There will be no lecture on 10/15/2024

The next lecture is next Thursday 10/17/2024

Erlang Distribution

Generalizes the Exponential Distribution:

waiting time between event 0 and event k

(constant rate process with rate=r)

$$P(X > x) = \sum_{m=0}^{k-1} \frac{e^{-rx}(rx)^m}{m!} = 1 - F(x)$$

Differentiating $F(x)$ we find that all terms in the sum except the last one cancel each other:

$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{(k-1)!} \quad \text{for } x > 0 \text{ and } k = 1, 2, 3, \ldots$$

Gamma Distribution

The random variable X with a probability density function:

$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{\Gamma(k)}, \text{ for } x > 0 \qquad (4\text{-}18)$$

has a gamma random distribution with parameters r > 0 and $k > 0$. If k is a positive integer, then X has an Erlang distribution.

$$f(x) = \frac{r^k x^{k-1} e^{-rx}}{\Gamma(k)}, \ \text{for} \ x > 0$$

$$\int_0^{+\infty} f(x)\,dx = 1 , \quad \text{Hence}$$

$$\Gamma(k) = \int_0^{+\infty} r^k x^{k-1} e^{-rx} dx = \int_0^{+\infty} y^{k-1} e^{-y} dy$$

Comparing with Erlang distribution

for integer k one gets

$$\Gamma(k) = (k-1)!$$

Gamma Function

The gamma function is the generalization of the factorial function for r > 0, not just non-negative integers.

$$\Gamma(k) = \int_0^{\infty} y^{k-1} e^{-y} dy, \qquad \text{for } r > 0 \qquad (4\text{−}17)$$

