Review for the Final Exam

Final exam

 The final exam for BIOE 505 will be held in this room 12/12/2023, 8am-11am

Course	Section	CRN	Date	Day
BIOE 505	СВ	54270	12/12/2023	Т

Start Time	End Time	Room
8:00 AM	11:00 AM	1302 Everitt Laboratory

Rules

- Closed book exam; no books, notes, laptops, smartphones, etc.
- However, calculators (not on a smartphone) can be used.
- You can prepare one cheat sheet (letter size, two-sided if needed)
- Printouts provided:
 - Distributions means/variances/pdfs
 - Standard normal distribution CDF table

Name	Probability Distribution	Mean	Variance	Section in Book
Discrete				
Uniform	$\frac{1}{n}$, $a \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a+1)^2-1}{12}$	3-5
Binomial	$\binom{n}{x}p^x(1-p)^{n-x},$	np	np(1-p)	3-6
	$x=0,1,\ldots,n,0\leq p\leq 1$			
Geometric	$(1-p)^{x-1}p,$ $x = 1, 2, \dots, 0 \le p \le 1$	1/p	$(1-p)/p^2$	3-7.1
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r}p^r$	r/p	$r(1-p)/p^2$	3-7.2
	$x = r, r + 1, r + 2, \dots, 0 \le p \le 1$			

This will be provided

Poisson
$$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda \qquad \lambda \qquad \lambda \qquad 3-9$$

$$\textbf{Continuous}$$
Uniform
$$\frac{1}{b-a}, a \leq x \leq b \qquad \frac{(b+a)}{2} \qquad \frac{(b-a)^2}{12} \qquad 4-5$$
Normal
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-1/2}(\frac{x-\mu}{\sigma})^2 \qquad \mu \qquad \sigma^2 \qquad 4-6$$

$$-\infty < x < \infty, -\infty < \mu < \infty, 0 < \sigma$$
Exponential
$$\lambda e^{-\lambda x}, 0 \leq x, 0 < \lambda \qquad 1/\lambda \qquad 1/\lambda^2 \qquad 4-8$$
Erlang
$$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}, 0 < x, r = 1, 2, \dots \qquad r/\lambda \qquad r/\lambda^2 \qquad 4-9.1$$
Gamma
$$\frac{\lambda^t x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda \qquad r/\lambda \qquad r/\lambda^2 \qquad 4-9.2$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

What may be on the final exam?

- Probability Multiplication, Combinatorics
- Bayes Theorem
- Discrete & Continuous Random Variables
- Joint Probability Distributions, Covariation/Correlations
- Sampling distributions and parameter point estimation
- Confidence Intervals
- Hypothesis testing for one and two samples
- Other topics
- Look at Homework 1-5 for examples of problems

One-sample hypothesis testing

What type of hypothesis should I apply?

A. Two-sided: $\mu_1 \neq \mu_0$

B. One-sided: $\mu_1 > \mu_0$

C. One-sided: $\mu_1 < \mu_0$

D. Three-sided

E. I have no idea

The standard deviation of \bar{x} in this sample is:

- A. \$4.50
- B. \$45
- C. \$0.45
 - D. I have no idea

Answer: Hypothesis: $\begin{cases} H_0: \mu = 52 \\ H_1: \mu > 52 \end{cases}$ The critical z-value can be obtained from $z^* = \frac{52.8 - 52}{4.5/10} = 1.78$ Since $z^* > z_{\alpha} = 1.65$, this test statistic lies in the rejection region for H₀. Thus

null hypothesis H₀ will be rejected and alternative hypothesis H₁ is accepted.

Two-sample hypothesis

Mating Calls. In a study of mating calls in the gray treefrogs *Hyla hrysoscelis* and *Hyla versicolor*, Gerhart (1994) reports that in a location in Lousiana the following data on the length of male advertisement calls have been collected:

	Sample	Average	SD of	Duration
	size	duration	duration	range
Hyla chrysoscelis	43	0.65	0.18	0.36-1.27
Hyla versicolor	12	0.54	0.14	0.36 - 0.75

The two species cannot be distinguished by external morphology, but *H. chrysoscelis* are diploids while *H. versicolor* are tetraploids. The triploid

crosses exhibit high mortality in larval stages, and if they attain sexual maturity, they are sterile. Females responding to the mating calls try to avoid mismatches.

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha = 0.05$.

	Sample	Average	SD of
	size	duration	duration
Hyla chrysoscelis	43	0.65	0.18
Hyla versicolor	12	0.54	0.14

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha=0.05$.

	Sample	Average	SD of
	size	duration	duration
Hyla chrysoscelis	43	0.65	0.18
Hyla versicolor	12	0.54	0.14

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha = 0.05$.

