

There will be no lecture on
10/15/2024

The next lecture is next Thursday
10/17/2024

Erlang Distribution

Generalizes the Exponential Distribution:
waiting time between event 0 and **event k**
(constant rate process with rate=**r**)

$$P(X > x) = \sum_{m=0}^{k-1} \frac{e^{-rx} (rx)^m}{m!} = 1 - F(x)$$

Differentiating $F(x)$ we find that all terms in the sum except the last one cancel each other:

$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{(k-1)!} \quad \text{for } x > 0 \quad \text{and } k = 1, 2, 3, \dots$$

Gamma Distribution

The random variable X with a probability density function:

$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{\Gamma(k)}, \text{ for } x > 0 \quad (4-18)$$

has a gamma random distribution with parameters $r > 0$ and $k > 0$. If k is a positive integer, then X has an Erlang distribution.



$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{\Gamma(k)}, \text{ for } x > 0$$

$$\int_0^{+\infty} f(x) dx = 1, \text{ Hence}$$

$$\Gamma(k) = \int_0^{+\infty} r^k x^{k-1} e^{-rx} dx = \int_0^{+\infty} y^{k-1} e^{-y} dy$$

Comparing with Erlang distribution
for integer k one gets

$$\Gamma(k) = (k-1)!$$

Gamma Function

The gamma function is the generalization of the factorial function for $r > 0$, not just non-negative integers.

$$\Gamma(k) = \int_0^{\infty} y^{k-1} e^{-y} dy, \quad \text{for } r > 0 \quad (4-17)$$