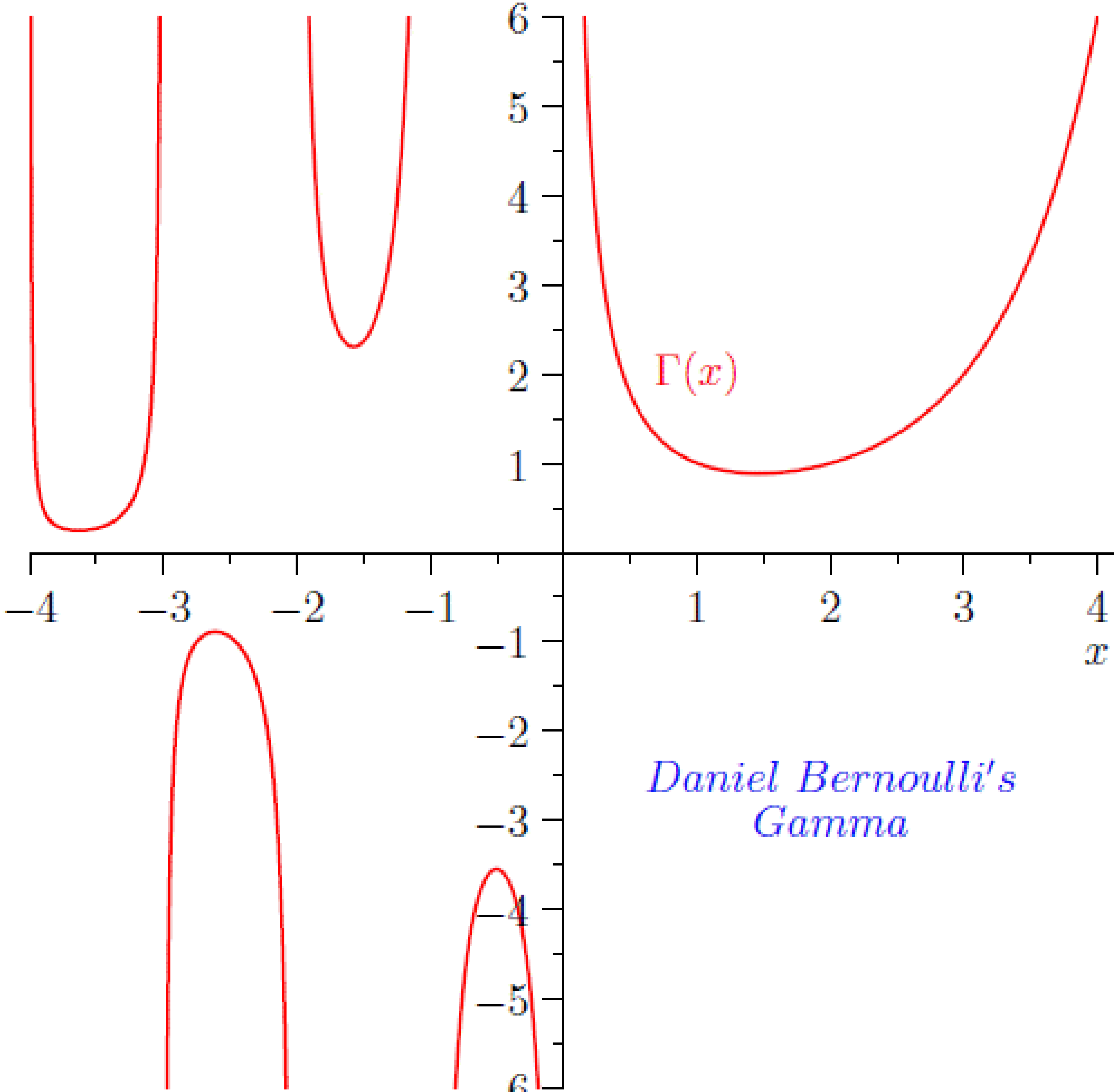
Properties of the gamma function

$$\Gamma(1) = 1$$

$$\Gamma(k) = (k-1)\Gamma(k-1) \quad \text{recursive property}$$

$$\Gamma(k) = (k-1)! \qquad \text{factorial function}$$

$$\Gamma\left(\frac{1}{2}\right) = \pi^{\frac{1}{2}} = 1.77 = \left(-\frac{1}{2}\right)! \qquad \text{interesting fact}$$



Γ(x)
x
Daniel Bernoulli's Gamma

SOLO

BERNOULLI FAMILY

Bernoulli trials

SOLO HERMELIN

http://www.solohermelin.com

The Bernoulli family

Nicolaus
1623-1708

Jacob 1654-1705 | Nicolaus 1662-1716 | **Johann** 1667-1748

Nicolaus (I) 1687-1759

Nicolaus (II) 1695-1726 | **Daniel** 1700-1782 | **Johann (II)** 1710-1790

Johann (III) 1744-1807 | Daniel (II) 1751-1834 | **Jacob (II)** 1759-1789

Those shown in **bold** above are in our archive

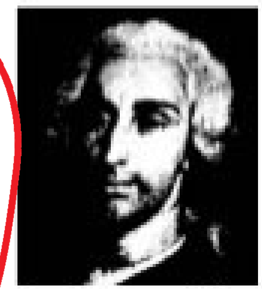
Run This

Jacob
1654-1705

Johann
1667-1748

Nicolaus II
1695-1720

Daniel
1700-1782



Johann II
1710-1790

Johann III
1744-1807

Jacob II
1759-1789

Gamma function

Mean & Variance of the Erlang and Gamma

- If X is an Erlang (or more generally Gamma) distributed random variable with parameters r and k,

  $\mu = E(X) = k/\mathrm{r}$ and $\sigma^2 = V(X) = \mathrm{k}/\mathrm{r}^2$ (4-19)

- Generalization of exponential results:

  $\mu = E(X) = 1/\mathrm{r}$ and $\sigma^2 = V(X) = 1/\mathrm{r}^2$ or

  Negative binomial results:

  $\mu = E(X) = k/p$ and $\sigma^2 = V(X) = k(1-p) / p^2$

Gamma RV: protein concentrations

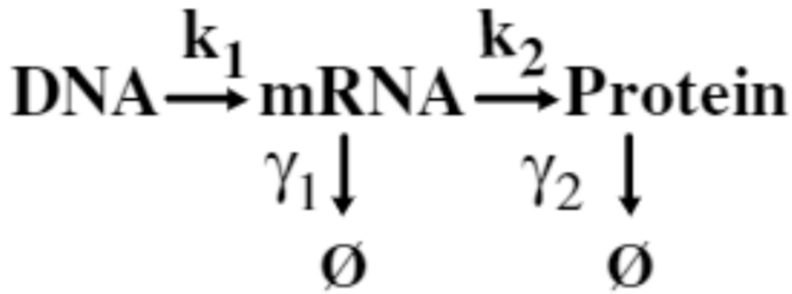
$$a = \frac{k_1}{\gamma_1}$$

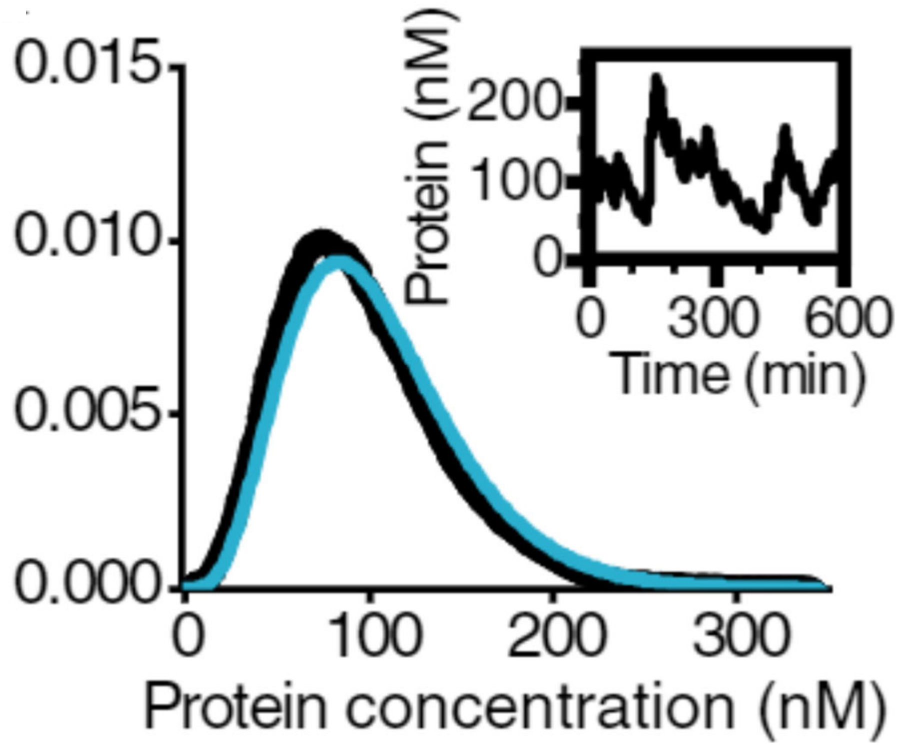
the mean number of mRNA per cell cycle (Poisson distribution)