- 1. Use two-sided hypothesis
- 2. z_{alpha/2}=1.96
- 3. $Z=(0.65-0.54)/sqrt(0.18.^2/43+0.14.^2/12)=2.2516$
- 4. Since Z> z_{alpha/2} null hypothesis can be rejected

Confidence intervals

What Z should I look up in the table?

A.
$$\Phi(Z)=0.9$$

B.
$$\Phi(Z)=0.05$$

C.
$$\Phi(Z)=0.95$$

D.
$$\Phi(Z)=0.1$$

What Z should I look up in the table?

A.
$$\Phi(Z)=0.9$$

B.
$$\Phi(Z)=0.05$$

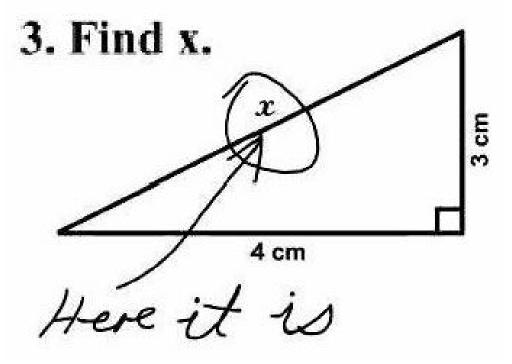
D.
$$\Phi(Z)=0.1$$

Answer: Let μ denote the mean assembly time (in minutes). We want a 90% confidence interval for μ based on the following information: n = 100, $\bar{X} = 16.2$, $\alpha = 0.1$, $\sigma = 3.6$. Since σ is known, we can use normal distribution to calculate confidence interval:

$$\overline{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 16.2 \pm (1.65) \frac{3.6}{10} = [15.61, 16.79]$$

What is X in this problem?

- What is X? Look for keywords:
 - Find the probability that....
 - What is the mean (or variance) of...
- What are the parameters?
 Look for keywords:
 - Given that...
 - Assuming that...
- <u>Is X discrete or</u> continuous?



Discrete Probability Distributions

(8 points) You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.

(A) (2 points) What is the probability that you first succeed on tenth experiment?

- **(B) (2 points)** What is the probability that it requires more than five experiments for you to succeed?
- (C) (2 points) What is the mean number of experiments needed to succeed once?

(D) (2 points) What is the probability that the <u>second</u> experiment that worked is the tenth one since you started?

- 2. (8 points, 2 points each) You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent.
 - (a) What is the probability that you first succeed on tenth experiment?

$$P(X=10) = (1-0.02)^9 *0.02 = 0.0167$$

(b) What is the probability that it requires more than five experiments for you to succeed?

$$P(X > 5) = 1 - P(X=1) - P(X=2) - P(X=3) - P(X=4) - P(X=5)$$

=1 - 0.98° *0.02 - 0.98¹ *0.02 - 0.98² *0.02 - 0.98³ *0.02 - 0.98⁴ *0.02 = 0.9039
Easier solution: $P(X>5)=0.98^5=0.9039$

(c) What is the mean number of experiments needed to succeed once?

Since X follows geometric distribution, the mean value of X is 1/0.02 = 50.

(d) What is the probability that the <u>second</u> experiment that worked is the tenth one since you started Probability = $9*0.02*0.98^8*0.02 = 0.0031$

Continuous Probability Distributions

(12 points) Time interval separating subsequent bus arrivals at a stop is an exponential random variable with mean 20 minutes. Steve and Andrew work at the same place and each will be late to work unless they board a bus on or before 8:40am. Steve comes to the bus stop exactly at 8am. Andrew also comes to the same bus stop but at a random time, uniformly distributed between 8am and 8:30am. Both of them take the first bus that arrives.

- (a) (4 points) What is the probability that Steve will be late for work tomorrow?
- (b) **(4 points)** What is the probability that Andrew will be late for work tomorrow?
- (c) **(4 points)** What is the probability that Steve and Andrew will ride the same bus

(12 points) Time interval separating subsequent bus arrivals at a stop is an exponential random variable with mean 20 minutes. Steve and Andrew work at the same place and each will be late to work unless they board a bus on or before 8:40am. Steve comes to the bus stop exactly at 8am. Andrew also comes to the same bus stop but at a random time, uniformly distributed between 8am and 8:30am. Both of them take the first bus that arrives.

(a) (4 points) What is the probability that Steve will be late for work tomorrow?

Answers:
$$P(Steve | ate) = 1 - P(T < 40) = 1 - \frac{1}{20} \int_{0}^{40} e^{-t/20} dt = e^{-2} = 0.1353$$

(b) (4 points) What is the probability that Andrew will be late for work tomorrow?