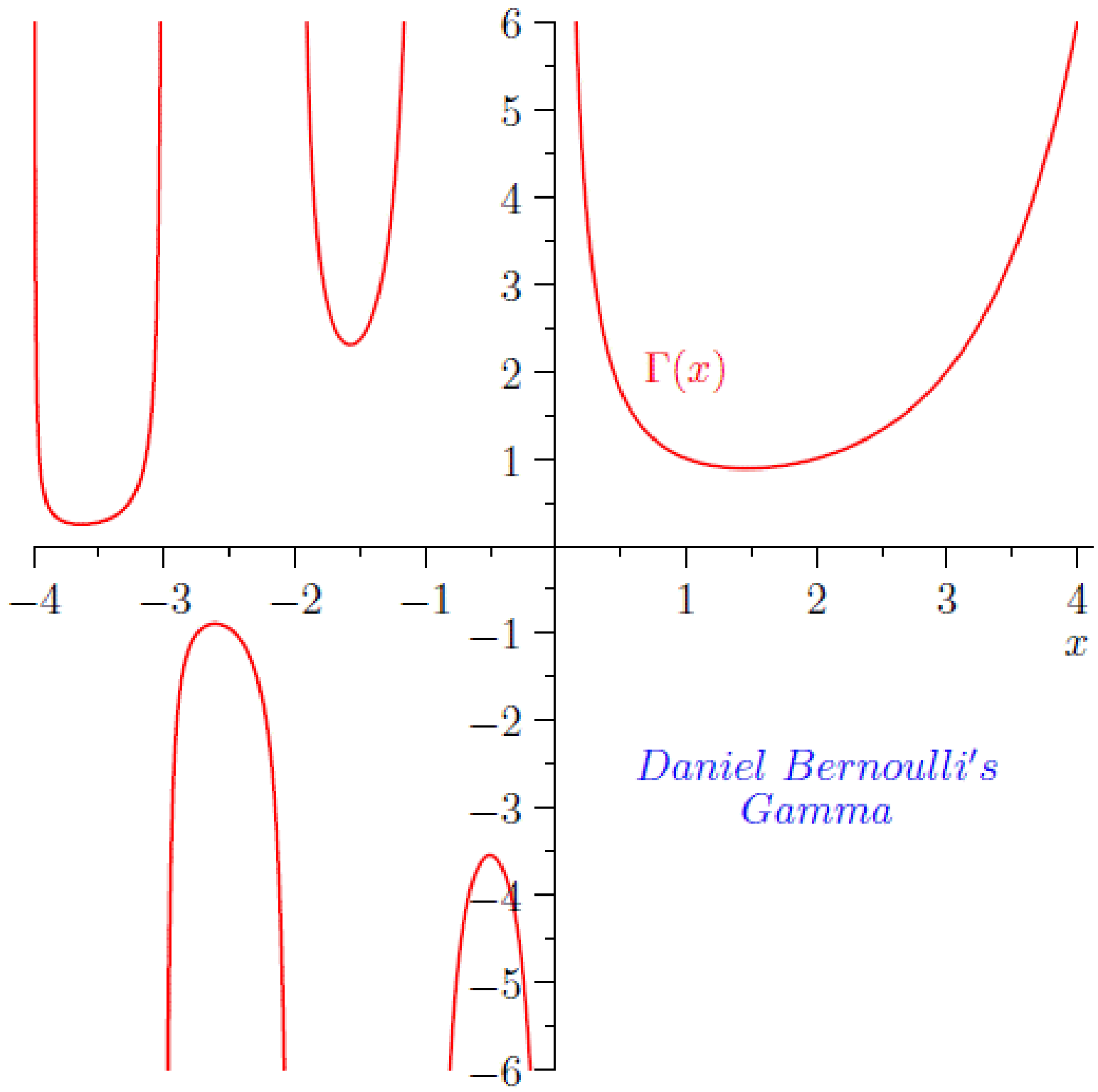
Properties of the gamma function

$$\Gamma(1) = 1$$

$$\Gamma(k) = (k - 1)\Gamma(k - 1) \quad \text{recursive property}$$

$$\Gamma(k) = (k - 1)! \quad \text{factorial function}$$

$$\Gamma\left(\frac{1}{2}\right) = \pi^{\frac{1}{2}} = 1.77 = \left(-\frac{1}{2}\right)! \quad \text{interesting fact}$$



*Daniel Bernoulli's
Gamma*

SOLO

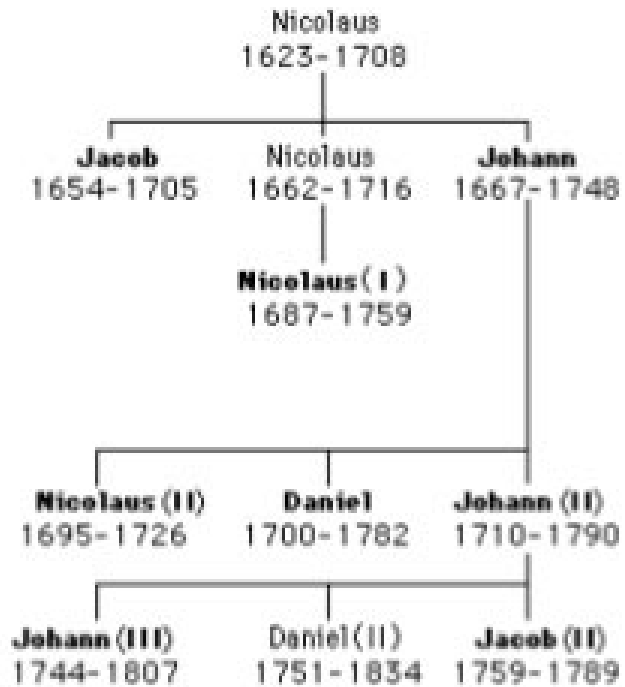
BERNOULLI FAMILY

Bernoulli trials

SOLO HERMELIN

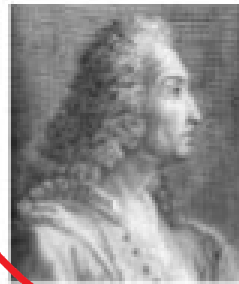
<http://www.solohermelin.com>

The Bernoulli family



Those shown in **bold** above are in our archive

See This



Jacob
1654-1705



Johann
1667-1748



Nicolaus II
1695-1720



Daniel
1700-1782



Johann II
1710-1790



Johann III
1744-1807



Jacob II
1759-1789

Gamma function

Mean & Variance of the Erlang and Gamma

- If X is an Erlang (or more generally Gamma) distributed random variable with parameters r and k ,

$$\mu = E(X) = k/r \quad \text{and} \quad \sigma^2 = V(X) = k/r^2 \quad (4-19)$$

- Generalization of exponential results:

$$\mu = E(X) = 1/r \quad \text{and} \quad \sigma^2 = V(X) = 1/r^2 \quad \text{or}$$

