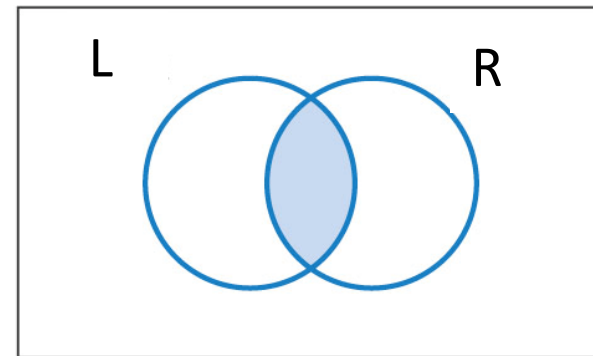


# Series Circuit

This circuit operates only if there is **at least one path of functional devices** from left to right. The **probability** that **each device functions** is shown on the graph. Assume that the **devices fail independently**. What is the probability that the circuit operates?

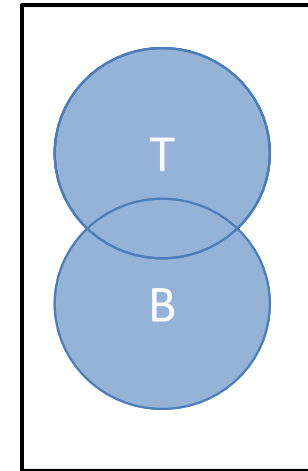
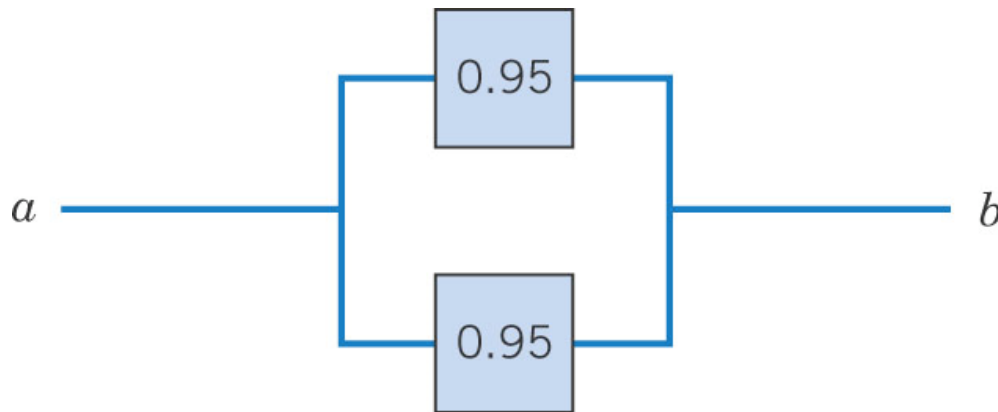


Let L & R denote the events that the left and right devices operate. The probability that the circuit operates is:

$$P(L \text{ and } R) = P(L \cap R) = P(L) * P(R) = 0.8 * 0.9 = 0.72.$$

# Parallel Circuit

This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.

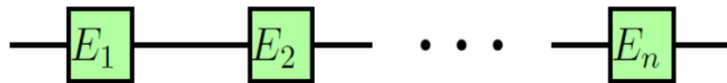


Let T & B denote the events that the top and bottom devices operate. The probability that the circuit operates is:

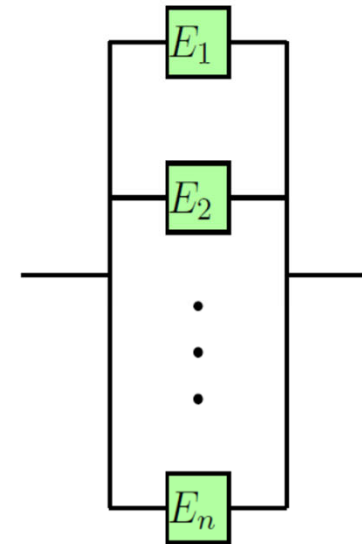
$$P(T \cup B) = 1 - P(T' \cap B') = 1 - P(T') * P(B') = 1 - 0.05^2 = 1 - 0.0025 = 0.9975.$$

# Duality between parallel and series circuits

$$q_i = 1 - p_i.$$



(a)

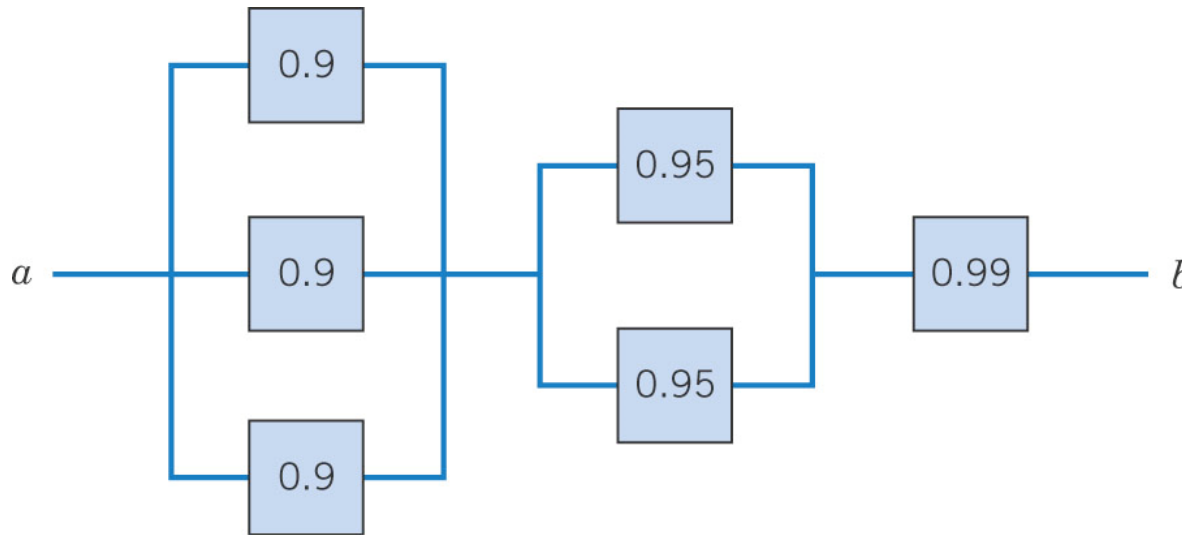


(b)