$$b = \frac{k_2}{\gamma_2}$$

the mean number of protein molecules produced per mRNA (Exponential distribution)

$$p(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-x/b}.$$





Friedman N, Cai L, Xie XS. Phys Rev Lett. 2006;97: 168302.

Matlab exercise:

- Generate a sample of 100,000 variables with "Harry Potter" Gamma distribution with r =0.1 and k=9 ¾ (9.75)
- Calculate mean and compare it to k/r (Gamma)
- Calculate standard deviation and compare it to sqrt(k)/r (Gamma)
- Plot semilog-y plots of PDFs **and CCDFs.**
- Hint: read the help page (better yet documentation webpage) for random('Gamma'…): one of their parameters is different than r

Matlab exercise: Gamma

```
• Stats=100000; r=0.1; k=9.75;
• r2=random('Gamma', k,1./r, Stats,1);
• disp([mean(r2),k./r]);
• disp([std(r2),sqrt(k)./r]);
• step=0.1; [a,b]=hist(r2,0:step:max(r2));
• pdf_g=a./sum(a)./step;
• figure;
• subplot(1,2,1); semilogy(b,pdf_g,'ko-'); hold on;
• x=0:0.01:max(r2); clear cdf_g;
• for m=1:length(x);
•     cdf_g(m)=sum(r2>x(m))./Stats;
• end;
• subplot(1,2,2); semilogy(x,cdf_g,'rd-');
```

Continuous Probability Distributions

Normal or Gaussian Distribution


PAY
ATTENTION

Normal or Gaussian Distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$-\infty < x < \infty$

is a normal random variable

with mean μ,

and standard dewviation σ

sometimes denoted as

$N(\mu, \sigma)$

Carl Friedrich Gauss (1777 –1855)
German mathematician

Normal Distribution

- The location and spread of the normal are independently determined by mean (μ) and standard deviation (σ)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

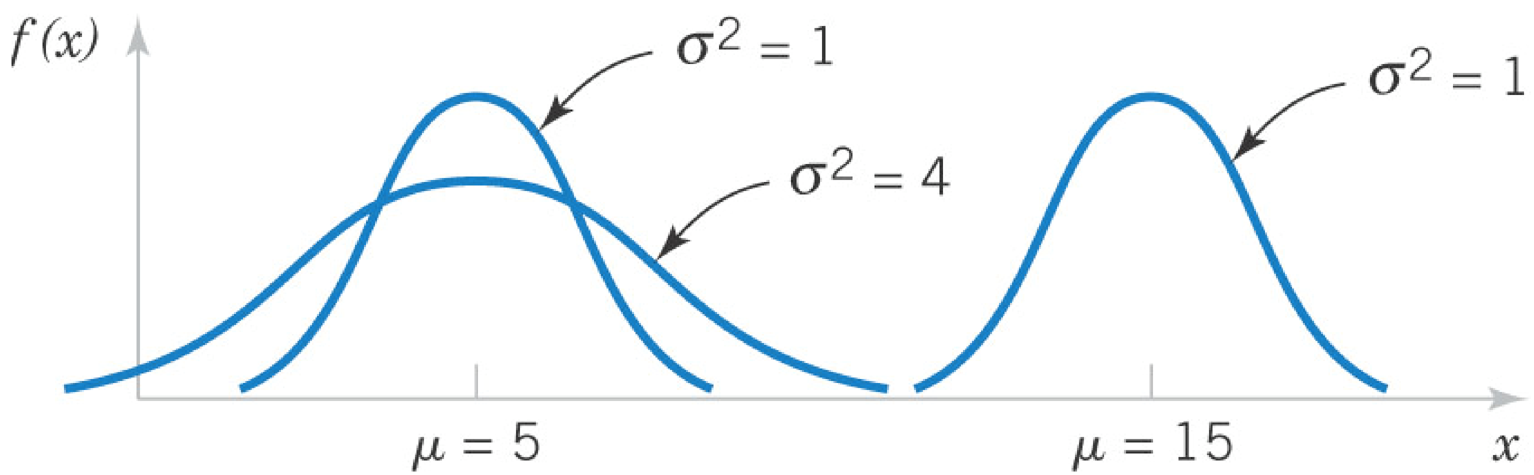


Figure 4-10 Normal probability density functions

Gaussian (Normal) distribution is very important because any sum of many independent random variables can be approximated with a Gaussian

Standard Normal Distribution

- A normal (Gaussian) random variable with

$$\mu = 0 \text{ and } \sigma^2 = 1$$

  is called a standard normal random variable and is denoted as Z.