Answers:

$$P(\text{Andrew late}) = \int_{0}^{30} \frac{dx}{30} P(T >= 40 \mid T > x) = \int_{0}^{30} \frac{dx}{30} e^{-(40-x)/20} = \frac{e^{-2}}{30} \int_{0}^{30} e^{x/20} dx = \frac{20e^{-2}}{30} \left(e^{30/20} - 1\right) = 0.3141$$

(c) (4 points) What is the probability that Steve and Andrew will ride the same bus?

Probability that Steve will not leave by the time x when Andrew comes is exp(-x/20).

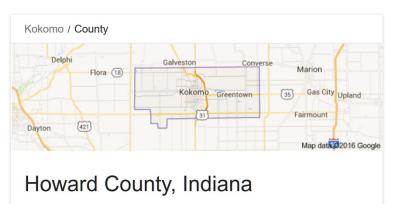
It needs to be integrated over Int_0^30 dx/30 exp(-x/20)=

Answers:
$$P(\text{Steve and Andrew meet}) = \int_{0}^{30} \frac{dx}{30} e^{-x/20} = \frac{20}{30} (1 - e^{-30/20}) = 0.5179$$

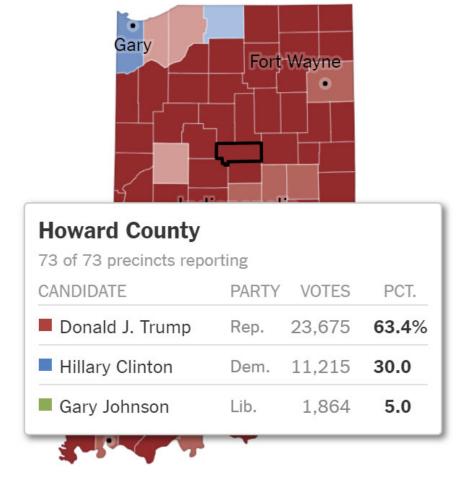


Bayes theorem

Kokomo, Indiana. In Kokomo, IN, 65% of the people are conservative, 20% are liberal, and 15% are independent. Records show that in a particular election, 82% of conservatives voted, 65% of liberals voted, and 50% of independents voted. If a person from the city is selected at random and it is learned that she did not vote, what is the probability that the person is liberal?



As of the 2010 census, the population was 82,752. The county seat is Kokomo, IN.



Kokomo, Indiana. In Kokomo, IN, 65% of the people are conservative, 20% are liberal, and 15% are independent. Records show that in a particular election, 82% of conservatives voted, 65% of liberals voted, and 50% of independents voted. If a person from the city is selected at random and it is learned that she did not vote, what is the probability that the person is liberal?

Kokomo, Indiana. In Kokomo, IN, 65% of the people are conservative, 20% are liberal, and 15% are independent. Records show that in a particular election, 82% of conservatives voted, 65% of liberals voted, and 50% of independents voted. If a person from the city is selected at random and it is learned that she did not vote, what is the probability that the person is liberal?

P(L|NV)=P(NV|L)*P(L)/P(NV)=0.35*0.2/(0.18*0.65+0.35*0.2+0.5*0.15)=0.2672

Joint Probability Distributions

- 1. (20 points) The joint probability mass function of discrete random variables X and Y taking values x = 1,
 - 2, 3 and y = 1, 2, 3, respectively, is given by $f_{XY}(x, y) = c^*(x + y)$. Determine the following:
 - a) (2 points) Find c
 - b) (2 points) Find probability of the event, where X = 1 and Y < 3
 - c) (2 points) Find marginal probability $P_Y(Y = 2)$
 - d) (2 points) Find marginal probability distribution of the random variable X
 - e) (2 points) Find E(X), E(Y), V(X), and V(Y)
 - f) (2 points) Find conditional probability distribution of Y given that X = 1
 - g) (2 points) Conditional probability distribution of X given that Y = 2
 - h) (2 points) Are X and Y independent?
 - i) (2 points) What is the covariance for X and Y?
 - i) (2 points) What is the correlation for X and Y?