Connection	Notation	Works with prob	Fails with prob
Serial	$E_1 \cap E_2 \cap \dots \cap E_n$	$p_1 p_2 \dots p_n$	$1 - p_1 p_2 \dots p_n$
Parallel	$E_1 \cup E_2 \cup \dots \cup E_n$	$1 - q_1 q_2 \dots q_n$	$q_1 q_2 \dots q_n$

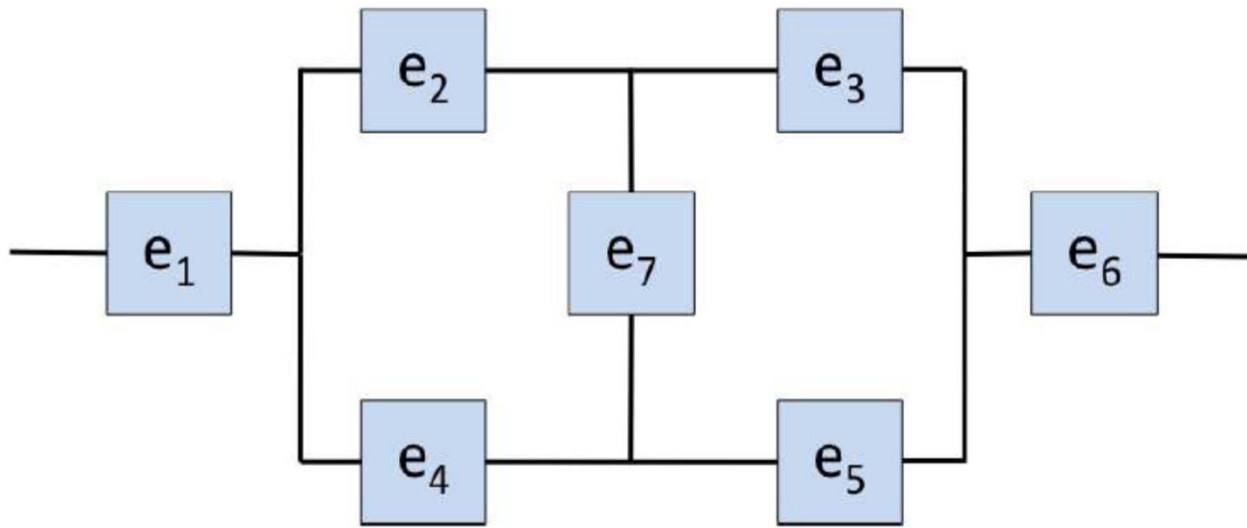
# Advanced Circuit

This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.

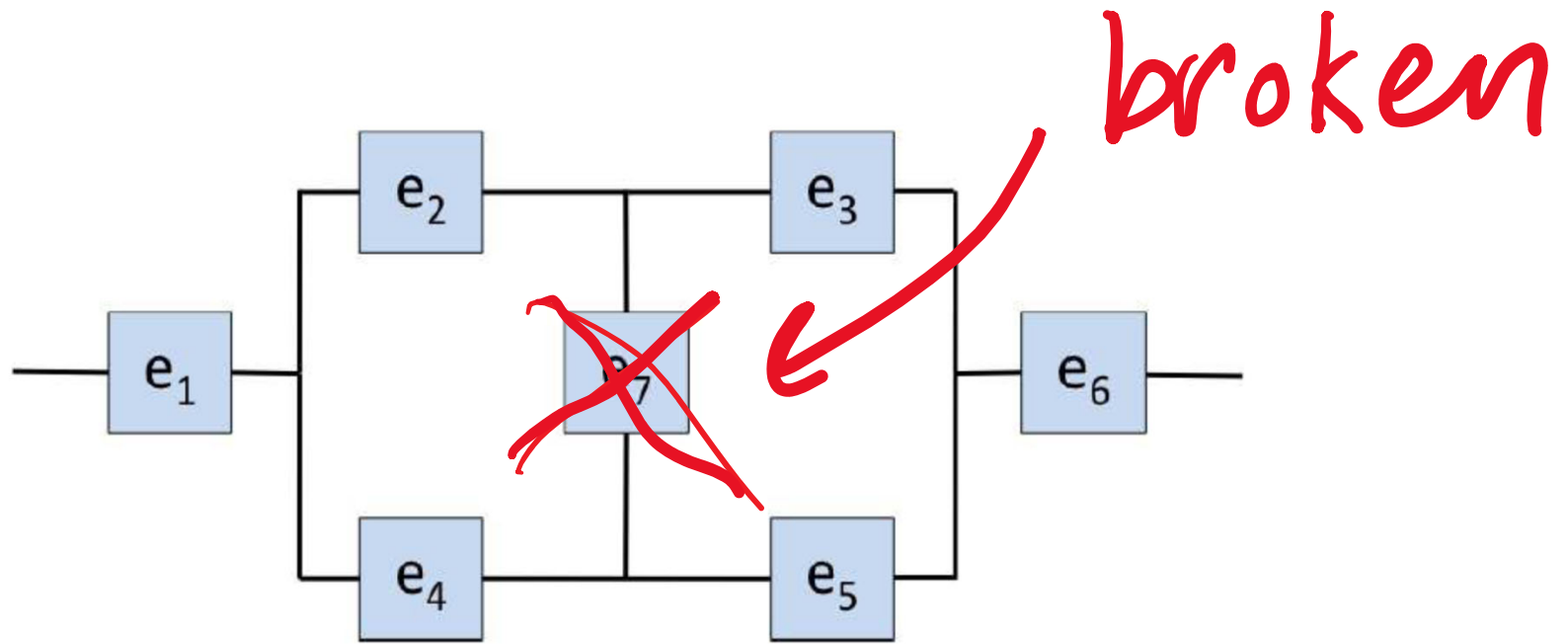


Partition the graph into 3 columns with L & M denoting the left & middle columns.