- Thed cumulative distribution function of a standard normal random variable is denoted as:

$$\Phi(z) = P(Z \leq z)$$

- Values are found in Appendix A Table III to Montgomery and Runger textbook

Standardizing

If X is a normal random variable with $E(X) = \mu$ and $V(X) = \sigma^2$, the random variable

$$Z = \frac{X - \mu}{\sigma} \tag{4-10}$$

is a normal random variable with $E(Z) = 0$ and $V(Z) = 1$. That is, Z is a standard normal random variable.

Suppose X is a normal random variable with mean μ and variance σ^2.

Then, $P(X \le x) = P\left(\frac{X-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) = P(Z \le z)$ (4-11)

where Z is a standard normal random variable, and

$z = \frac{(x-\mu)}{\sigma}$ is the z-value obtainedby standardizing x.

The probability is obtained by using Appendix Table III

$$P(X < \mu - \sigma) = P(X > \mu + \sigma) = (1\text{-}0.68)/2 = 0.16 = 16\%$$

$$P(X < \mu - 2\sigma) = P(X > \mu + 2\sigma) = (1\text{-}\ 0.95)/2 = 0.023 = 2.3\%$$

$$P(X < \mu - 3\sigma) = P(X > \mu + 3\sigma) = (1\text{-}\ 0.997)/2 = 0.0013 = 0.13\%$$

