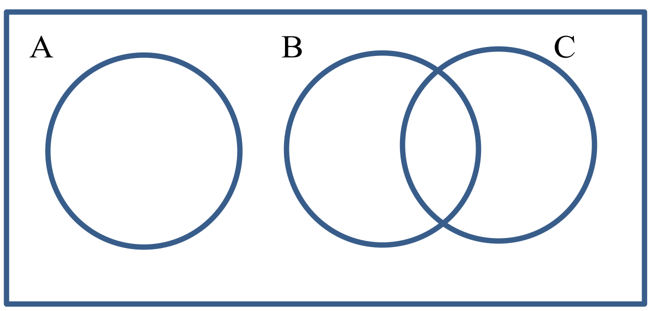
Homework #1

1. (**10 points**) If A and B are independent, their complements A′ and B′ are independent as well.
   (a) State the mathematical relationship that you will need to show to prove this.
   $P(A' \cap B') = P(A')*P(B')$

   (b) Use the following identity for sets $A' \cap B' = (A \cup B)'$ to prove this relationship.
   Also use the rules for complementary probability and addition that follow from the axioms of probability to help you.
   $$\begin{aligned} P(A' \cap B') &= P((A \cup B)') \\ &= 1 - P(A \cup B) \\ &= 1 -(P(A) + P(B) - P(A \cap B)) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= (1 - P(A)) (1 - P(B)) \\ &= P(A') * P(B') \end{aligned}$$

2. (**10 points**) Three events are shown on the Venn diagram in the following figure:



Reproduce the figure and shade the region that corresponds to each of the following events.
(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cup B) \cap C$ (d) $(B \cup A)'$ (e) $(B \cap C)' \cup A$

(a) Everything but A (b) Just A (c) Intersection of B and C (d) Everything but A and B
(e) Everything but the intersection of B and C

3. **(10 points)** Consider the hospital emergency department data in the following table. Let A denote the event that a visit is to Hospital 1 and let B denote the event that a visit results in admittance to any hospital. Determine the number of persons in each of the following events.

Hospital	1	2	3	4	total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

LWBS: People leave without being seen by a physician.

(a) $A' \cap B$
(b) B'
(c) $A \cup B$
(d) $A \cup B'$

(e) $A' \cap B'$

Answers:

a) 1558+666+984=3208

b) 22252 -4485 = 17767

c) 195 + 1277 + 3820 + 1558 + 666 + 984 = 8500

d) 195 + 270 + 246 + 242+ 3820 + 5163 + 4728 + 3103 + 1277 =19044

e) 270 + 246 + 242 + 5163 + 4728 + 3103 =13752

4. **(10 points)** There are 4 red balls and 6 white balls in a box. One draws two balls simultaneously. What is the probability that they are the same color?

Answer: 7/15

5. **(10 points)** You enter a room with three arcade games. One of the games is rigged so that you always lose. The other two games allow you to win with probability 0.2.

(a) What is the probability that if you play a game at random, you will win?

P(W) = P(W|F) P(F) + P(W|F') P(F')

= 0.2*(2/3) + 0

= 0.1333

(b) You play a game at random and lose. What is the probability that it is a fair game?

P(F|W') = P(W'|F) (F)/P(W')

= 0.8 * (2/3) / (1 - 0.1333)

= 0.6154

6. **(10 points)** Pet rats commonly suffer from chronic upper respiratory infections (URIs). Suppose that the probability that an adult rat has a URI is 0.5. If at least one parent of a rat has a URI, the baby will also have a URI with probability 0.8 and the probability that a baby rat has a URI given that neither parent has a URI is 0.05. What is the probability that a baby rat will have a URI? Assume that the probability that the parents have URI are independent of each other.

The probability that neither parent has a URI is P(P') = 0.5*0.5 = 0.25 and probability that at least one parent has a URI is P(P) = 1-0.25 = 0.75

P(B|P) = 0.8

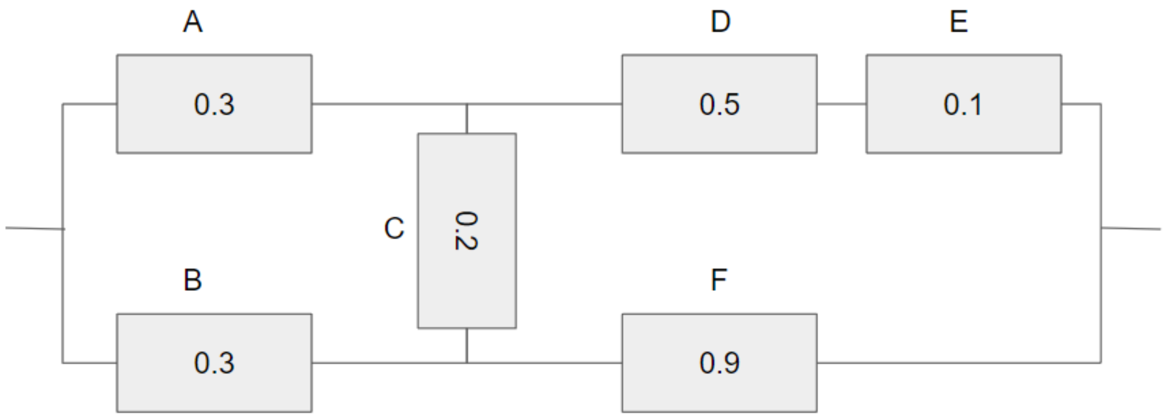
P(B|P') = 0.05

P(B) = P(B|P)P(P) + P(B|P')P(P')

= 0.8*0.75 + 0.05*0.25

= 0.6125

7. (**10 points**) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Answer:

P(W) = P(W|C')P(C') +P(W|C)P(C)

= 0.28095*0.8 + 0.46155*0.2

= 0.31707