$P(L) = 1 - 0.1^3$ , and  $P(M) = 1 - 0.05^2$ , so the probability that the circuit operates is:  $(1 - 0.1^3)(1 - 0.05^2)(0.99) = 0.9875$  (this is a series of parallel circuits).



Component	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

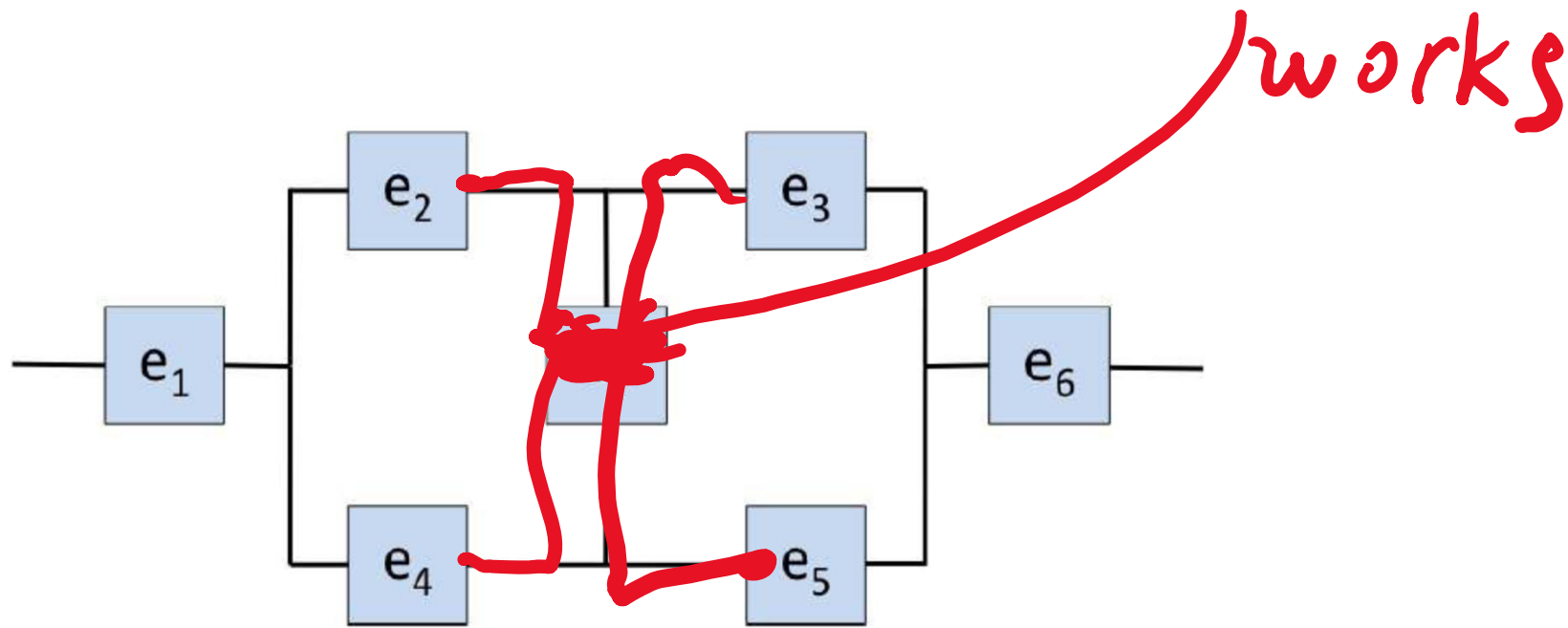


Component	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

$$P(\text{circuit works} \mid e_7 \text{ is broken}) = P(e_1 \text{ works}) * [1 - (1 - P(e_2 \text{ works}) * P(e_3 \text{ works})) * (1 - P(e_4 \text{ works}) * P(e_5 \text{ works}))] * P(e_6 \text{ works}) = 0.3 * (1 - (1 - 0.8 * 0.2) * (1 - 0.2 * 0.5)) * 0.6 = 0.0439$$

The contribution to total probability:

$$P(\text{circuit works} \mid e_7 \text{ is broken}) * P(e_7 \text{ is broken}) = 0.6 * 0.0439 = 0.0264$$



Component	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

$$P(\text{circuit works} \mid e_7 \text{ works}) = P(e_1 \text{ works}) \cdot$$

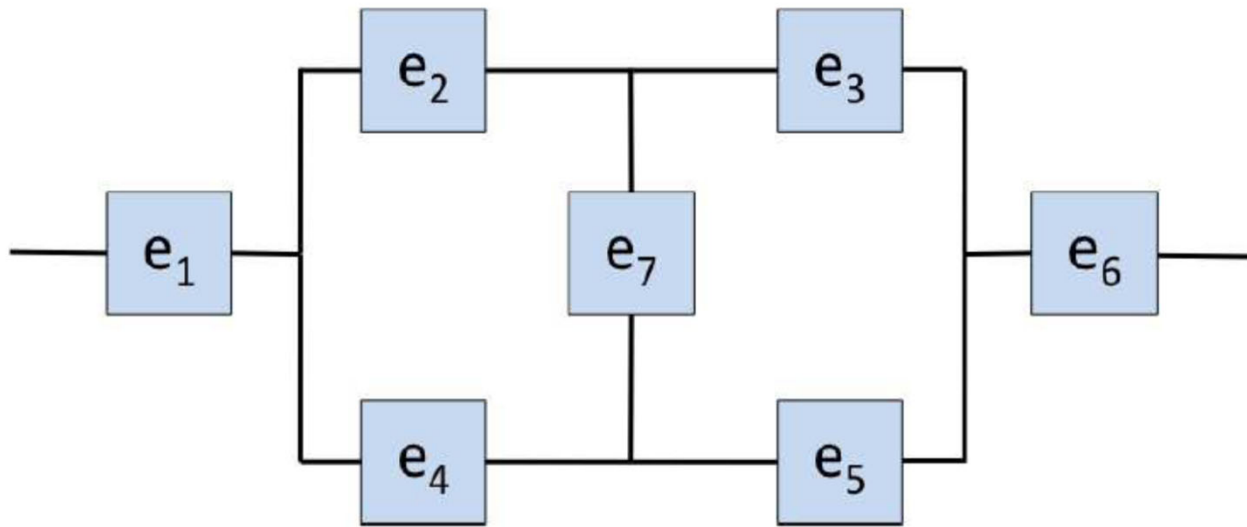
$$[1 - (1 - P(e_2 \text{ works})) \cdot (1 - P(e_3 \text{ works}))]$$