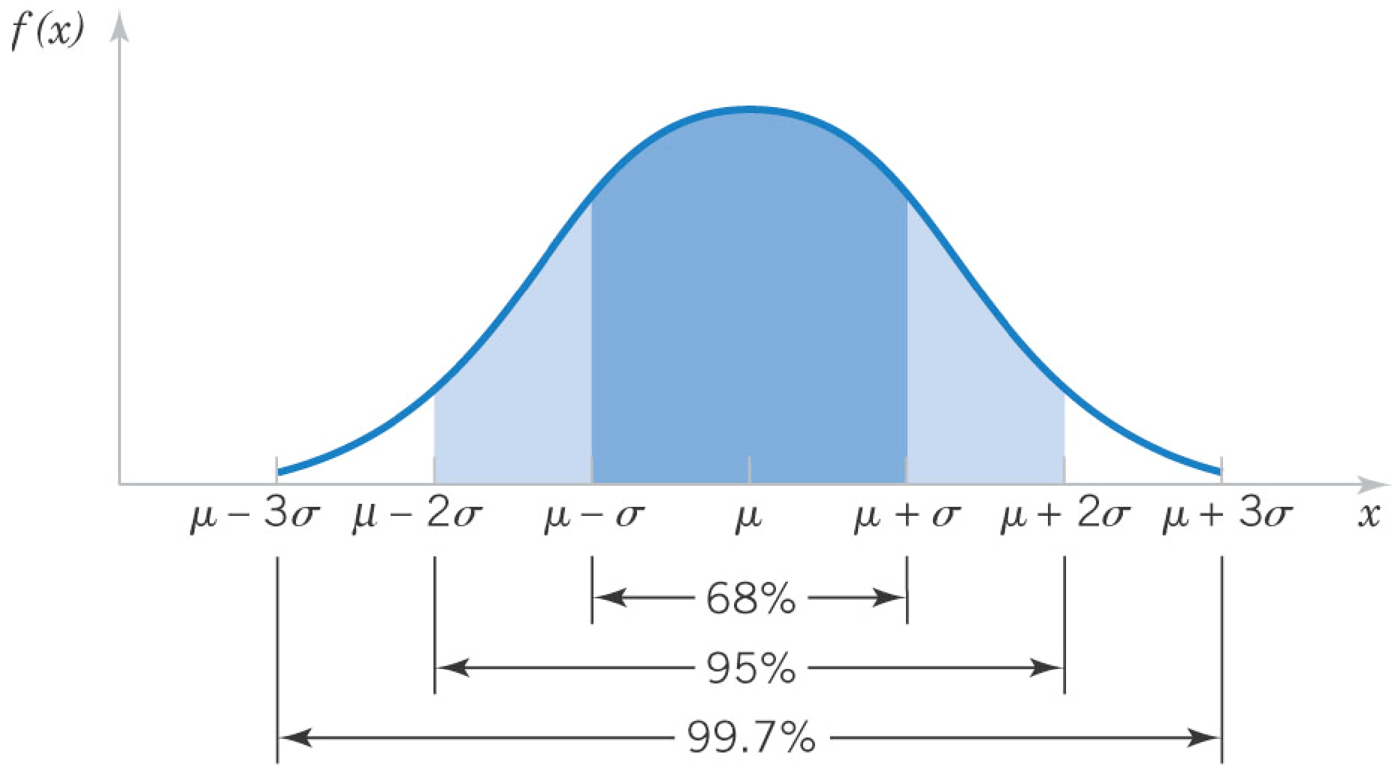


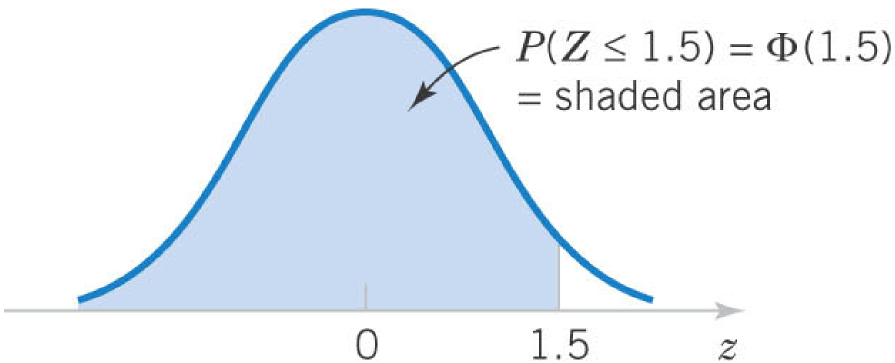
Figure 4-12 Probabilities associated with a normal distribution – well worth remembering to quickly estimate probabilities.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Standard Normal Distribution Tables

Assume Z is a standard normal random variable.

Find $P(Z \leq 1.50)$. Answer: 0.93319



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

Figure 4-13 Standard normal PDF

Table III from, Appendix A in Montgomery & Runger

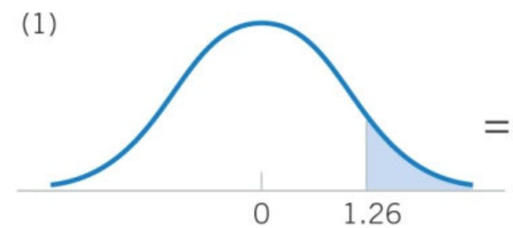
Find $P(Z \leq 1.53)$. Answer: 0.93699

Find $P(Z \leq 0.02)$. Answer: 0.50398

Standard Normal Exercises

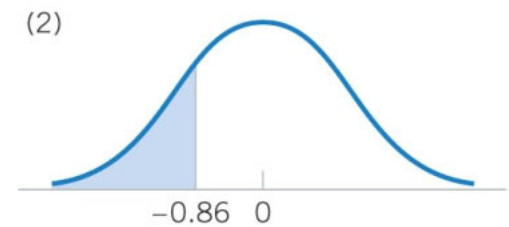
1. $P(Z > 1.26) = 1 - P(Z < 1.26) = 1 - 0.8962 =$

   $= \underline{0.1038}$

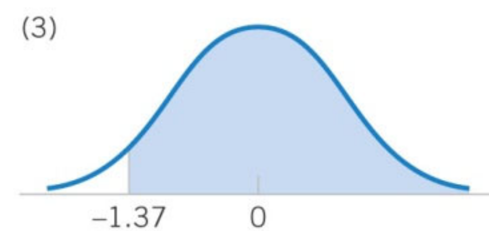


2. $P(Z < -0.86) = P(Z > 0.86) = 1 - P(Z < 0.86) =$

   $1 - 0.815 = \underline{0.195}$



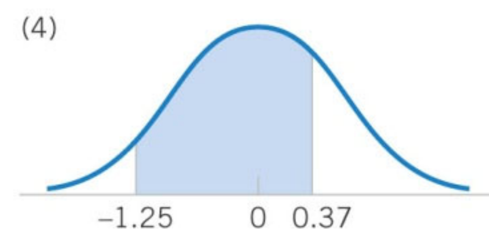
3. $P(Z > -1.37) = P(Z < 1.37) = \underline{0.915}$



