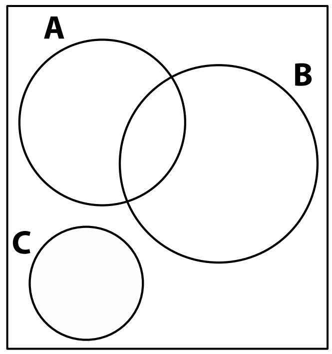
Homework #1

1. (**10 points**) If $P(A) = 0.2$, $P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

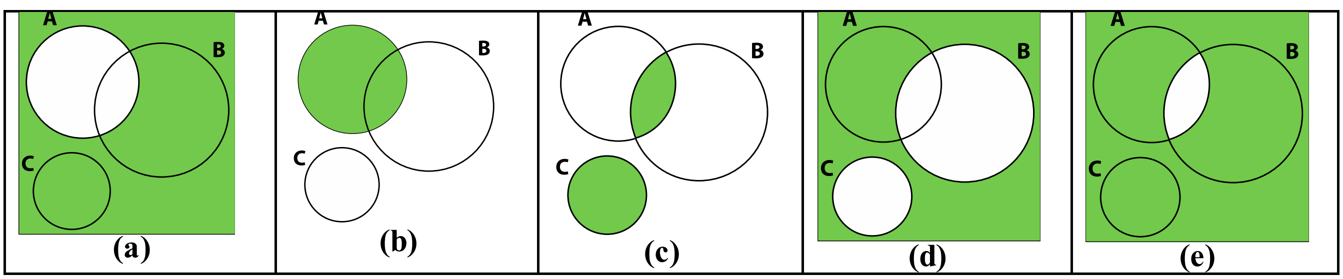
   Answer: If A and B are mutually exclusive, then $P(A \cap B) = 0$ and $P(A)P(B) = 0.04 \neq 0$. Therefore, A and B are not independent.

2. (**10 points**) Three events are shown on the Venn diagram in the following figure:



Reproduce the figure and shade the region that corresponds to each of the following events.
(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

Answers are shown in green



3. **(10 points)** Consider the hospital emergency department data in the following table. Let A denote the event that a visit is to the hospital 1 and let B denote the event that a visit results in admittance to any of 4 hospitals. Determine the number of people involved in each of the following events.

   (a) $A \cap B$ (b) A' c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

Hospital	1	2	3	4	total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

LWBS: People leave without being seen by a physician.

Answers:

a) $A \cap B = 1277$
b) $A' = 22252 - 5292 = 16960$
c) $A \cup B = 195 + 1277 + 3820 + 1558 + 666 + 984 = 8500$
d) $A \cup B' = 195 + 270 + 246 + 242 + 3820 + 5163 + 4728 + 3103 + 1277 = 19044$
e) $A' \cap B' = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752$

4. **(10 points)** There are 4 red balls and 6 white balls in a box. Someone draws two balls simultaneously. What is the probability that they are of the same color?

Answer:

P(they are both red) = P(the first one is red)*P(the second one is red given the first one is red) = (4/10)*(3/9) = 2/15

P(they are both white) = P(the first one is white)*P(the second one is white given the first one is white) = (6/10)*(5/9) = 1/3

P(they are the same color) = P(they are reds) + P(they are white) = 2/15+1/3 = 7/15

5. **(10 points)** George asked his professor for a recommendation letter for the graduate school. He estimates that the probability that the letter will be strong is 0.5, the probability that the letter will be weak is 0.2, and that it will be mediocre is 0.3. He also estimates that if the letter is strong, the probability that he will be accepted to the graduate school of his choice is 0.8; if it is weak – it is exactly 0; and if it is mediocre, the probability is 0.4. Given that he did get accepted to the school of his choice, find the probability that: (a) the letter was strong and (b) the letter was weak?

Answer: Let us denote S/W/M to be events of strong/weak/mediocre recommendation letters. We represent the event whether or not George did get the job as Y/N.

According to the problem, we have $P(S) = 0.5$, $P(W) = 0.2$, $P(M) = 0.3$, $P(Y|S) = 0.8$, $P(Y|W) = 0.0$, $P(Y|M) = 0.4$

Applying Bayes' theorem, the probability that the letter was strong given he did get to the school of his choice is $P(S|Y) = P(Y|S)P(S)/(P(Y|S)P(S)+P(Y|M)P(M)+P(Y|W)P(W)) = 0.8*0.5/(0.8*0.5+0.4*0.3+0.0*0.2) = 0.769$

Obviously, the probability that the letter was weak given he did get the job is 0: $P(W|Y) = 0$

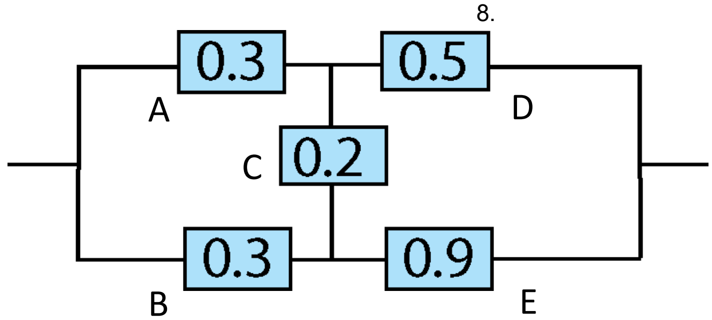
6. **(10 points)** Suppose that a bag contains 10 coins, 3 of which are fair, while the remaining 7 are biased: they have a probability of 0.6 of heads when flipped. A coin was taken at random from the bag and flipped five times. All five flips were heads. What's the probability that this coin was fair?

Answer: Let H_1 denote the hypothesis that a coin was fair and H_2 that it was biased. These hypotheses are mutually exclusive and cover the whole set of possible outcomes. The data that all five flips were heads is denoted as D. $P(D)=P(D|H_1)*P(H_1)+P(D|H_2)*P(H_2)= 0.5^5 * 0.3 + 0.6^5 * 0.7$. Therefore,

$P(H_1|D) = P(D|H_1)P(H_1)/P(D) = 0.5^5 * 0.3/(0.5^5 * 0.3 + 0.6^5 * 0.7) = 0.147$

7. (**10 points**) The following circuit works if and only if there is at least one path of functional devices from left to right. The probability that each device is functional is independent of others and is shown inside each box. What is the probability that the circuit works?

8.



Answer:

P(W) = P(A)P(not B)P(D or (C and E)) + P(not A)P(B)P(E or (C and D)) + P(A)P(B)P(C or D)
= 0.3*(1-0.3)*(1-(1-0.5)*(1-0.2*0.9)) +(1-0.3)*0.3*(1-(1-0.9)*(1-0.2*0.5))+ + 0.3*0.3*(1-(1-0.5)*(1-0.9))=0.4005