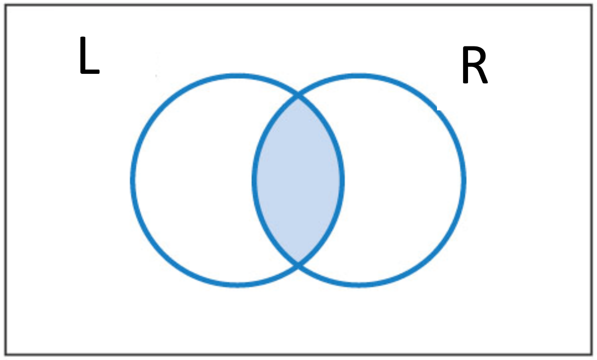
Series Circuit

This circuit operates only if there is at least one path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that the devices fail independently. What is the probability that the circuit operates?

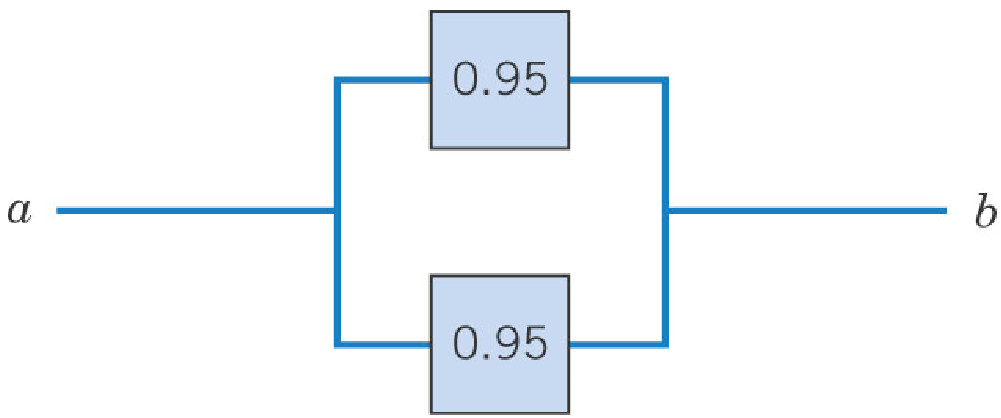


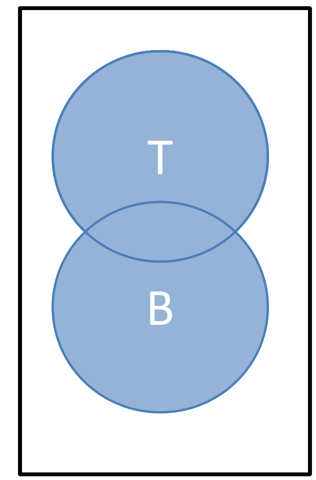
Let L & R denote the events that the left and right devices operate. The probability that the circuit operates is:

$$P(L \text{ and } R) = P(L \cap R) = P(L) * P(R) = 0.8 * 0.9 = 0.72.$$

Parallel Circuit

This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.



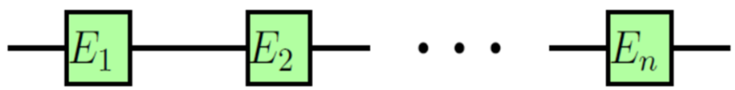


Let T & B denote the events that the top and bottom devices operate. The probability that the circuit operates is:

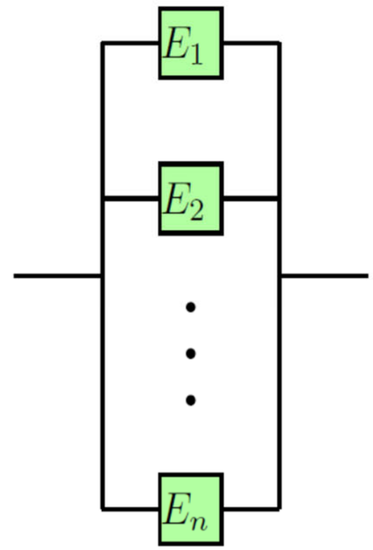
$P(T \cup B) = 1 - P(T' \cap B') = 1 - P(T')*P(B') = 1 – 0.05^2 = 1 – 0.0025 – 0.9975.$

Duality between parallel and series circuits

$$q_i = 1 - p_i.$$



(a)

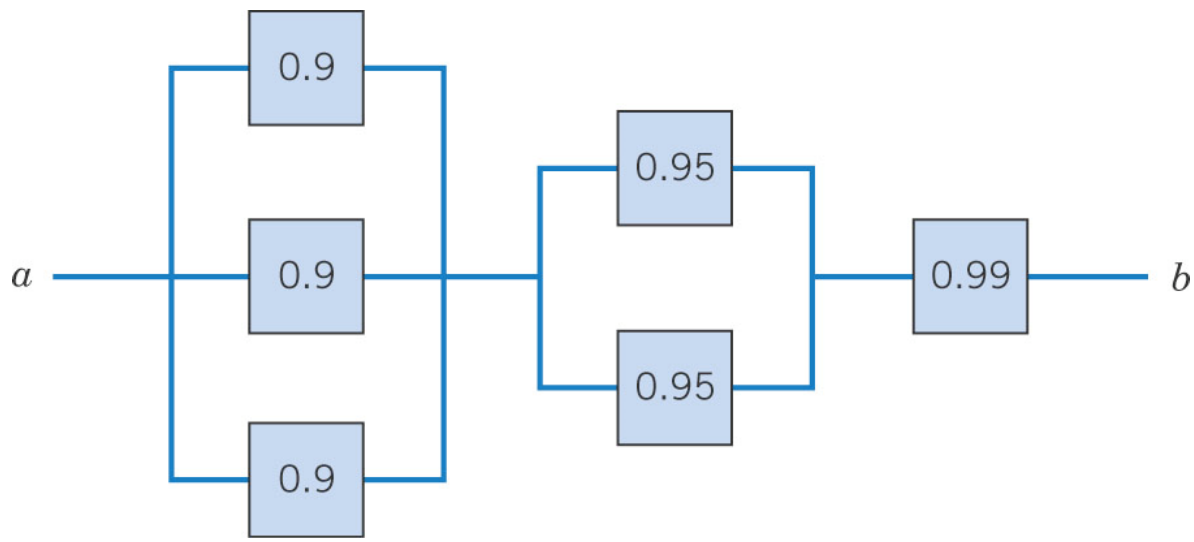


(b)

