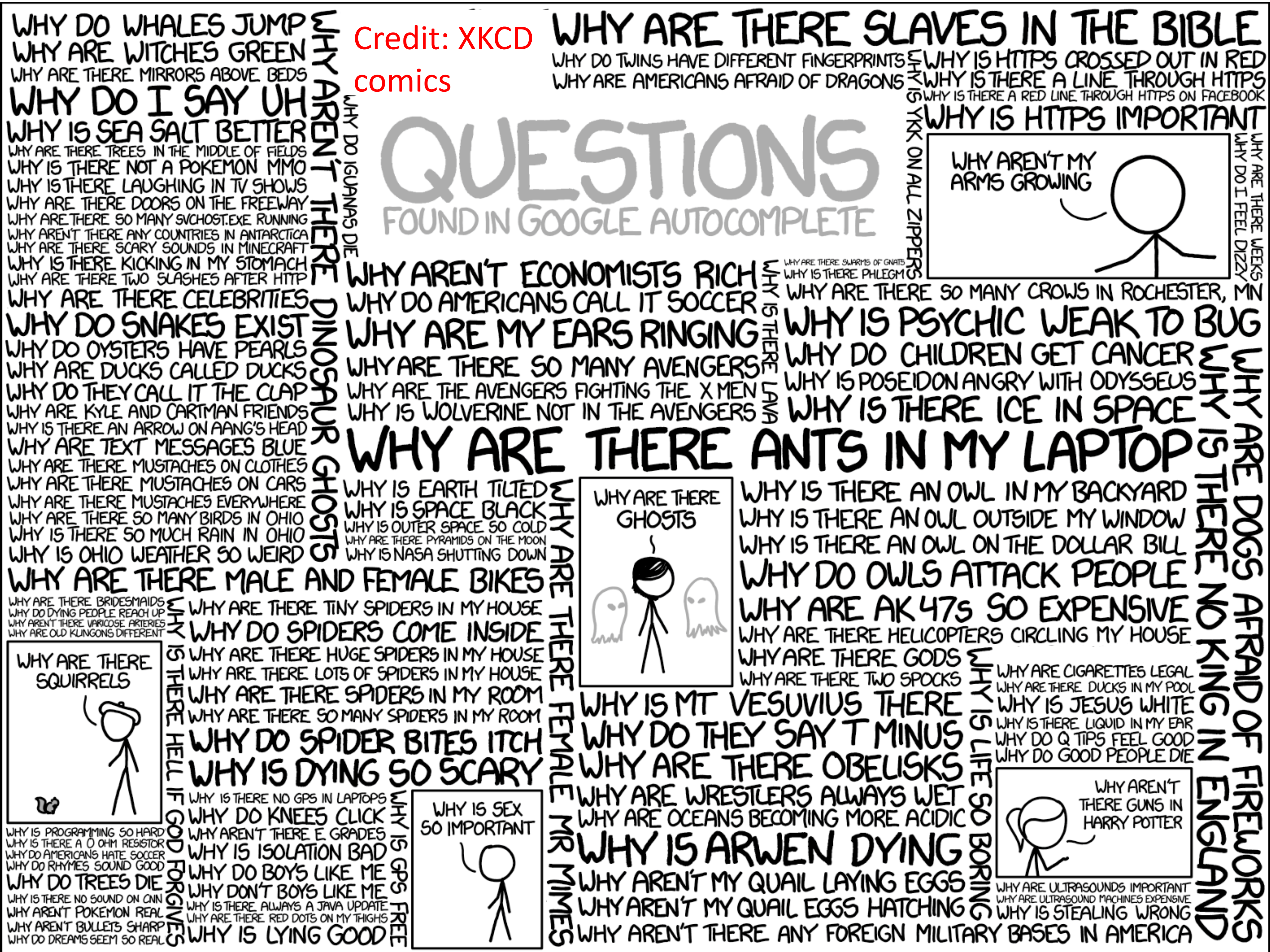
Answer: Using table one finds Prob($Z<0.5$)=0.6914. Thus the answer is 0.6914-(1-0.6914)=0.3829

(B) (4 points) On average, how many cells does one have to sample until there will be exactly 2 cells with such "close to average" *TP53* expression?

Answer: Using the negative binomial distribution one gets 2./0.3829=5.22

I can show you how to solve any HW1-HW2 problem.

Which one do you choose?



Credit: XKCD comics