Negative binomial results:

$$\mu = E(X) = k/p \quad \text{and} \quad \sigma^2 = V(X) = k(1-p) / p^2$$

Gamma RV: protein concentrations

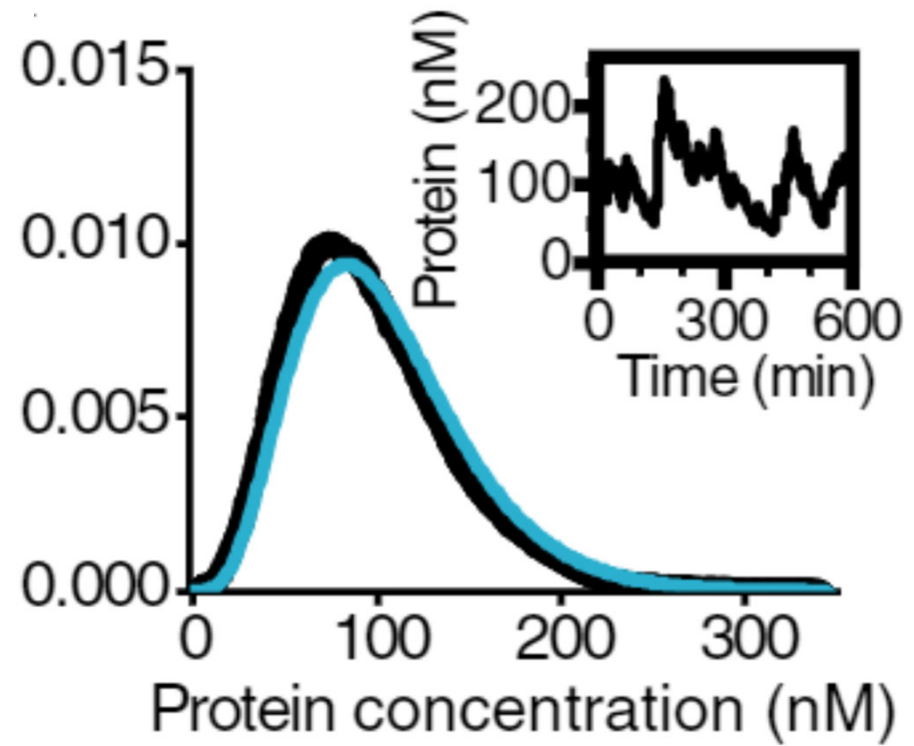
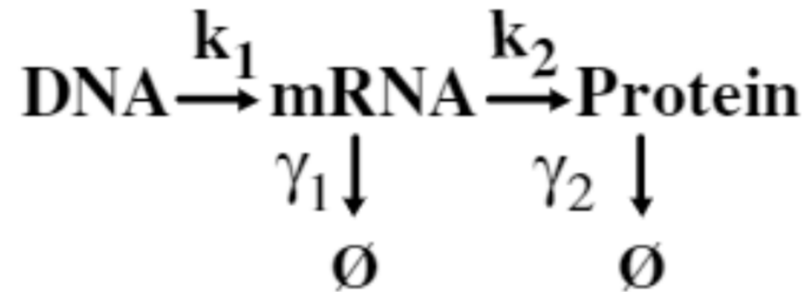
$$a = \frac{k_1}{\gamma_1}$$

the mean number of mRNA per cell cycle
(Poisson distribution)

$$b = \frac{k_2}{\gamma_2}$$

the mean number of protein molecules produced per mRNA
(Exponential distribution)

$$p(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-x/b}$$



Matlab exercise:

- Generate a sample of 100,000 variables with “Harry Potter” Gamma distribution with $r = 0.1$ and $k = 9 \frac{3}{4}$ (9.75)
- Calculate mean and compare it to k/r (Gamma)
- Calculate standard deviation and compare it to \sqrt{k}/r (Gamma)
- Plot semilog-y plots of **PDFs** and **CCDFs**.
- **Hint:** read the help page (better yet documentation webpage) for `random('Gamma'...)`: one of **their parameters is different than r**

Matlab exercise: Gamma

- `Stats=100000; r=0.1; k=9.75;`
- `r2=random('Gamma', k,1./r, Stats,1);`
- `disp([mean(r2),k./r]);`
- `disp([std(r2),sqrt(k)./r]);`
- `step=0.1; [a,b]=hist(r2,0:step:max(r2));`
- `pdf_g=a./sum(a)./step;`
- `figure;`
- `subplot(1,2,1); semilogy(b,pdf_g,'ko-'); hold on;`
- `x=0:0.01:max(r2); clear cdf_g;`
- `for m=1:length(x);`
- `cdf_g(m)=sum(r2>x(m))./Stats;`
- `end;`
- `subplot(1,2,2); semilogy(x,cdf_g,'rd-');`

Continuous Probability Distributions

Normal or Gaussian Distribution



**PAY
ATTENTION**

Normal or Gaussian Distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < \infty$$

is a **normal random variable**

with mean μ ,

and standard deviation σ

sometimes denoted as

$$N(\mu, \sigma)$$



Carl Friedrich Gauss (1777 –1855)
German mathematician

Normal Distribution

- The location and spread of the normal are independently determined by mean (μ) and standard deviation (σ)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

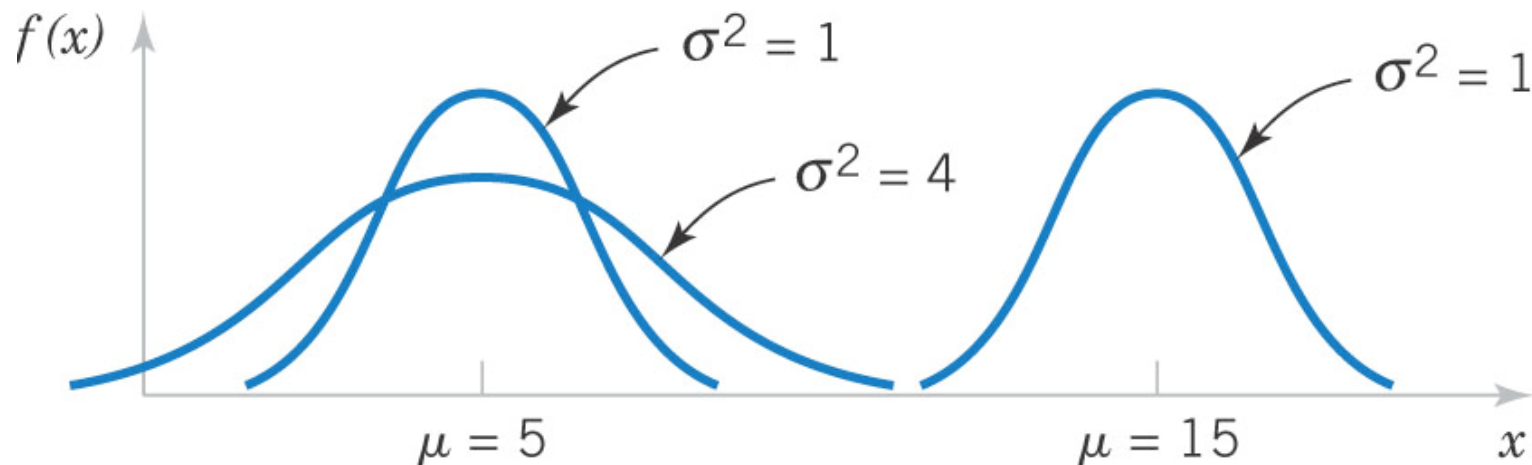


Figure 4-10 Normal probability density functions

Gaussian (Normal) distribution is very important because **any sum of many independent random variables** can be **approximated with a Gaussian**

Standard Normal Distribution

- A normal (Gaussian) random variable with

$$\mu = 0 \text{ and } \sigma^2 = 1$$

is called a **standard normal random variable** and is denoted as Z .