Connection	Notation	Works with prob	Fails with prob
Serial	$E_1 \cap E_2 \cap \cdots \cap E_n$	$p_1 p_2 \ldots p_n$	$1 - p_1 p_2 \ldots p_n$
Parallel	$E_1 \cup E_2 \cup \cdots \cup E_n$	$1 - q_1 q_2 \ldots q_n$	$q_1 q_2 \ldots q_n$

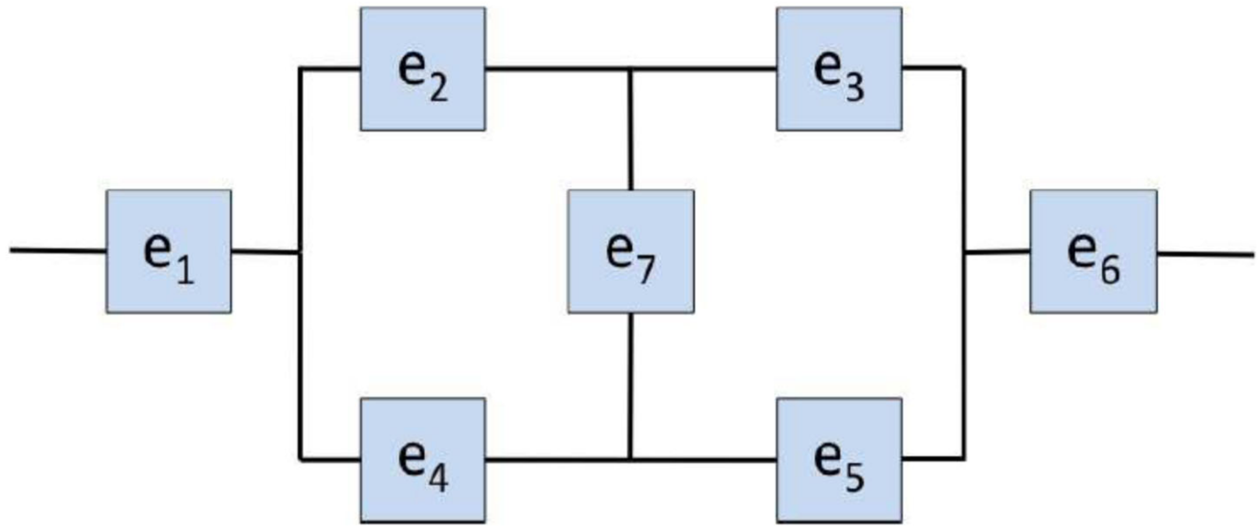
Advanced Circuit

This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.