$$\cdot [1 - (1 - P(e_4 \text{ works})) \cdot (1 - P(e_5 \text{ works}))] \cdot$$

$$P(e_6 \text{ works}) = 0.3 \cdot (1 - (1 - 0.8) \cdot (1 - 0.2)) \cdot (1 - (1 - 0.2) \cdot (1 - 0.5)) \cdot 0.6 = 0.0907$$

The contribution to total probability:

$$P(\text{circuit works} \mid e_7 \text{ works}) \cdot P(e_7 \text{ works}) = 0.4 \cdot 0.0907 = 0.0363$$



Component	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

$P(\text{circuit works}) =$

$P(\text{circuit works} \mid e_7 \text{ works}) * P(e_7 \text{ works}) +$

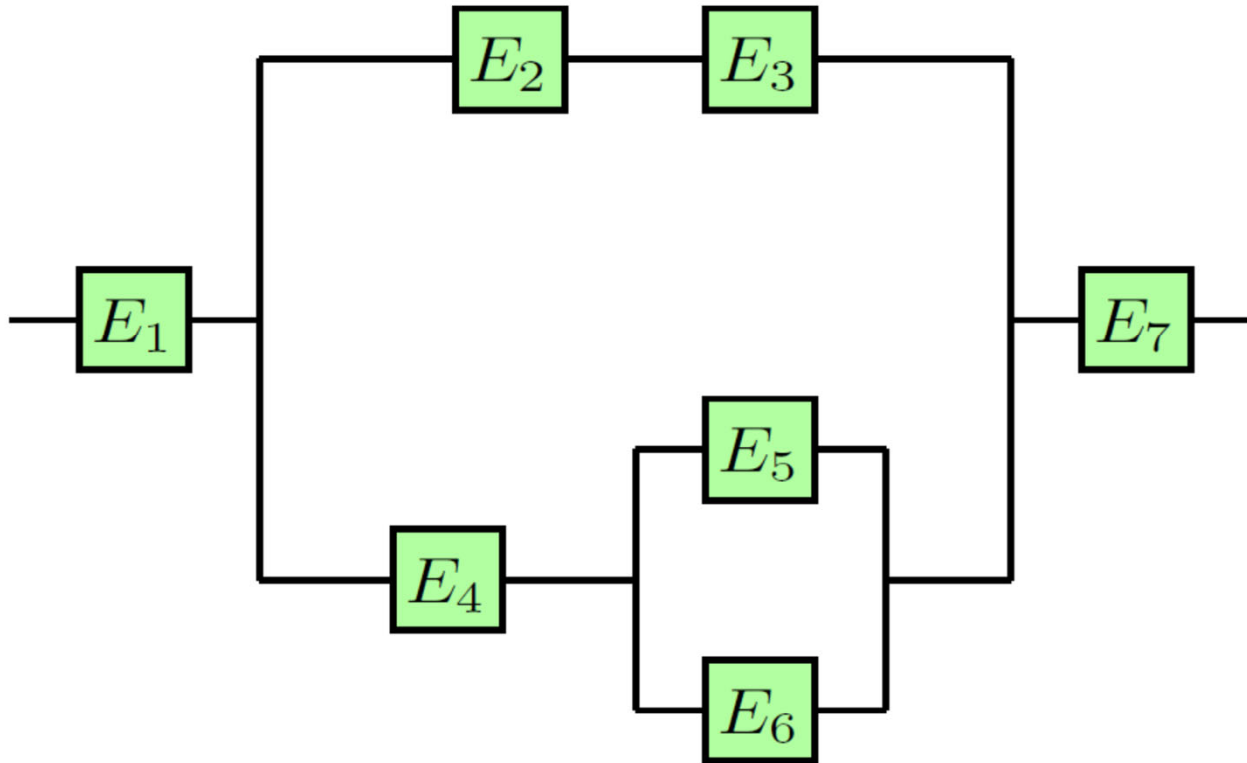
$P(\text{circuit works} \mid e_7 \text{ is broken}) * P(e_7 \text{ is broken}) =$