- The cumulative distribution function of a **standard normal random variable** is denoted as:

$$\Phi(z) = P(Z \leq z)$$

- Values are found in **Appendix A Table III** to **Montgomery and Runger textbook**

Standardizing

If X is a normal random variable with $E(X) = \mu$ and $V(X) = \sigma^2$, the random variable

$$Z = \frac{X - \mu}{\sigma} \quad (4-10)$$

is a normal random variable with $E(Z) = 0$ and $V(Z) = 1$. That is, Z is a standard normal random variable.

Suppose X is a normal random variable with mean μ and variance σ^2 .

$$\text{Then, } P(X \leq x) = P\left(\frac{X - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) = P(Z \leq z) \quad (4-11)$$

where Z is a **standard normal random variable**, and

$z = \frac{(x - \mu)}{\sigma}$ is the z-value obtained by **standardizing** x .

The probability is obtained by using Appendix Table III

$$P(X < \mu - \sigma) = P(X > \mu + \sigma) = (1 - 0.68) / 2 = 0.16 = 16\%$$

$$P(X < \mu - 2\sigma) = P(X > \mu + 2\sigma) = (1 - 0.95) / 2 = 0.023 = 2.3\%$$

$$P(X < \mu - 3\sigma) = P(X > \mu + 3\sigma) = (1 - 0.997) / 2 = 0.0013 = 0.13\%$$