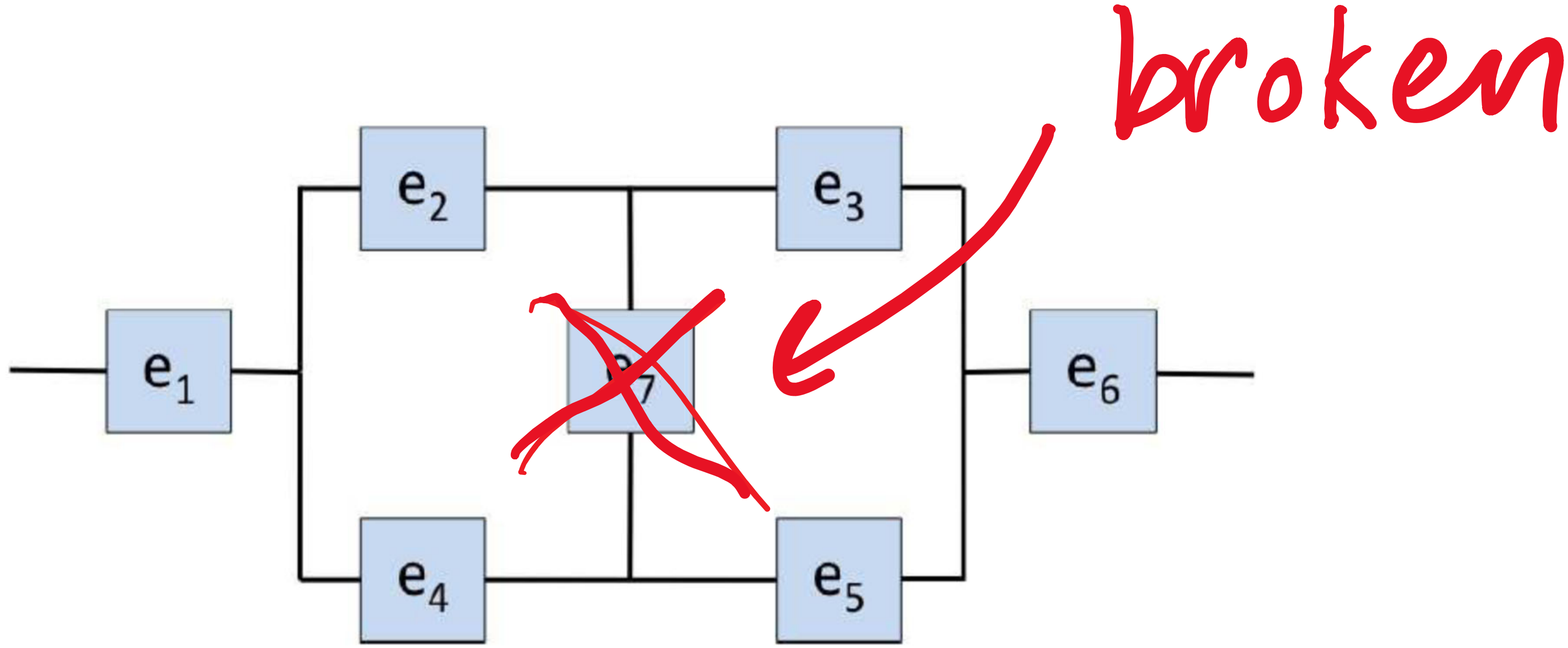


Partition the graph into 3 columns with L & M denoting the left & middle columns.

$P(L) = 1- 0.1^3$, and $P(M) = 1- 0.05^2$, so the probability that the circuit operates is: $(1 - 0.1^3)(1-0.05^2)(0.99) = 0.9875$ (this is a series of parallel circuits).



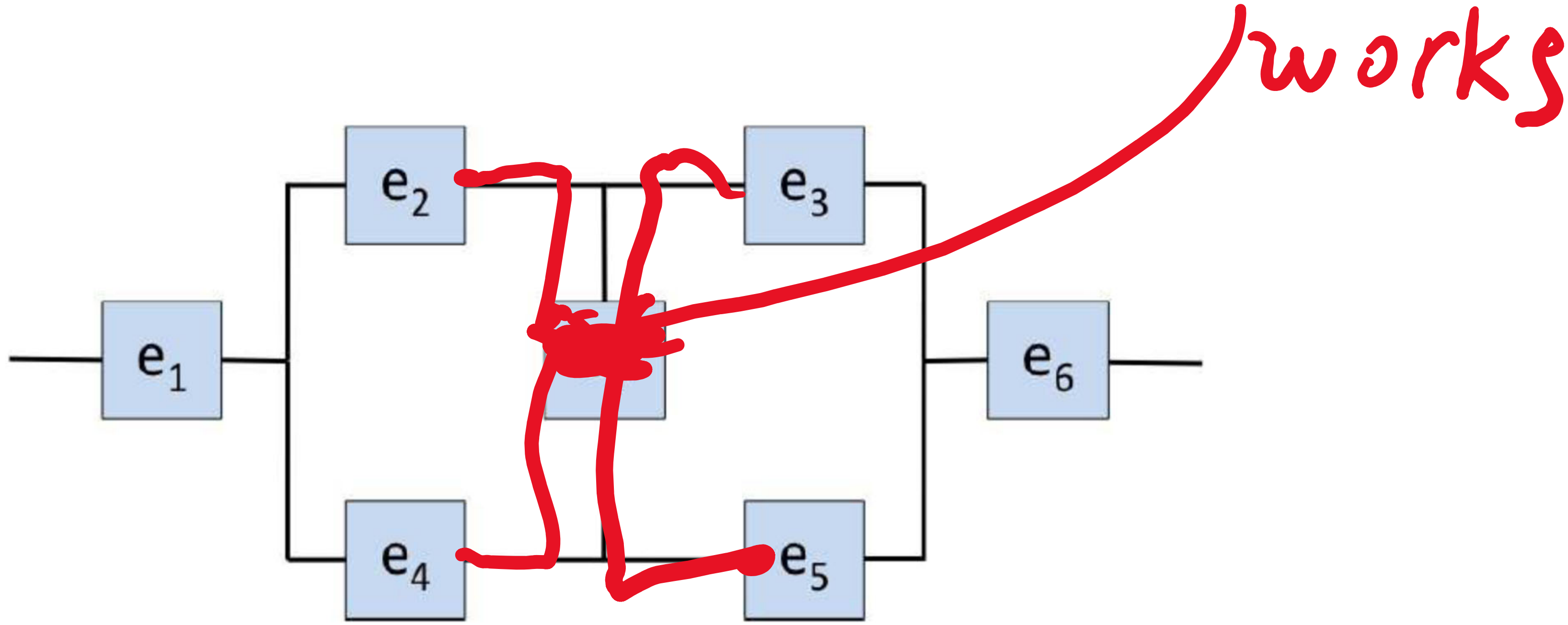
Component	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4



Component	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

P(circuit works |e7 is broken)=P(e1 works)*
[1-(1-P(e2 works)*P(e3 works))*(1-P(e4 works)*P(e5 works))]*
P(e6 works)=0.3*(1-(1-0.8*0.2)*(1-0.2*0.5))*0.6=0.0439