$= 0.0264 + 0.0363 = 0.0627$

**Answer: 6.27%**

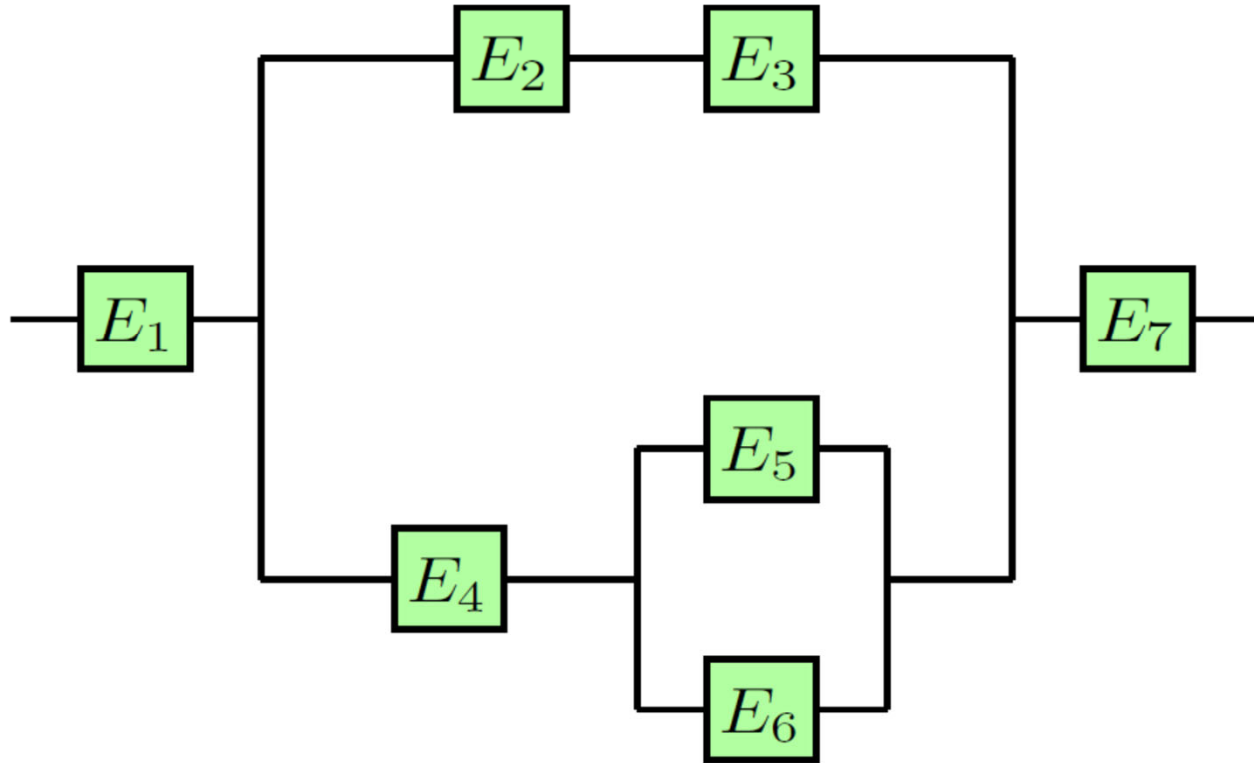


# Circuit $\rightarrow$ Set equation



Component	$E_1$	$E_2$	$E_3$	$E_4$	$E_5$	$E_6$	$E_7$
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

# Circuit → Set equation

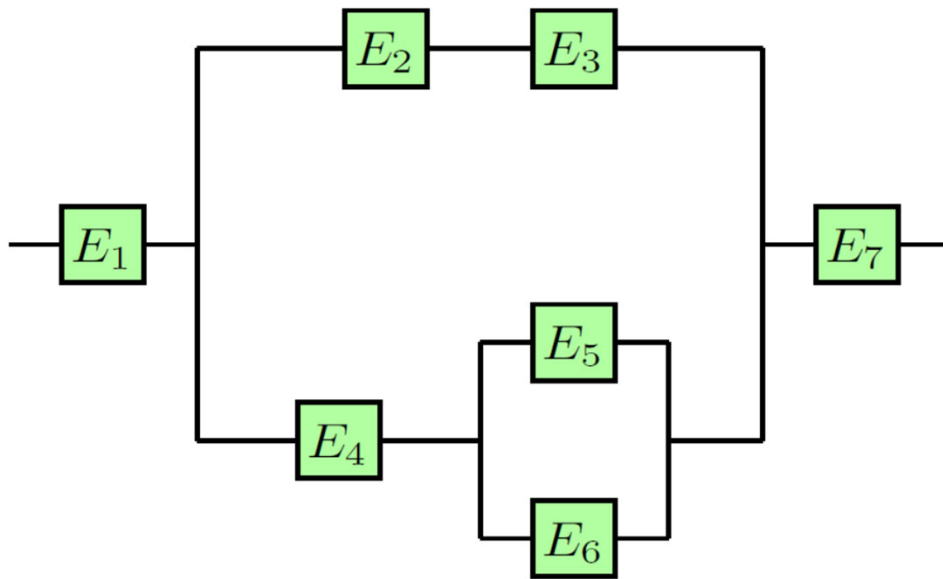


Component	$E_1$	$E_2$	$E_3$	$E_4$	$E_5$	$E_6$	$E_7$
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

$$P(\text{Works}) = 0.9 \cdot (1 - (1 - 0.5 \cdot 0.3) \cdot (1 - 0.1 \cdot (1 - 0.6 \cdot 0.5))) \cdot 0.8 = 0.15084$$

# Matlab group exercise

- Test our result for this circuit.
- Download `circuit_template.m` from the website



Component	$E_1$	$E_2$	$E_3$	$E_4$	$E_5$	$E_6$	$E_7$
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

$$P(\text{Works}) = 0.9 \cdot (1 - (1 - 0.5 \cdot 0.3)) \cdot (1 - 0.1 \cdot (1 - 0.6 \cdot 0.5)) \cdot 0.8 = 0.15084$$

# Here is how I did it

- Stats=1e6;
- count= 0;
- for i = 1: Stats
- e1 = rand < 0.9; e2 = rand < 0.5; e3 = rand < 0.3;
- e4 = rand < 0.1; e5 = rand < 0.4; e6 = rand < 0.5;
- e7 = rand < 0.8;
- s1 = min(e2,e3); % or s1 = e2\*e3;
- s2 = max(e5,e6); % or s2= e5+e6>0;
- s3 = min(e4,s2); % or s3 = e4\*s2;
- s4 = max(s1,s3); % or s4 = s1+s3 > 0;
- s5= min([e1;s4;e7]); % or s5=e1\*s4\*e7;
- count = count + s5;
- End;
- P\_circuit\_works = count/Stats
- **% our calculation: P(circuit\_works)= 0.9.\*(1-(1-0.5.\*0.3)).\*(1-0.1.\*(1-0.6.\*0.5))).\*0.8==0.15084**

Credit: XKCD  
comics

WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS  
WHY ARE AMERICANS AFRAID OF DRAGONS

WHY IS HTTPS CROSSED OUT IN RED  
WHY IS THERE A LINE THROUGH HTTPS  
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK  
WHY IS HTTPS IMPORTANT