4. $P(-1.25 < Z < 0.37) = P(Z < 0.37) - P(Z < -1.25)$

   $= P(Z < 0.37) - P(Z > 1.25) = P(Z < 0.37) -$

   $(1 - P(Z < 1.25)) = 0.6443 - (1 - 0.8944) = \underline{0.5387}$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
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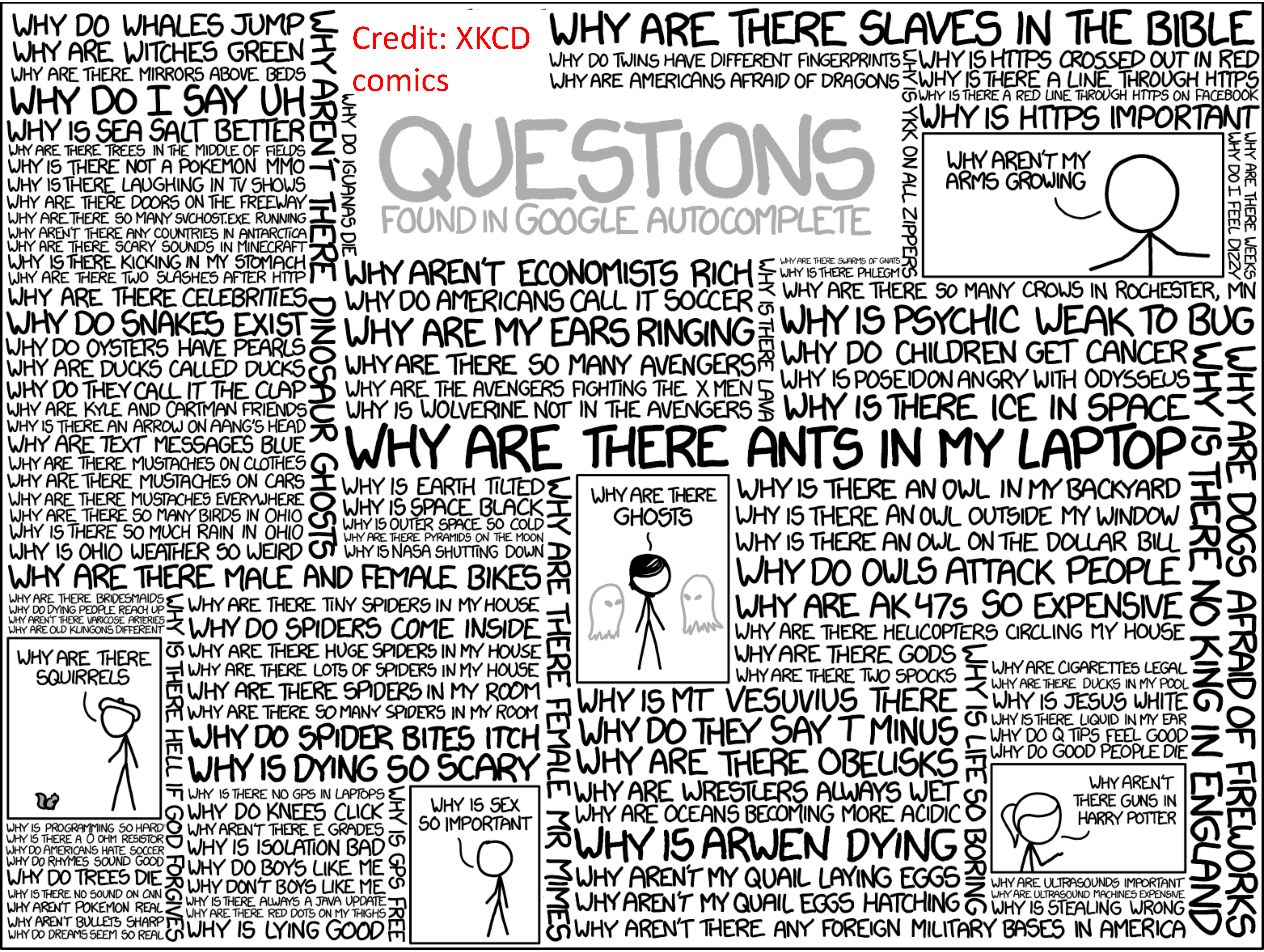
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Credit: XKCD comics



Matlab exercise:

plot PDF of the Gaussian distribution

with **mu=3; sigma=2**

calculate mean, standard deviation and variance,

Linear-y and Semilog-y plots of PDF

Hint:

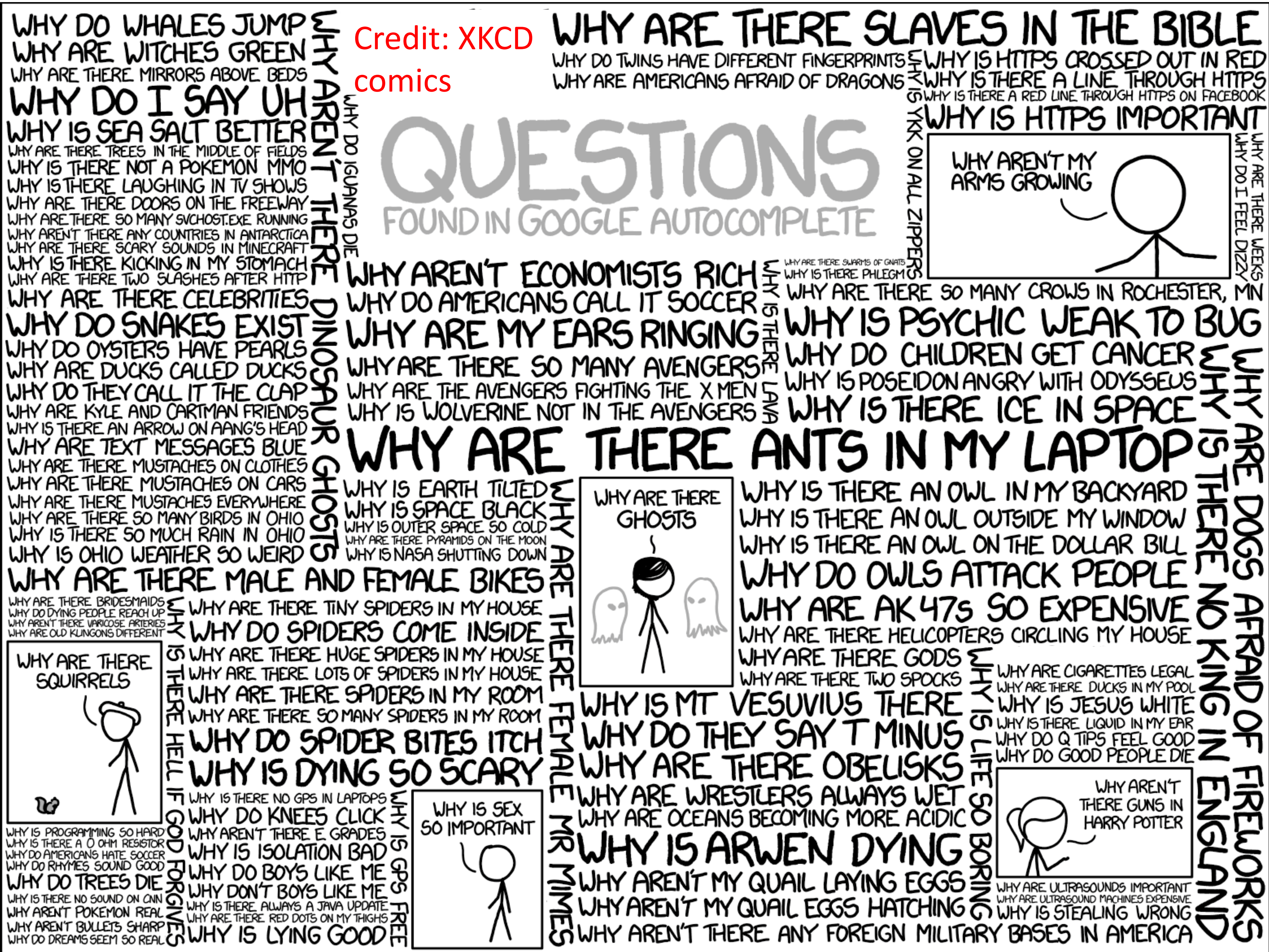
Generate Standard normal distribution using

randn(Stats,1) then

multiply and add using sigma, mu

Matlab exercise solution

- **Stats=100000;**
- **mu=3; sigma=2;**
- **r1=sigma.*randn(Stats,1)+mu;**
- **step=0.1;**
- **[a,b]=hist(r1,(mu-10.*sigma):step:(mu+10.*sigma));**
- **pdf_n=a./sum(a)./step;**
- **figure; subplot(1,2,1); plot(b,pdf_n,'ko-');**
- **subplot(1,2,2); semilogy(b,pdf_n,'ko-');**