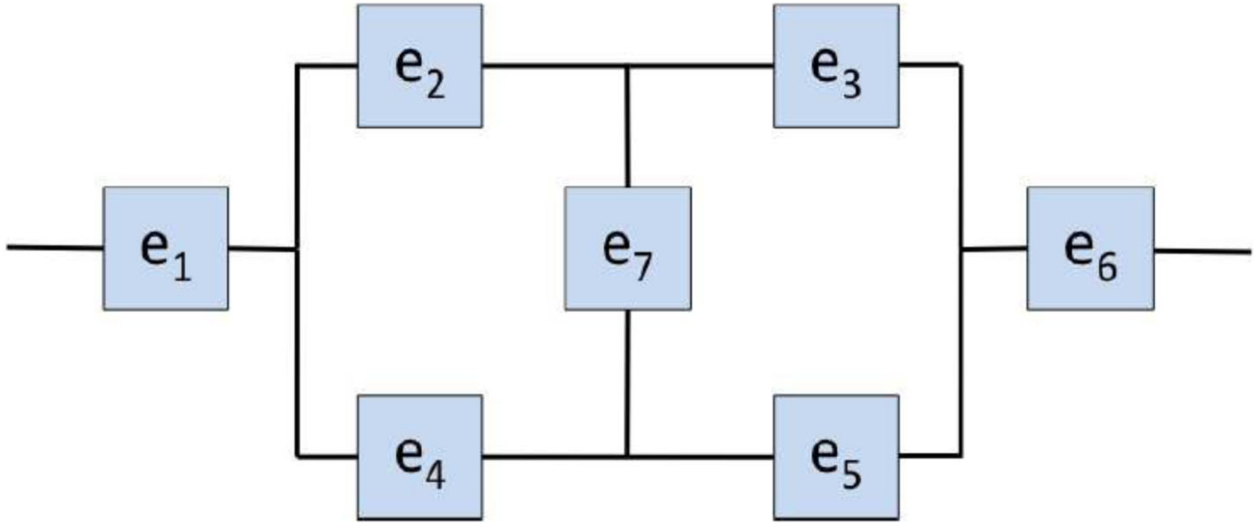
The contribution to total probability:
P(circuit works |e7 is broken)*P(e7 is broken)=0.6*0.0439=0.0264



Component	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

P(circuit works |e7 works)=P(e1 works)*
[1-(1-P(e2 works))*(1-P(e3 works))]
[1-(1-P(e4 works))(1-P(e5 works))]*
P(e6 works)=0.3*(1-(1-0.8)*(1-0.2))*(1-(1-0.2)*(1-0.5)))*0.6=0.0907

The contribution to total probability:
P(circuit works |e7 works)*P(e7 works)=0.4*0.0907=0.0363

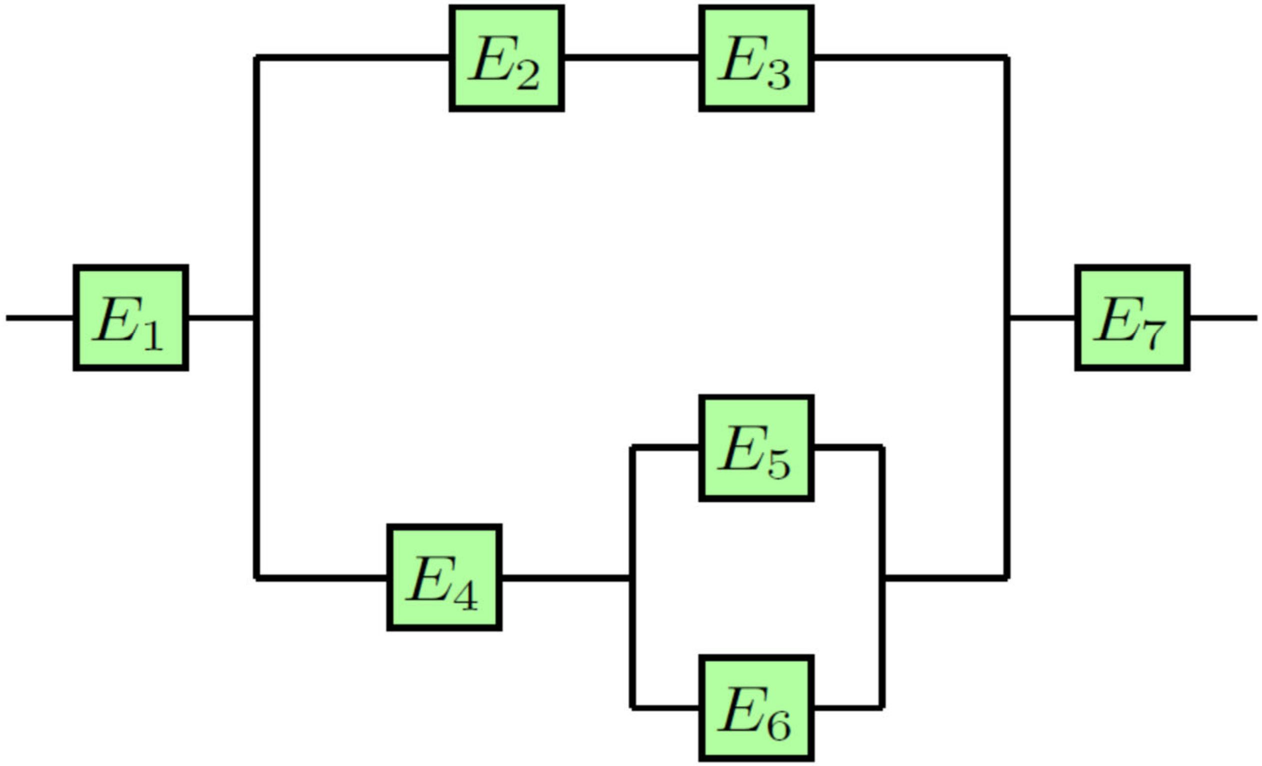


Component	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

P(circuit works)=
P(circuit works |e7 works)*P(e7 works)+
P(circuit works |e7 is broken)*P(e7 is broken)=
=0.0264+0.0363=0.0627