# QUESTIONS

FOUND IN GOOGLE AUTOCOMplete



WHY ARE THERE WEEKS  
WHY DO I FEEL DIZZY

WHY AREN'T ECONOMISTS RICH

WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN  
WHY IS THERE PHLEGM

WHY DO AMERICANS CALL IT SOCCER

WHY IS PSYCHIC WEAK TO BUG

WHY ARE MY EARS RINGING

WHY DO CHILDREN GET CANCER

WHY ARE THERE SO MANY AVENGERS

WHY IS POSEIDON ANGRY WITH ODYSSEUS

WHY ARE THE AVENGERS FIGHTING THE X MEN

WHY IS THERE ICE IN SPACE

## WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED

WHY ARE THERE GHOSTS



WHY IS THERE AN OWL IN MY BACKYARD

WHY IS THERE AN OWL OUTSIDE MY WINDOW

WHY IS THERE AN OWL ON THE DOLLAR BILL

WHY DO OWLS ATTACK PEOPLE

WHY ARE AK 47s SO EXPENSIVE

WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE

WHY ARE THERE GODS

WHY ARE THERE TWO SPOCKS

WHY ARE CIGARETTES LEGAL

WHY ARE THERE DUCKS IN MY POOL

WHY IS JESUS WHITE

WHY IS THERE LIQUID IN MY EAR

WHY DO Q TIPS FEEL GOOD

WHY DO GOOD PEOPLE DIE

WHY ARE THERE TINY SPIDERS IN MY HOUSE

WHY DO SPIDERS COME INSIDE

WHY ARE THERE HUGE SPIDERS IN MY HOUSE

WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE

WHY ARE THERE SPIDERS IN MY ROOM

WHY ARE THERE SO MANY SPIDERS IN MY ROOM

WHY DO SPIDER BITES ITCH

WHY IS DYING SO SCARY

WHY IS THERE NO GPS IN LAPTOPS

WHY DO KNEES CLICK

WHY AREN'T THERE E GRADES

WHY IS SEX SO IMPORTANT



WHY IS MT VESUVIUS THERE

WHY DO THEY SAY T MINUS

WHY ARE THERE OBELISKS

WHY ARE WRESTLERS ALWAYS WET

WHY ARE OCEANS BECOMING MORE ACIDIC

WHY IS ARWEN DYING

WHY AREN'T MY QUAIL LAYING EGGS

WHY AREN'T MY QUAIL EGGS HATCHING

WHY IS LIFE SO BORING



WHY ARE ULTRASOUNDS IMPORTANT

WHY ARE ULTRASOUND MACHINES EXPENSIVE

WHY IS STEALING WRONG

WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA

WHY ARE DOGS AFRAID OF FIREWORKS  
WHY IS THERE NO KING IN ENGLAND

WHY DO WHALES JUMP  
WHY ARE WITCHES GREEN  
WHY ARE THERE MIRRORS ABOVE BEDS  
WHY DO I SAY UH  
WHY IS SEA SALT BETTER  
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS  
WHY IS THERE NOT A POKEMON MMO  
WHY IS THERE LAUGHING IN TV SHOWS  
WHY ARE THERE DOORS ON THE FREEWAY  
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING  
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA  
WHY ARE THERE SCARY SOUNDS IN MINECRAFT  
WHY IS THERE KICKING IN MY STOMACH  
WHY ARE THERE TWO SLASHES AFTER HTTP  
WHY ARE THERE CELEBRITIES  
WHY DO SNAKES EXIST  
WHY DO OYSTERS HAVE PEARLS  
WHY ARE DUCKS CALLED DUCKS  
WHY DO THEY CALL IT THE CLAP  
WHY ARE KYLE AND CARTMAN FRIENDS  
WHY IS THERE AN ARROW ON AANG'S HEAD  
WHY ARE TEXT MESSAGES BLUE  
WHY ARE THERE MUSTACHES ON CLOTHES  
WHY ARE THERE MUSTACHES ON CARS  
WHY ARE THERE MUSTACHES EVERYWHERE  
WHY ARE THERE SO MANY BIRDS IN OHIO  
WHY IS THERE SO MUCH RAIN IN OHIO  
WHY IS OHIO WEATHER SO WEIRD  
WHY ARE THERE MALE AND FEMALE BIKES

WHY AREN'T THERE DINOSAUR GHOSTS

WHY ARE THERE SQUIRRELS



WHY IS PROGRAMMING SO HARD

WHY IS THERE A 0 OHM RESISTOR

WHY DO AMERICANS HATE SOCCER

WHY DO RHYMES SOUND GOOD

WHY DO TREES DIE

WHY IS THERE HELL IF GOD FORGIVES

WHY ARE THERE TINY SPIDERS IN MY HOUSE

WHY DO SPIDERS COME INSIDE

WHY ARE THERE HUGE SPIDERS IN MY HOUSE

WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE

WHY ARE THERE SPIDERS IN MY ROOM

WHY ARE THERE SO MANY SPIDERS IN MY ROOM

WHY DO SPIDER BITES ITCH

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WHY IS THERE NO GPS IN LAPTOPS