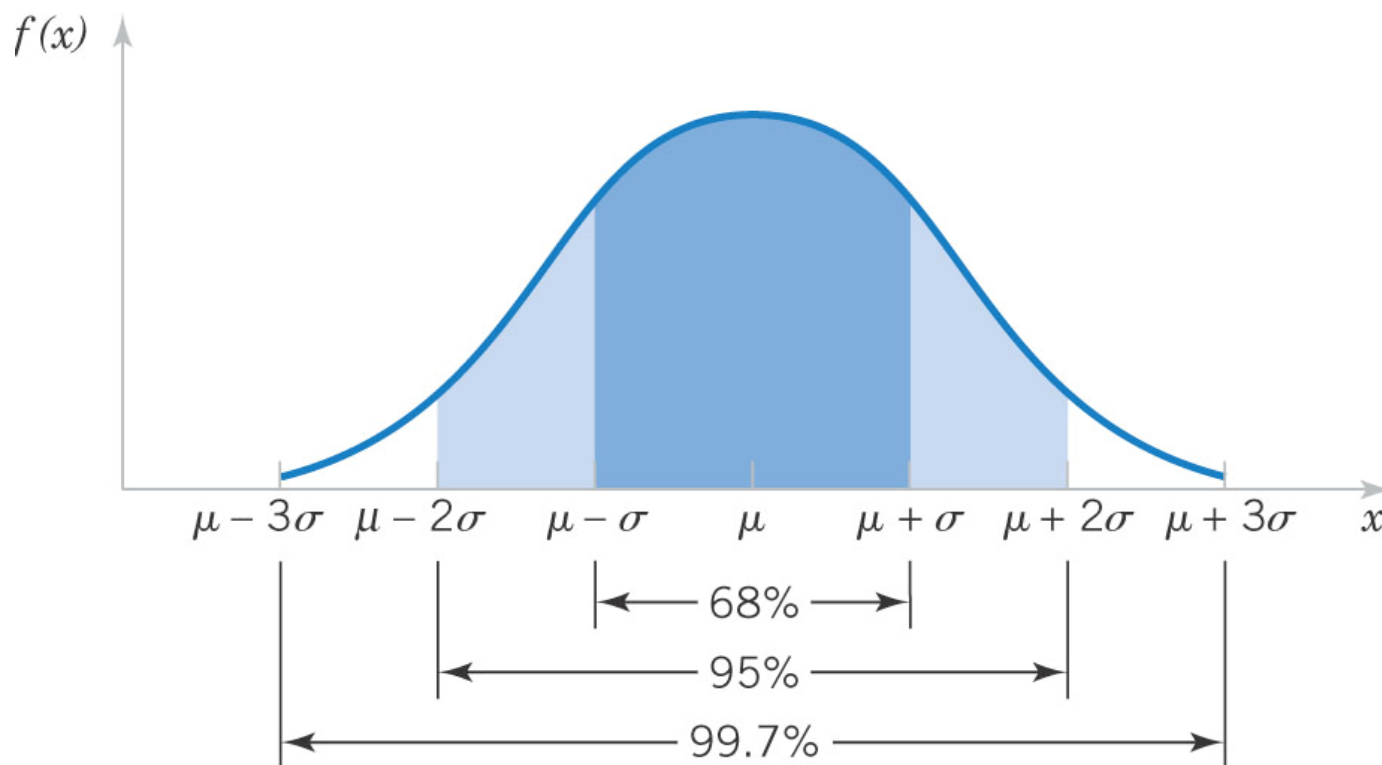


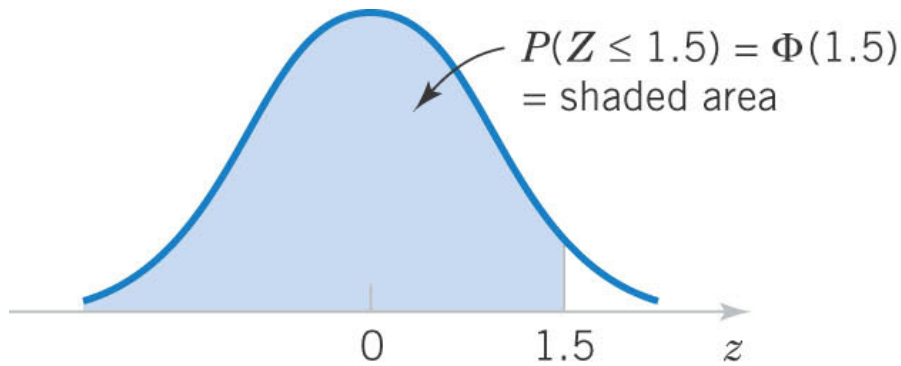
Figure 4-12 Probabilities associated with a normal distribution – well worth remembering to quickly estimate probabilities.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Standard Normal Distribution Tables

Assume Z is a standard normal random variable.

Find $P(Z \leq 1.50)$. Answer: 0.93319



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
\vdots		\vdots		
1.5	0.93319	0.93448	0.93574	0.93699

Figure 4-13 Standard normal PDF

Table III from,
Appendix A in
Montgomery
& Runger

Find $P(Z \leq 1.53)$.

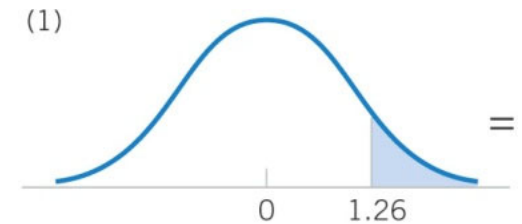
Answer: 0.93699

Find $P(Z \leq 0.02)$.

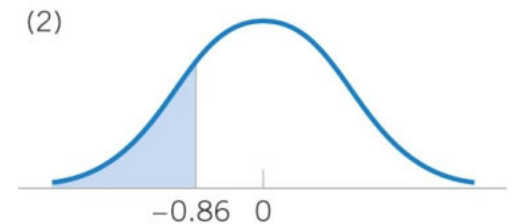
Answer: 0.50398

Standard Normal Exercises

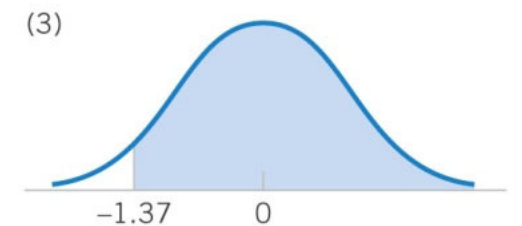
1. $P(Z > 1.26) = 1 - P(Z < 1.26) = 1 - 0.8962 =$
 $= \underline{0.1038}$