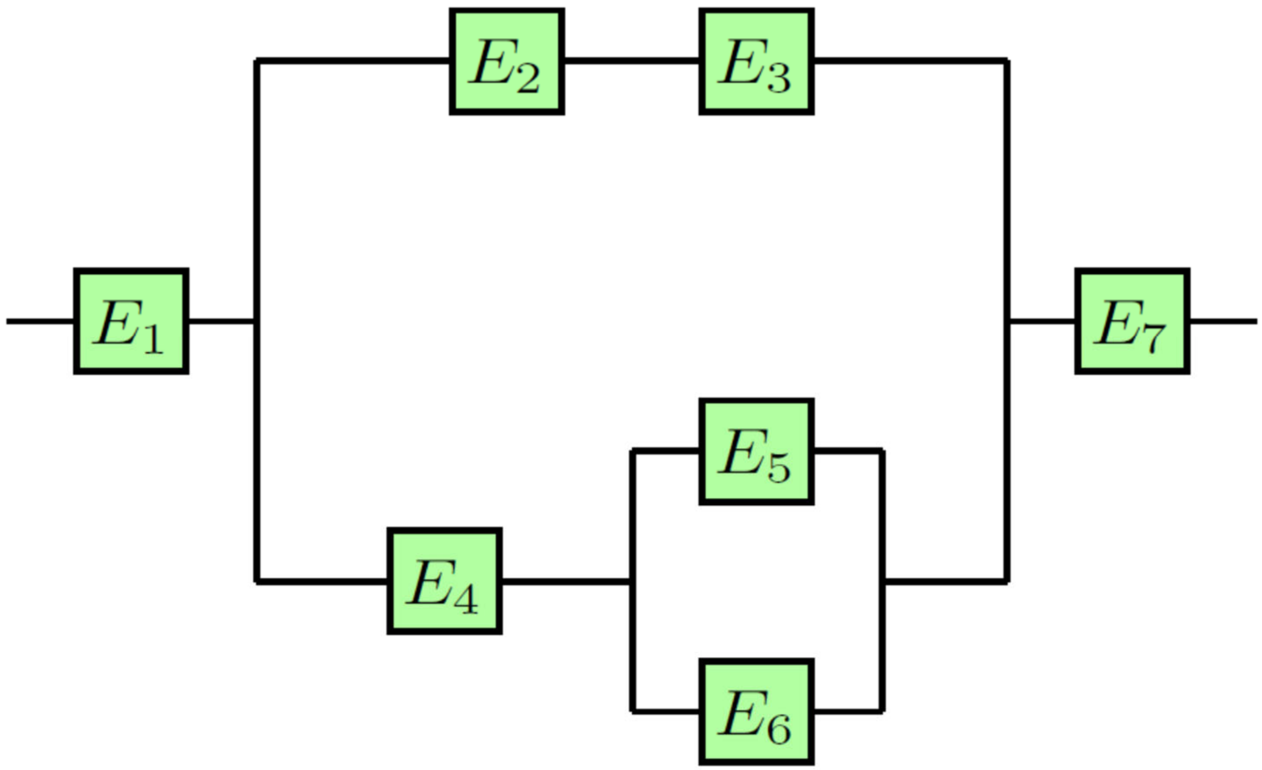
Answer: 6.27%

Circuit → Set equation



Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

Circuit → Set equation

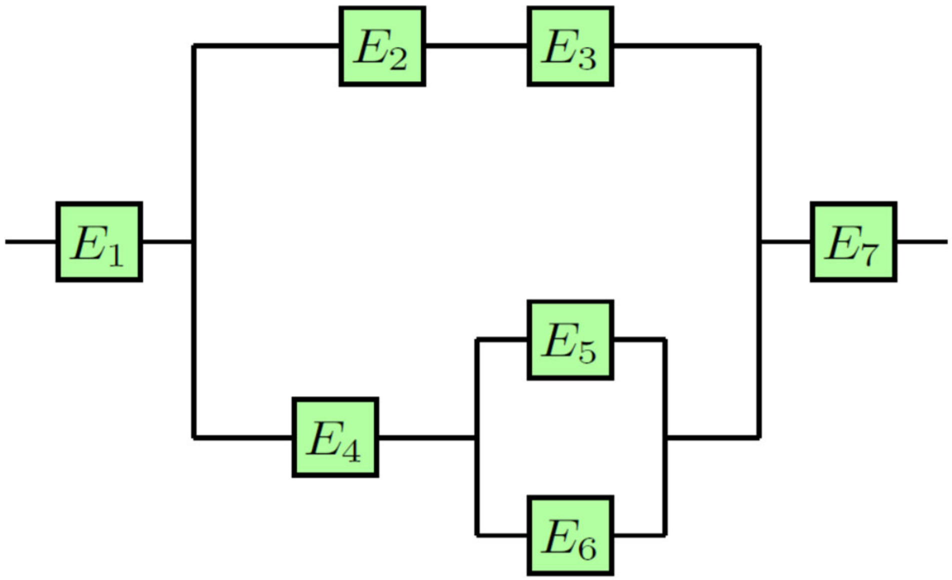


Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

P(Works) = 0.9.*(1-(1-0.5.*0.3).*(1-0.1.*(1-0.6.*0.5))).*0.8=0.15084

Matlab group exercise

- Test our result for this circuit.
- Download circuit_template.m from the website



