Homework #3

Please present 4 significant figures in your final answers for probabilities

1. **(20 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by a formula $f_{XY}(x, y) = c*(x + y)$. Determine the following:
   a) **(2 points)** Find c

   b) **(2 points)** Find probability of the event where X = 1 and Y $\leqslant$ 2

   c) **(2 points)** Find marginal probability $P_Y(Y = 3)$

   d) **(2 points)** Marginal probability distribution of the random variable X

   e) **(2 points)** E(X), E(Y), V(X), and V(Y)

   f) **(2 points)** Find conditional probability distribution of Y given that X = 1

   g) **(2 points)** Conditional probability distribution of X given that Y = 3

   h) **(2 points)** Are X and Y independent?

   i) **(2 points)** What is the covariance for X and Y?

   j) **(2 points)** What is the correlation for X and Y?

2. **(6 points)** A random variable X has density function $f(X = x) = c(x + x^3)$ for $x \in [0,1]$ and $f(X = x) = 0$ otherwise.
   a) **(3 points)** Determine c.

   b) **(3 points)** Compute E(1/X)

3. **(10 points)** Toss an unfair coin 4 times. The probability of head is 0.6 and that of tail is 0.4. Let X be the total number of heads among the first two tosses and Y the total number of heads among the last two tosses.

   a) **(4 points)** Write down the joint probability mass fraction of X and Y.

   b) **(2 points)** Are X and Y independent? Please explain.

c) **(4 points)** Compute the conditional probability $P(X \geq Y | X \geq 1)$

4. **(6 points)** A random variable X is the average of p independent random variables X_k, i.e., $X = \frac{1}{p}\sum_{k=0}^{p} X_k$, Calculate the expectation and the variance of X for three different cases:

a) **(2 points)** When all X_k are independent uniform continuous random variables in the interval (0,1)

b) **(2 points)** When all X_k are independent exponential random variables with PDF $P(X_k = x) = \lambda_k e^{-\lambda_k x}$

c) **(2 points)** When all X_k are Independent normal random variables but each one has its own mean μ_k and its own standard deviation σ_k

5. **(4 points)** Suppose random variables X, Y have standard derivations, $\sigma_X = 2$ and $\sigma_Y = 6$, respectively, and correlation coefficient corr(X, Y) = −1/3.

(a) **(2 points)** Find cov(X, Y).

(b) **(2 points)** Find Var(3X − 2Y).

6. **(12 points)** Time interval separating subsequent bus arrivals at a stop is an exponential random variable with mean 20 minutes. Steve and Andrew work at the same place and each will be late to work unless they board a bus on or before 8:40am. Steve comes to the bus stop exactly at 8am. Andrew also comes to the same bus stop but at a random time, uniformly distributed between 8am and 8:30am. Both of them take the first bus that arrives.

(a) **(4 points)** What is the probability that Steve will be late for work tomorrow?

(b) **(4 points)** What is the probability that Andrew will be late for work tomorrow?

(c) **(4 points)** What is the probability that Steve and Andrew will ride the same bus?