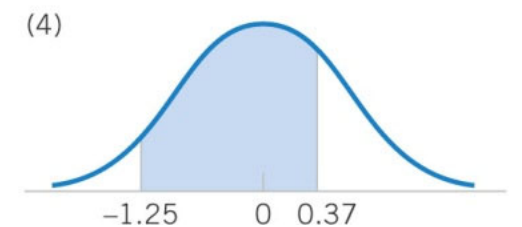
2. $P(Z < -0.86) = P(Z > 0.86) = 1 - P(Z < 0.86) =$
 $1 - 0.815 = \underline{0.195}$



3. $P(Z > -1.37) = P(Z < 1.37) = \underline{0.915}$



4. $P(-1.25 < Z < 0.37) = P(Z < 0.37) - P(Z < -1.25)$
 $= P(Z < 0.37) - P(Z > 1.25) = P(Z < 0.37) -$
 $(1 - P(Z < 1.25)) = 0.6443 - (1 - 0.8944) = \underline{0.5387}$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
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1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
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1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Credit: XKCD
comics

WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS

WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT

QUESTIONS FOUND IN GOOGLE AUTOCOMPLETE



WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY

WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD

WHY DO IGUANAS DIE
WHY AREN'T THERE DINOSAUR GHOSTS

WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS

WHY ARE THERE SWARMS OF GNATS
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE

WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN



WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
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WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS

WHY ARE DOGS AFRAID OF FIREWORKS
WHY IS THERE NO KING IN ENGLAND

WHY ARE THERE BRIDESMAIDS
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WHY AREN'T THERE VARICOSE ARTERIES
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WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY



WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD

WHY IS SEX SO IMPORTANT



WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
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WHY ARE WRESTLERS ALWAYS WET
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WHY IS ARWEN DYING
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WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA

WHY IS LIFE SO BORING



WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG

WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL

Matlab exercise:
plot PDF of the Gaussian distribution
with mu=3; sigma=2

calculate mean, standard deviation
and variance,

Linear-y and Semilog-y plots of PDF

Hint:

Generate Standard normal
distribution using

randn(Stats,1) then

multiply and add using sigma, mu

Matlab exercise solution

- **Stats=100000;**
- **mu=3; sigma=2;**
- **r1=sigma.*randn(Stats,1)+mu;**
- **step=0.1;**
- **[a,b]=hist(r1,(mu-10.*sigma):step:(mu+10.*sigma));**
- **pdf_n=a./sum(a)./step;**
- **figure; subplot(1,2,1); plot(b,pdf_n,'ko-');**
- **subplot(1,2,2); semilogy(b,pdf_n,'ko-');**

Credit: XKCD
comics

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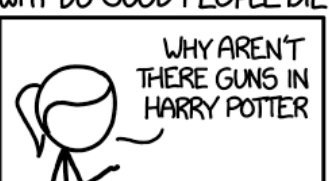
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WHY ARE CIGARETTES LEGAL
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WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE



WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG

Range	The expected fraction of population inside the range	Approximate expected frequency outside the range	The approximate frequency for daily event
$\mu \pm 0.5\sigma$	0.382924922548026	2 in 3	Four or five times a week
$\mu \pm 1\sigma$	0.682689492137086	1 in 3	Twice a week
$\mu \pm 1.5\sigma$	0.866385597462284	1 in 7	Weekly
$\mu \pm 2\sigma$	0.954499736103642	1 in 22	Every month (three weeks)
$\mu \pm 2.5\sigma$	0.987580669348448	1 in 81	Quarterly
$\mu \pm 3\sigma$	0.997300203936740	1 in 370	Yearly
$\mu \pm 3.5\sigma$	0.999534741841929	1 in 2149	Every six years
$\mu \pm 4\sigma$	0.999936657516334	1 in 15787	Every 43 years (twice in a lifetime)
$\mu \pm 4.5\sigma$	0.999993204653751	1 in 147160	Every 403 years (once in the modern era)
$\mu \pm 5\sigma$	0.999999426696856	1 in 1744278	Every 4776 years (once in recorded history of civilization)
$\mu \pm 5.5\sigma$	0.999999962020875	1 in 26330254	Every 72090 years (thrice in history of modern humankind)
$\mu \pm 6\sigma$	0.999999998026825	1 in 506797346	Every 1.38 million years (twice in history of humans and their ancestors)
$\mu \pm 6.5\sigma$	0.999999999919680	1 in 12450197393	Every 34 million years (twice since the extinction of dinosaurs)
$\mu \pm 7\sigma$	0.99999999997440	1 in 390682215445	Every 1.07 billion years (four times in history of Earth)