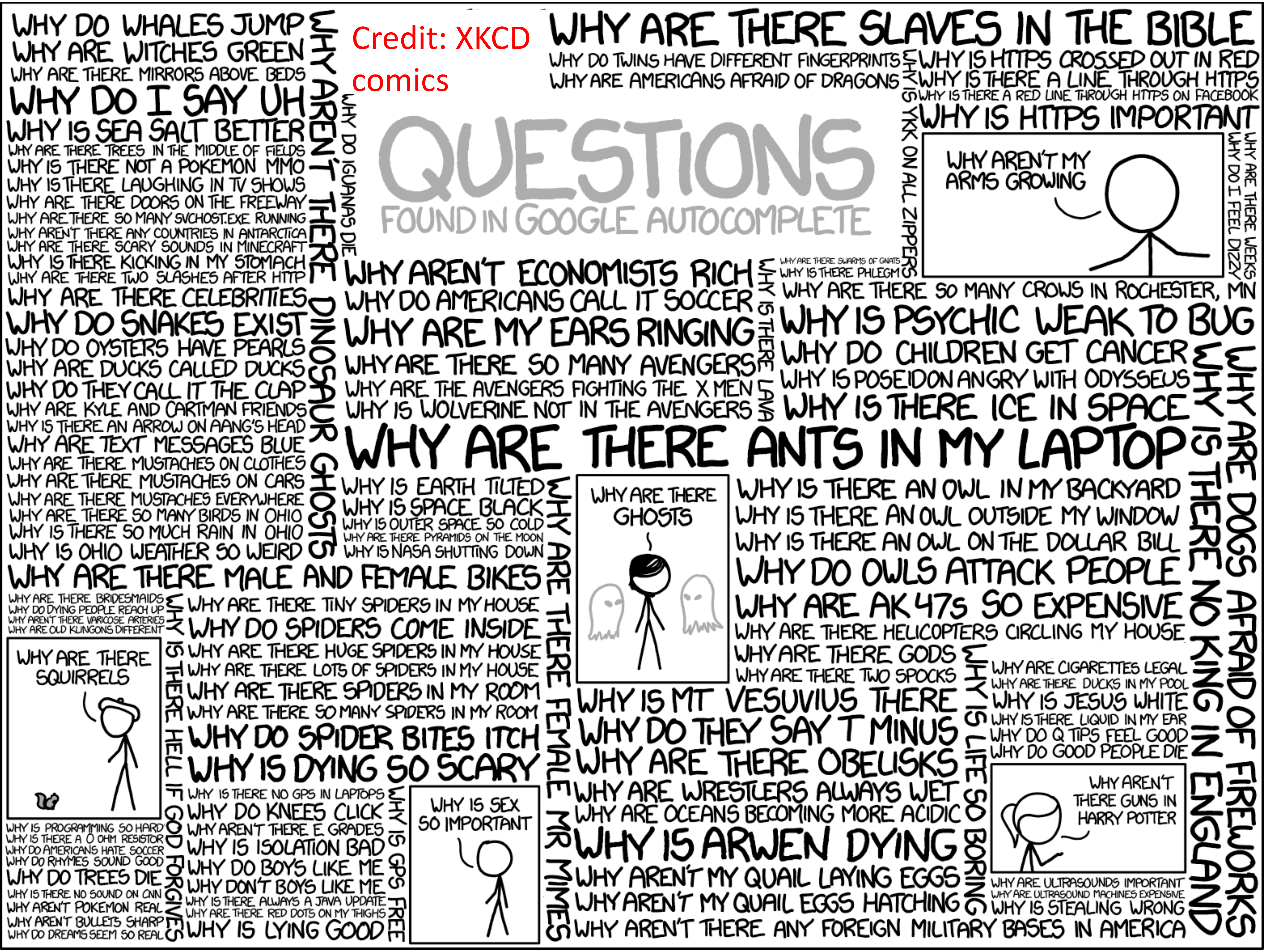
Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