WHY DO KNEES CLICK

WHY AREN'T THERE E GRADES

WHY IS SEX SO IMPORTANT

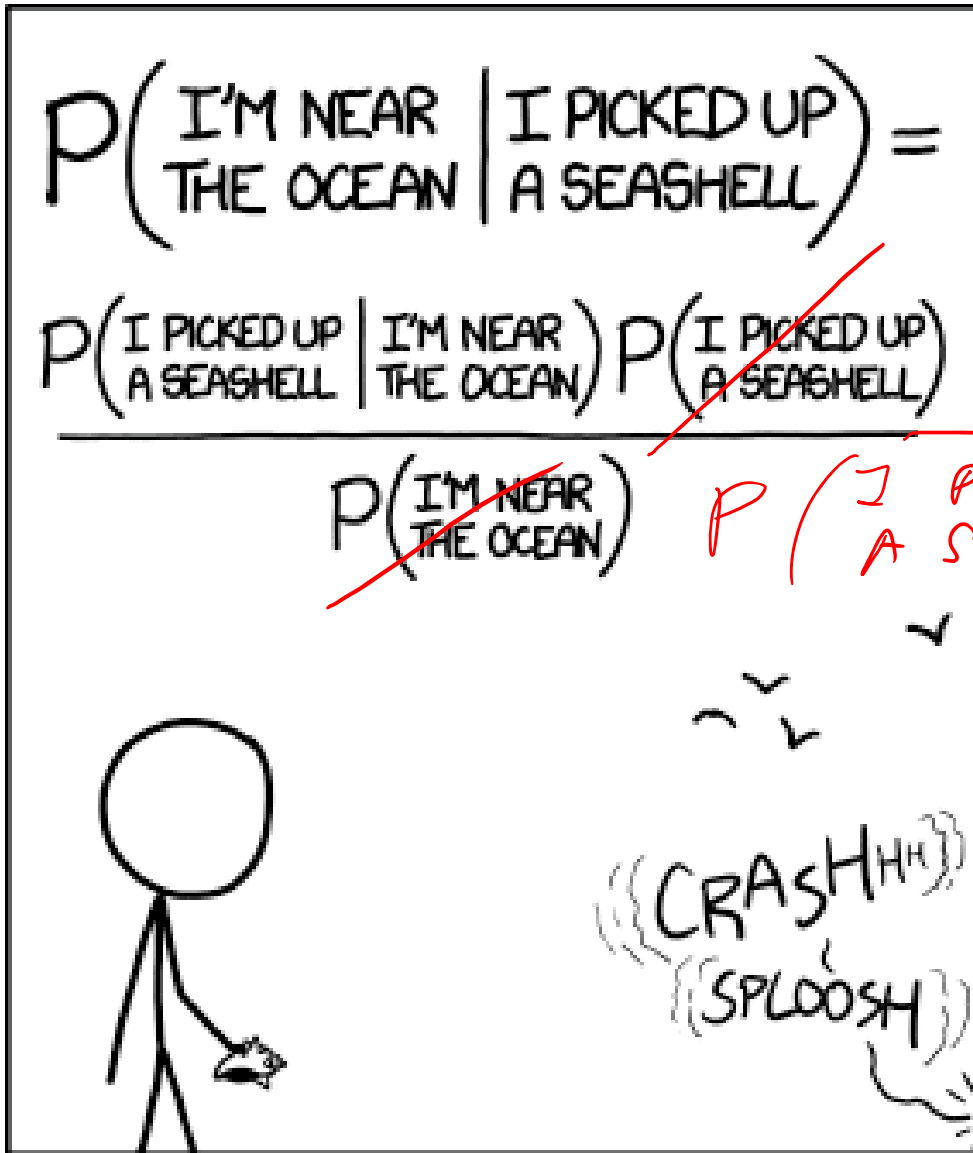


WHY ARE THERE FEMALE MR NIMES

WHY IS GPS FREE



Reminder:  
Conditional probability



STATISTICALLY SPEAKING, IF YOU PICK UP A SEASHELL AND DON'T HOLD IT TO YOUR EAR, YOU CAN PROBABLY HEAR THE OCEAN.

What is wrong in this comics?

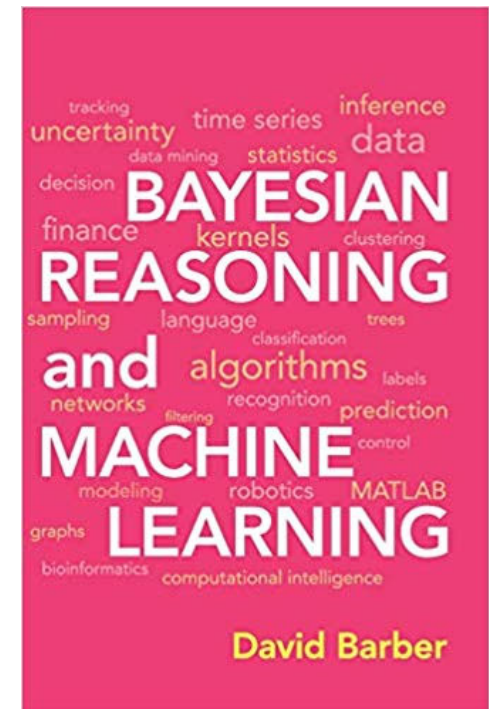
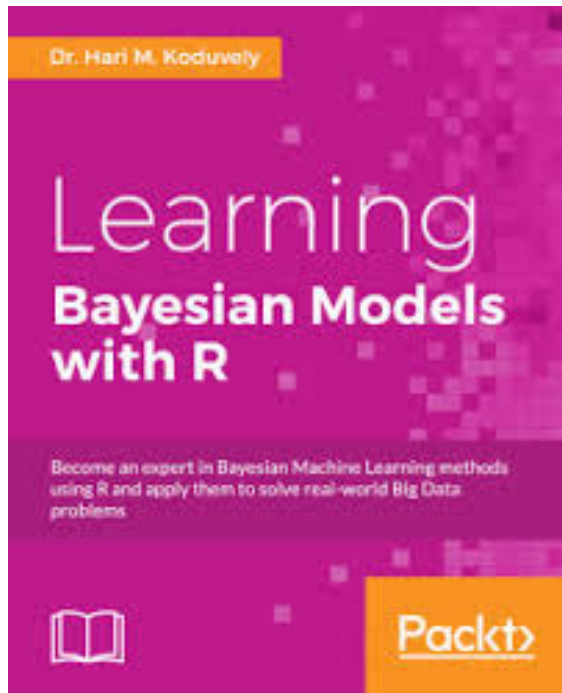
$P(I'M NEAR THE OCEAN)$   
 $P(I PICKED UP A SEASHELL)$

If you are not yet reading XKCD comics <https://xkcd.com/> you should start

# Bayes Theorem



# Bayes' theorem



Thomas Bayes (1701-1761)

English statistician, philosopher, and Presbyterian minister

Bayes' theorem was presented in "An Essay towards solving a Problem in the Doctrine of Chances" which was read to the Royal Society in 1763 already after Bayes' death.