- 1. (20 points) The joint probability mass function of discrete random variables X and Y taking values x = 1,
 - 2, 3 and y = 1, 2, 3, respectively, is given by $f_{XY}(x, y) = c^*(x + y)$. Determine the following:
 - a) (2 points) Find c
 - b) (2 points) Find probability of the event, where X = 1 and Y < 3
 - c) (2 points) Find marginal probability $P_Y(Y = 2)$
 - f) (2 points) Find conditional probability distribution of Y given that X = 1

(a)
$$1 = c \cdot (2+3+4+3+4+5+4)$$
 $c = 1/36$

(b) $P(x=1, 4/3) = \frac{2+3}{36} = \frac{5}{36}$

(c) $P_{Y}(Y=2) = \frac{3+4+5}{36} = \frac{17}{36} = \frac{1}{3}$

$$(f) P(Y=1) = P(Y=2, X=1) = \frac{3/36}{P_X(X=1)} = \frac{3/36}{(2+3+4)/36} = \frac{1}{3}$$

- **1. (20 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by a formula $f_{XX}(x, y) = c^*(x + y)$. Determine the following:
 - a) (2 points) Find c

Answer:
$$\sum_{R} f(x, y) = c*(2+3+4+3+4+5+4+5+6) = 1$$
, $c*36 = 1$. Thus, $c = 1/36$

b) (2 points) Find probability of the event where X = 1 and Y < 3

Answer:
$$P(X = 1, Y < 3) = f_{XY}(1,1) + f_{XY}(1,2) = \frac{1}{36}(2+3) = 5/36$$

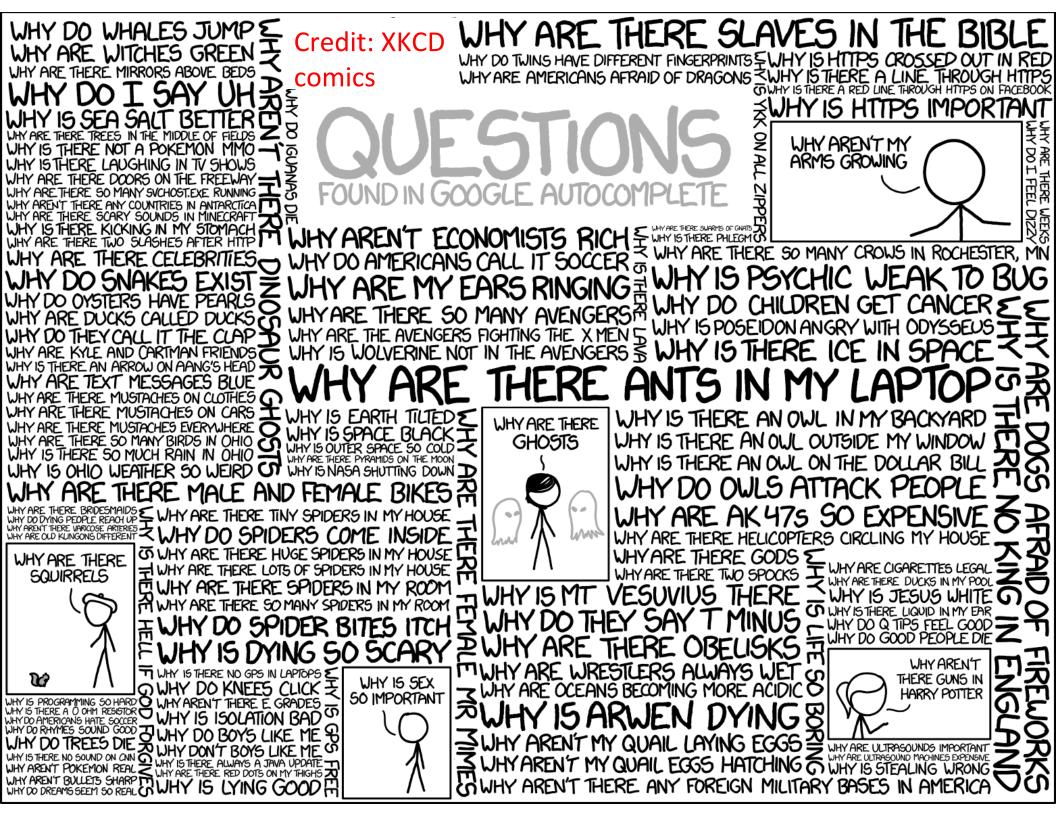
c) (2 points) Find marginal probability $P_Y(Y = 2)$

Answers:
$$P(Y = 2) = f_{XY}(1,2) + f_{XY}(2,2) + f_{XY}(3,2) = \frac{1}{36}(3+4+5) = 1/3$$

f) (2 points) Find conditional probability distribution of Y given that X = 1

Answers:
$$f_{Y|X}(y) = \frac{f_{XY}(1,y)}{f_{X}(1)}$$

- y f_{Y|X}(y)
- 1 (2/36)/(1/4)=2/9
- $2 \frac{(3/36)/(1/4)=1/3}{}$
- $3 \frac{(4/36)/(1/4)=4/9}{}$



Problem I from the midterm (Simplified) Probability of erranging letters BLOBSO, that hot a single position has the same) effer

BLOB JBB 0 variants 0 BB) how man BLOB7 BOBL OBBL 3 BBOL BBLD