P(Works) = 0.9.*(1-(1-0.5.*0.3).*(1-0.1.*(1-0.6.*0.5))).*0.8=0.15084

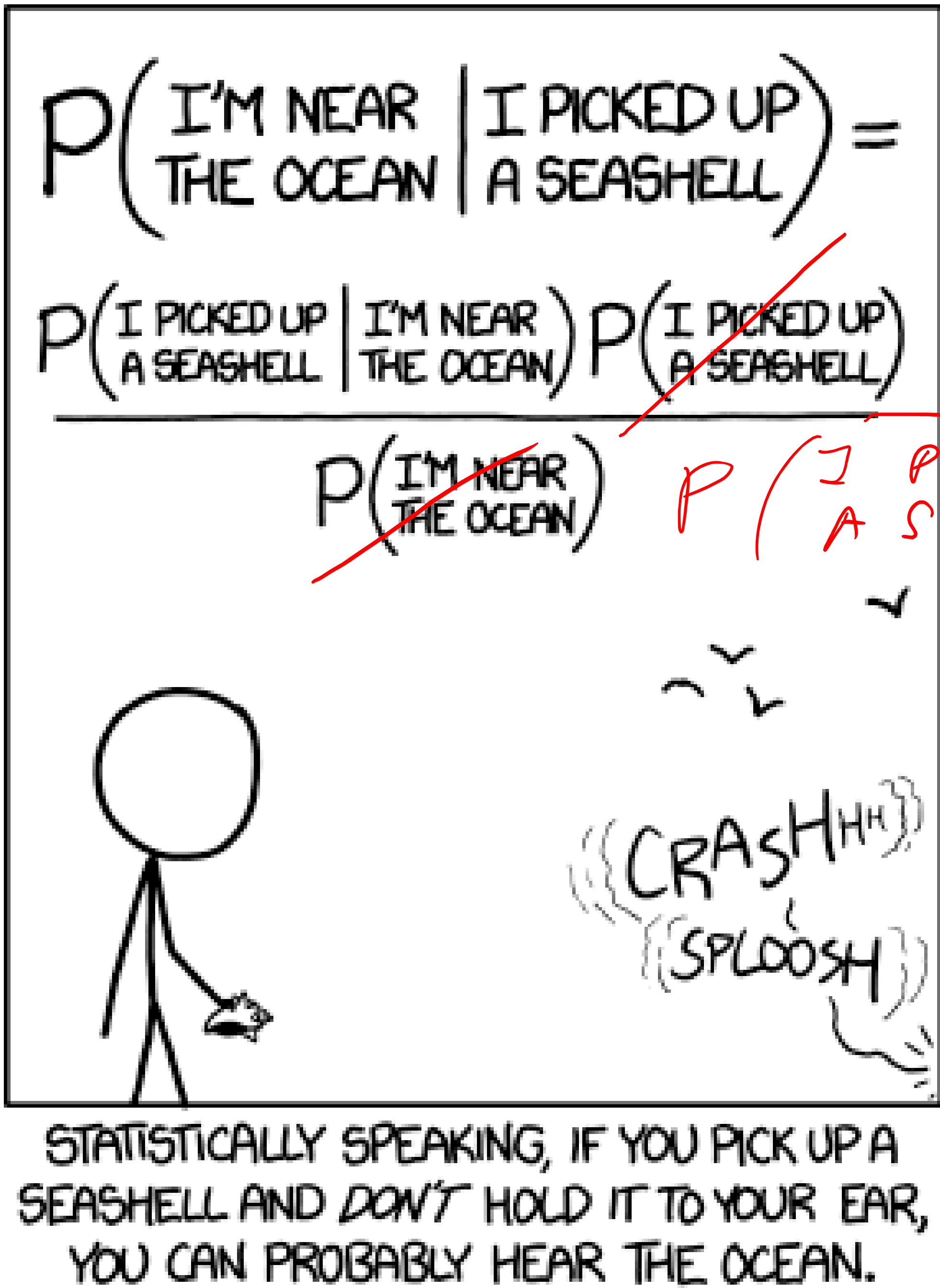
Here is how I did it

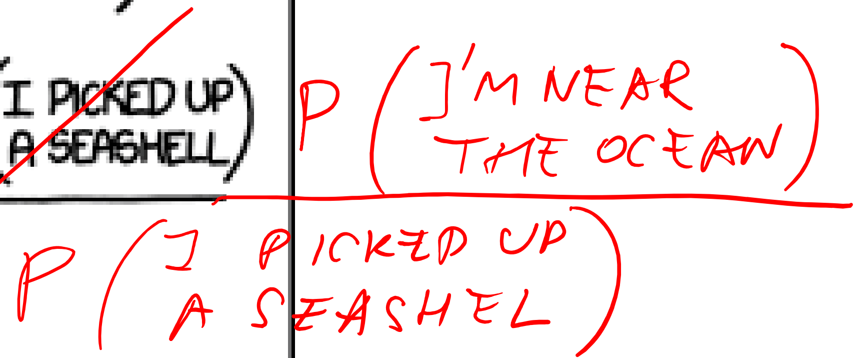
- Stats=1e6;
- count= 0;
- for i = 1: Stats
- e1 = rand < 0.9; e2 = rand < 0.5; e3 = rand < 0.3;
- e4 = rand < 0.1; e5 = rand < 0.4; e6 = rand < 0.5;
- e7 = rand < 0.8;
- s1 = min(e2,e3); % or s1 = e2*e3;
- s2 = max(e5,e6); % or s2= e5+e6>0;
- s3 = min(e4,s2); % or s3 = e4*s2;
- s4 = max(s1,s3); % or s4 = s1+s3 > 0;
- s5= min([e1;s4;e7]); % or s5=e1*s4*e7;
- count = count + s5;
- End;
- P_circuit_works = count/Stats
- **% our calculation: P(circuit_works)= 0.9.*(1-(1-0.5.*0.3).*(1-0.1.*(1-0.6.*0.5))).*0.8==0.15084**

Credit: XKCD comics



Reminder:
Conditional probability



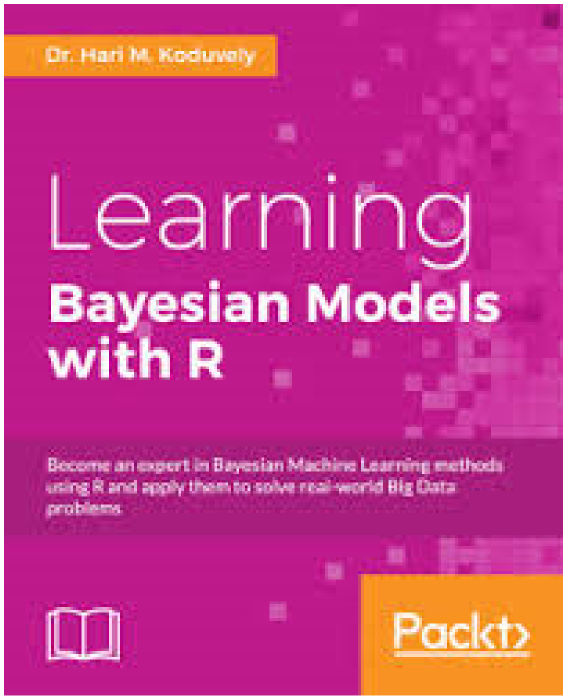


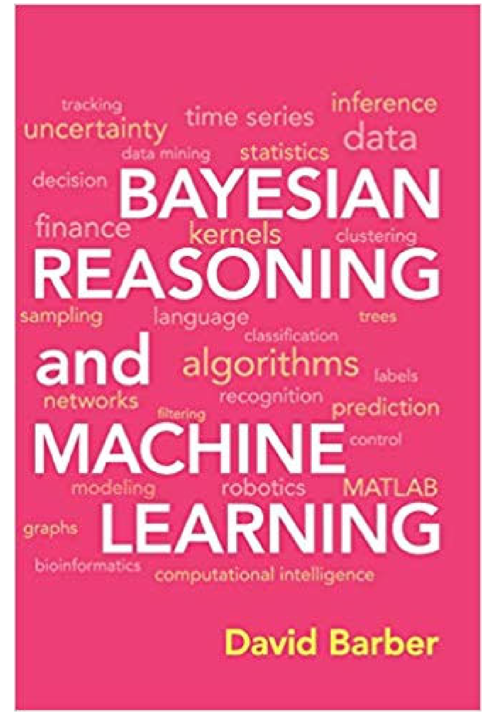
What is wrong in this comics?