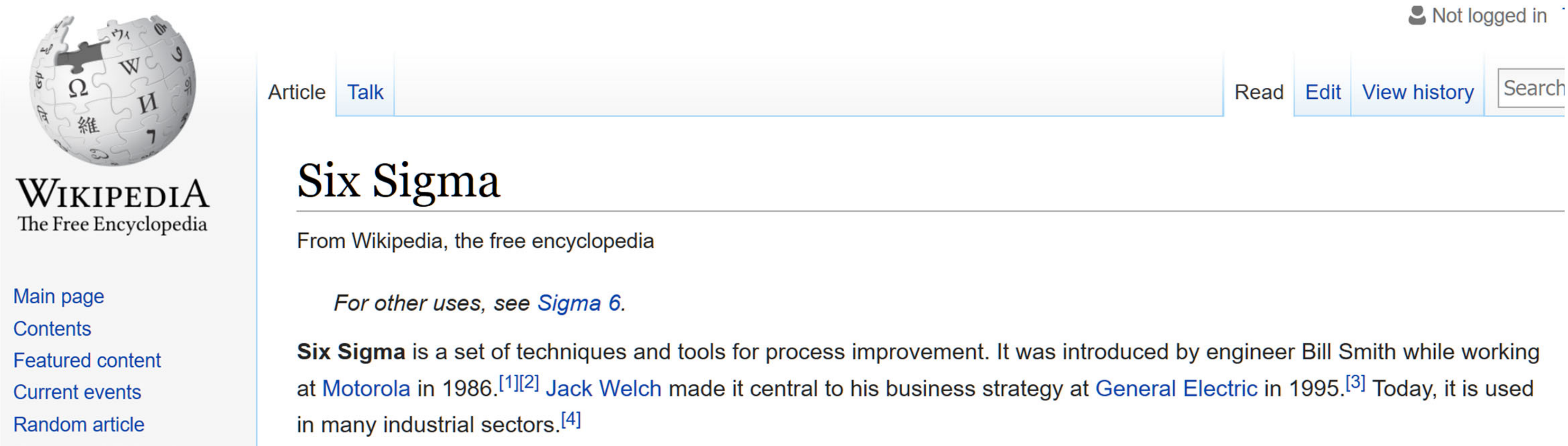


Credit: XKCD comics

Range	The expected fraction of population inside the range	Approximate expected frequency outside the range	The approximate frequency for daily event
μ ± 0.5σ	0.382924922548026	2 in 3	Four or five times a week
μ ± 1σ	0.682689492137086	1 in 3	Twice a week
μ ± 1.5σ	0.866385597462284	1 in 7	Weekly
μ ± 2σ	0.954499736103642	1 in 22	Every month (three weeks)
μ ± 2.5σ	0.987580669348448	1 in 81	Quarterly
μ ± 3σ	0.997300203936740	1 in 370	Yearly
μ ± 3.5σ	0.999534741841929	1 in 2149	Every six years
μ ± 4σ	0.999936657516334	1 in 15787	Every 43 years (twice in a lifetime)
μ ± 4.5σ	0.999993204653751	1 in 147160	Every 403 years (once in the modern era)
μ ± 5σ	0.999999426696856	1 in 1744278	Every 4776 years (once in recorded history of civilization)
μ ± 5.5σ	0.999999962020875	1 in 26330254	Every 72090 years (thrice in history of modern humankind)
μ ± 6σ	0.999999998026825	1 in 506797346	Every 1.38 million years (twice in history of humans and their ancestors)
μ ± 6.5σ	0.999999999919680	1 in 12450197393	Every 34 million years (twice since the extinction of dinosaurs)
μ ± 7σ	0.999999999997440	1 in 390682215445	Every 1.07 billion years (four times in history of Earth)

Source: Wikipedia

Business buzzword: Six Sigma



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Six Sigma

From Wikipedia, the free encyclopedia

For other uses, see Sigma 6.

Six Sigma is a set of techniques and tools for process improvement. It was introduced by engineer Bill Smith while working at Motorola in 1986.[1][2] Jack Welch made it central to his business strategy at General Electric in 1995.[3] Today, it is used in many industrial sectors.[4]

Business literature defined six sigma as no more than 3.4 defective products per million

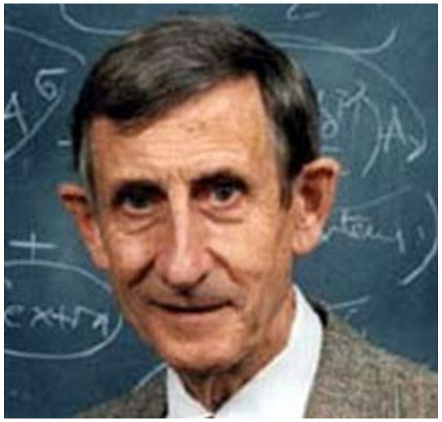
Fact checking Six Sigma

- 1-normcdf(z) computes CCDF in Matlab
- I expected Prob(X-μ>6σ)=3.4 defective products per million
- Prob(X-μ>6σ)=1-normcdf(6) = 9.8659e-10 ~1 per billion
- What Six Sigma should be called? Find z such that Prob(X-μ>z · σ)=3.4 per million
- 5 is not enough: 1-normcdf(5) = 2.8665e-07
- 4 is too much: 1-normcdf(4) = 3.1671e-05
- 4.5 is perfect: 1-normcdf(4.5) =3.3977e-06
- Should be called Four-point-Five Sigma but not as catchy

What’s wrong with Six Sigma?

- “Motorola has determined, through years of process and data collection, that processes vary and drift over time – what they call the Long-Term Dynamic Mean Variation. This variation typically falls between 1.4 and 1.6.” They shifted their sigma down by 1.5.
- The statistician Donald J. Wheeler has dismissed the 1.5 sigma shift as "goofy" because of its arbitrary nature.
- A *Fortune* article stated that "of 58 large companies that have announced Six Sigma programs, 91 percent have trailed (performed below) the S&P 500 index since"

- Freeman Dyson (a famous theoretical physicist) once sat on a committee reviewing the Department of Energy Joint Genomics Institute (DOE JGI)
- Motorola sent their six-sigma preacher Freeman Dyson asked him:
  - D: Can you explain me what is six–sigma?
    - P: Mumbling something about it being the gold standard of reliability
  - D: Can you at least define one-sigma?
    - P: Silence
- Six-sigma was never implemented at JGI



Born:
December 15, 1923,
Crowthorne, UK
Died:
February 28, 2020
Princeton, NJ USA

Dyson's legacy

- **<u>Seminal contributions to quantum mechanics</u>**
- <u>The Origin of Life:</u>
  Cells → Enzymes → DNA/RNA later
  First proposed by Alexander Oparin in 1922
- Dyson sphere:
  Completely
  captures light from a star
- Dyson tree:
  genetically engineered
  tree growing inside a
  comet