## Homework \#1

1. (10 points) Events $A$ and $B$ are mutually exclusive and have probabilities $P(A)=0.2, P(B)=0.2$. Are they independent?
2. ( $\mathbf{1 0}$ points) Three events are shown on the Venn diagram in the following figure:


For each of the questions (a-e) reproduce the figure and shade the region corresponds to the following events.
(a) $A^{\prime}$
(b) $(A \cap B) \cup\left(A \cap B^{\prime}\right)$
(c) $(A \cap B) \cup C$
(d) $(B \cup C)^{\prime}$
(e) $(A \cap B)^{\prime} \cup C$

3. ( $\mathbf{1 0}$ points) Consider the hospital emergency department data in the following table. Let A denote the event that a visit is to the hospital 1 and let B denote the event that a visit results in admittance to any of 4 hospitals. Determine the number of people involved in each of the following events.
(a) $A \cap B$
(b) $\mathrm{A}^{\prime}$
c) $\mathrm{A} \cup \mathrm{B}$
(d) $A \cup B^{\prime}$
(e) $\mathrm{A}^{\prime} \cap \mathrm{B}^{\prime}$

| Hospital | 1 | 2 | 3 | 4 | total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Total | 5292 | 6991 | 5640 | 4329 | 22,252 |
| Left before seen by a doctor | 195 | 270 | 246 | 242 | 953 |
| Admitted | 1277 | 1558 | 666 | 984 | 4485 |
| Not admitted | 3820 | 5163 | 4728 | 3103 | 16,814 |

4. ( $\mathbf{1 0}$ points) There are 4 red balls and 6 white balls in a box. One draws two balls simultaneously. What is the probability that they are of the same color?
5. (10 points) George asked his professor for a recommendation letter for the graduate school. He estimates that the probability that the letter will be strong is 0.5 , the probability that the letter will be weak is 0.2 , and that it will be mediocre is 0.3 . He also estimates that if the letter is strong, the probability that he will be accepted to the graduate school of his choice is 0.8 ; if it is weak - it is exactly 0 ; and if it is mediocre, the probability is 0.4 . Given that he did get accepted to the school of his choice, find the probability that: (a) the letter was strong and (b) the letter was weak?
6. ( $\mathbf{1 0}$ points) Suppose that a bag contains 10 coins, 3 of which are fair, while the remaining 7 are biased: they have a probability of 0.6 of heads when flipped. A coin was taken at random from the bag and flipped five times. All five flips were heads. What's the probability that this coin is fair?
7. ( $\mathbf{1 0}$ points) The following circuit works if and only if there is at least one path of functional devices from left to right. The probability that each device is functional is independent of others and is shown inside each box. What is the probability that the circuit works?