Source: Wikipedia

DATA SCIENCE
DISCOVERY

Human Impact of Probabilities
STAT 107: Data Science Discovery

Business buzzword: Six Sigma



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Six Sigma

From Wikipedia, the free encyclopedia

For other uses, see [Sigma 6](#).

Six Sigma is a set of techniques and tools for process improvement. It was introduced by engineer Bill Smith while working at [Motorola](#) in 1986.^{[1][2]} [Jack Welch](#) made it central to his business strategy at [General Electric](#) in 1995.^[3] Today, it is used in many industrial sectors.^[4]

Business literature defined **six sigma**
as no more than **3.4 defective products**
per million

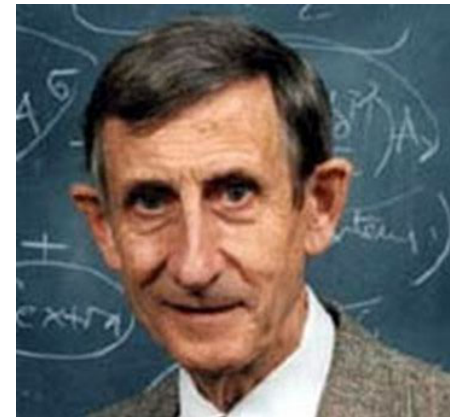
Fact checking Six Sigma

- $1-\text{normcdf}(z)$ computes CCDF in Matlab
- I expected $\text{Prob}(X-\mu > 6\sigma) = 3.4$ defective products per million
- $\text{Prob}(X-\mu > 6\sigma) = 1-\text{normcdf}(6) = 9.8659e-10$
~1 per billion
- What Six Sigma should be called? Find z such that $\text{Prob}(X-\mu > z \cdot \sigma) = 3.4$ per million
- 5 is not enough: $1-\text{normcdf}(5) = 2.8665e-07$
- 4 is too much: $1-\text{normcdf}(4) = 3.1671e-05$
- 4.5 is perfect: $1-\text{normcdf}(4.5) = 3.3977e-06$
- Should be called **Four-point-Five Sigma** but not as catchy

What's wrong with Six Sigma?

- “Motorola has determined, through years of process and data collection, that processes vary and drift over time – what they call the Long-Term Dynamic Mean Variation. This variation typically falls **between 1.4 and 1.6.**” They shifted their sigma down by **1.5.**
- The statistician [Donald J. Wheeler](#) has dismissed the **1.5 sigma shift** as "goofy" because of its arbitrary nature.
- A [Fortune](#) article stated that "of **58 large companies** that have announced Six Sigma programs, **91 percent have trailed (performed below)** the S&P 500 index since"

- **Freeman Dyson** (a famous theoretical physicist) once sat on a committee reviewing the Department of Energy Joint Genomics Institute (DOE JGI)
- Motorola sent their **six-sigma preacher** Freeman Dyson asked him:
 - **D: Can you explain me what is six-sigma?**
 - P: Mumbling something about it being the gold standard of reliability
 - **D: Can you at least define one-sigma?**
 - P: Silence
- Six-sigma was **never implemented at JGI**



Born:
December 15, 1923,
Crowthorne, UK
Died:
February 28, 2020
Princeton, NJ USA

Dyson's legacy

- **Seminal contributions to quantum mechanics**
- The Origin of Life:
Cells → Enzymes → DNA/RNA later
First proposed by Alexander Oparin in 1922
- Dyson sphere:
Completely
captures light from a star
- Dyson tree:
genetically engineered
tree growing inside a
comet

