Homework #4

1. (10 points) A sample of size 100, which has the sample mean $\bar{X} = 400$, was drawn from a population with an unknown mean μ and the standard deviation $\sigma = 60$.

a) What is the probability that the population mean will be in the interval (410, 420)?

b) Give the 95% confidence interval for the population mean.

2. (10 points) You play on *n* identical arcade games. On each game, you play the game until you win it once and record as x_i the number of times you had to play until you won it. Find the maximum likelihood estimate for the probability of winning a game on one of these arcade games, p using the MLE method. You can leave your answer in terms if *n* and x_i.

3. **12 points)** All cigarettes presently on the market have an average nicotine content of 1.6 mg per cigarette. A company that produces cigarettes want to test if the average nicotine content of a cigarette is 1.6 mg. To test this, a sample of 36 of the company's cigarettes were analyzed.

   a) If it is known that the standard deviation of a cigarette's nicotine content is 0.3 mg, what conclusions can be drawn, at the 1 percent level of significance, if the average nicotine content of the 36 cigarettes is 1.45?

   b) What is the P-value for the hypothesis test in (a)?

4. (10 points) The true mean height of adult women is 64 inches with a standard deviation of 2.2, and the mean height of men is 69 inches with a standard deviation of 2.5. Random samples of sizes 20 and 10 correspondingly are taken, find the probability that the $\bar{X}_{men} - \bar{X}_{women} \geq 7$.

5. (12 points) A lab measures the viral load (virions per mililiter) of SARS-CoV-2 of infected patients:

   6.48366553e+05, 5.85064552e+03, 6.09144634e+05, 5.86114118e+03, 8.12354732e+08, 1.92888061e+06, 2.43946293e+07, 4.48598119e+06, 7.33095635e+03, 4.58773594e+06, 2.60489048e+04, 1.52270429e+07, 3.21471724e+05, 6.79572147e+08, 1.21258820e+05, 6.35426652e+07, 5.24408559e+06, 1.11827164e+06, 1.31108135e+06, 9.29018085e+05

(a) Find a point estimate of the mean log10 viral load. You can use a computer for this.

b) Find a point estimate of the standard deviation of the log10 viral load. You can use a computer for this.

c) What is approximately the standard error of the estimate of the mean log10 viral load number obtained in part a)

d) Find a point estimate for the proportion of readings that are less than 10000000.

e) Find 95% confidence intervals for the point estimate in part d)

f) Use a computer to plot the histogram and the box-and-whisker for the sample and the log base 10 of the sample.

g) What can you observe in comparing the sample to the log of the sample?