If you are not yet reading XKCD comics https://xkcd.com/ you should start

Bayes Theorem

Bayes’ theorem





Thomas Bayes (1701-1761)

English statistician, philosopher, and Presbyterian minister

Bayes’ theorem was presented in "An Essay towards solving a Problem in the Doctrine of Chances" which was read to the Royal Society in 1763 already after Bayes' death.

Bayes' theorem (simple)

$$P(A \cap B) = P(A|B)P(B) = P(B \cap A) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- In Science we often want to know:
  "How much faith should I put into hypothesis, given the data?"
  or $P(H|D)$ (see also the inductive definition of probability)
- What we usually can calculate if the hypothesis/model is OK:
  "Assuming that this hypothesis is true, what is the probability of the observed data?" or $P(D|H)$
- Bayes' theorem can help: $P(H|D)=P(D|H) \bullet P(H)/P(D)$
- The problem is $P(H)$ (so-called prior) is often not known

Bayes' theorem (continued)

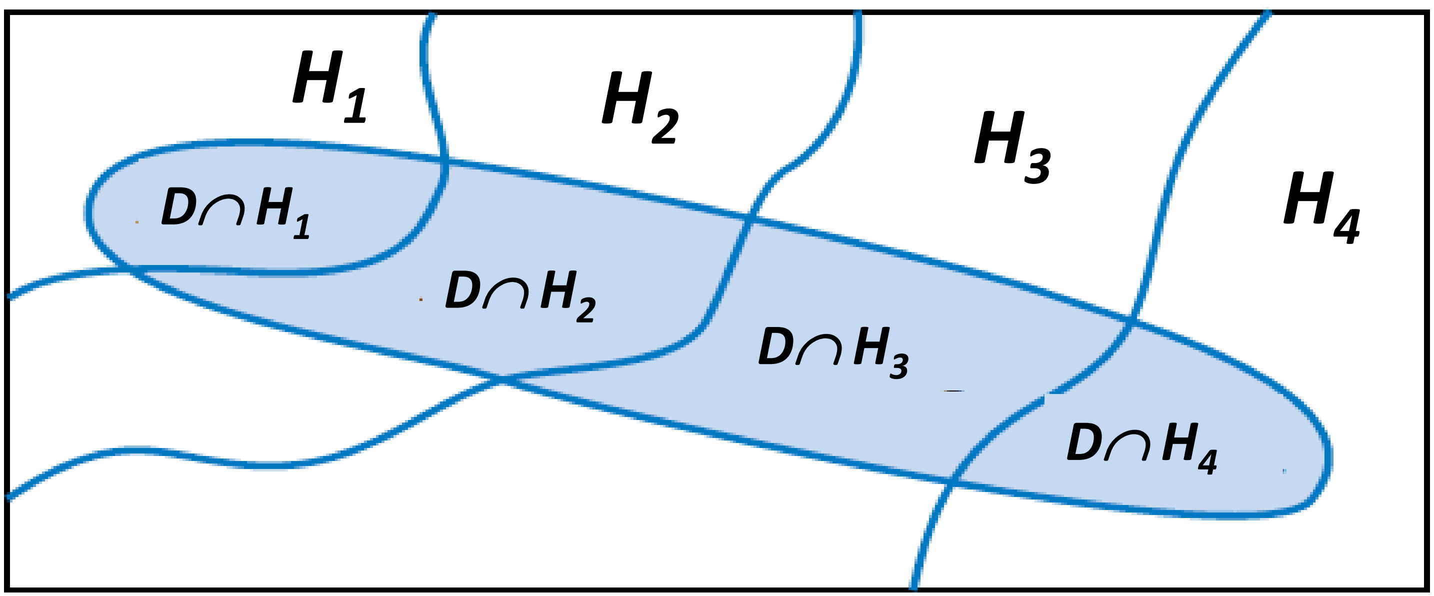
Works best with exhaustive and mutually-exclusive hypotheses:

$H_1, H_2, \ldots H_n$ such that $H_1 \cup H_2 \cup H_3 \ldots \cup H_n = S$ and $H_i \cap H_j = \emptyset$ for $i \neq j$

$P(H_k|D)=P(D|H_k) \bullet P(H_k)/P(D)$

where:

$P(D)= P(D|H_1) \bullet P(H_1) + P(D|H_2) \bullet P(H_2) + \ldots P(D|H_n) \bullet P(H_n)$



An **awesome new test** has been invented for an early detection of cancer. The probability that it correctly identifies someone with cancer as positive is 95%, and the probability that it correctly identifies someone without cancer as negative is 99%. The incidence of this type of cancer in the general population is 10^{-4}. A random person in the population takes the test, and the result is positive.

What is the probability that he/she has cancer?

A. 99%
B. 95%
C. 30%
D. 1%

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participants

10^6 → 100 - cancer → 95 positive tests

10^6 → $10^6 - 100 \approx 10^6$ no cancer

10^6 participants with no cancer → 10,000 positive tests

$$P(C|P) = \frac{95}{10{,}000 + 95} \approx 1\%$$

Events: C - cancer, C' - no cancer

Test events Y - positive, N - negative

We know:

$P(C) = 10^{-4}$, $P(Y|C) = 0.95$

$P(N|C') = 0.99$

We need

$P(C|Y)$

Bayes:

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)}$$

?

$P(Y)$ – probability that a random person will test positive

$$P(Y) = P(Y \cap C) + P(Y \cap C') =$$

$$= P(Y|C)P(C) + P(Y|C')P(C') =$$

$$= 0.95 \times 10^{-4} + (1 - 0.99) \times (1 - 10^{-4}) \approx$$

$$\approx 10^{-4} + 10^{-2} \approx 10^{-2} = 1\%$$

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)} = 0.95 \times \frac{10^{-4}}{10^{-2}} \approx$$

$$\approx 1\%$$

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What is the probability that he/she has cancer?

A. 99%
B. 95%
C. 30%
D. 1%

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