# Bayes' theorem (simple)

$$P(A \cap B) = \underline{P(A|B)P(B)} = P(B \cap A) = \underline{P(B|A)P(A)}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- In Science **we often want to know**:  
“**How much faith** should I put into **hypothesis, given the data?**”  
or  $P(H|D)$  (see also the inductive definition of probability)
- What **we usually can calculate** if the hypothesis/model is OK:  
“Assuming that this **hypothesis is true**, what is the **probability of the observed data?**” or  $P(D|H)$
- Bayes' theorem can help:  $P(H|D) = P(D|H) \cdot P(H) / P(D)$
- The problem is  $P(H)$  (so-called **prior**) is often **not known**

# Bayes' theorem (continued)

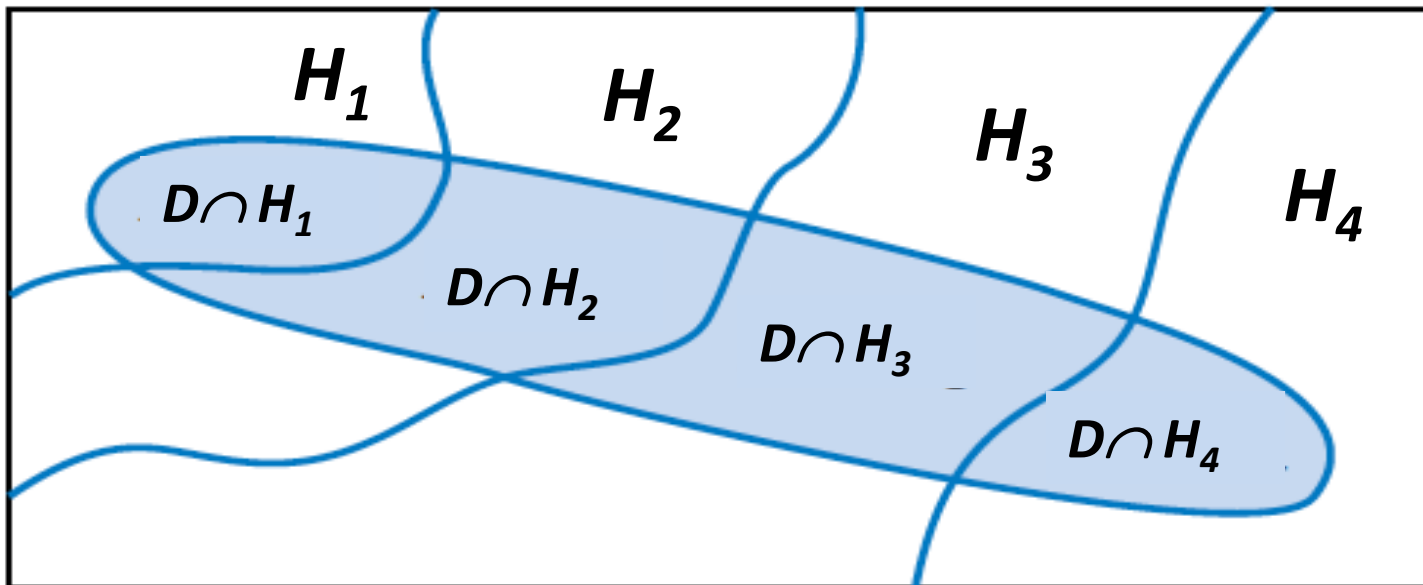
Works best with **exhaustive** and **mutually-exclusive** hypotheses:

$H_1, H_2, \dots, H_n$  such that  $H_1 \cup H_2 \cup H_3 \dots \cup H_n = S$  and  $H_i \cap H_j = \emptyset$  for  $i \neq j$

$$P(H_k|D) = P(D|H_k) \cdot P(H_k) / P(D)$$

where:

$$P(D) = P(D|H_1) \cdot P(H_1) + P(D|H_2) \cdot P(H_2) + \dots + P(D|H_n) \cdot P(H_n)$$



An awesome new test has been invented for an early detection of cancer. The probability that it **correctly identifies someone with cancer as positive is 95%**, and the probability that it **correctly identifies someone without cancer as negative is 99%**. The **incidence** of this type of cancer in the general population is  $10^{-4}$ . A random person in the population takes the test, and the result is positive.

What is the probability that he/she has cancer?

- A. 99%
- B. 95%
- C. 30%
- D. 1%

Get your i-clickers

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- C. 30%
- D. 1%

Get your i-clickers



participants  
 $10^6$  ← 100 - cancer — 95 positive tests  
 $10^6 - 100 \approx 10^6$  no cancer

$10^6$  participants with no cancer → 10,000 positive tests

$$P(C|P) = \frac{95}{10,000 + 95} \approx 1\%$$





Events:  $C$  - cancer,  $C'$  - no cancer  
Test events  $Y$  - positive,  $N$  - negative

We know:

$$P(C) = 10^{-4}, \quad P(Y|C) = 0.95$$
$$P(N|C') = 0.99$$

We need

$$P(C|Y)$$

Bayes =

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)} ?$$



$P(Y)$  - probability that a random person will test positive

$$\begin{aligned} P(Y) &= P(Y \cap C) + P(Y \cap C') = \\ &= P(Y|C)P(C) + P(Y|C')P(C') = \\ &= 0.95 \times 10^{-4} + (1 - 0.99) \times (1 - 10^{-4}) \approx \\ &\approx 10^{-4} + 10^{-2} \approx 10^{-2} = 1\% \end{aligned}$$

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)} = 0.95 \times \frac{10^{-4}}{10^{-2}} \approx 1\%$$

An awesome new test has been invented for an early detection of cancer. The probability that it **correctly identifies someone with cancer as positive is 95%**, and the probability that it **correctly identifies someone without cancer as negative is 99%**. The **incidence** of this type of cancer in the general population is  $10^{-4}$ . A suspected cancer patient with likelihood of cancer 50% takes the test, and the result is positive.

What is the probability that he/she has cancer?

- A. 99%
- B. 95%
- C. 30%
- D. 1%

Get your i-clickers

An awesome new test has been invented for an early detection of cancer. The probability that it **correctly identifies someone with cancer as positive is 95%**, and the probability that it **correctly identifies someone without cancer as negative is 99%**. The **incidence** of this type of cancer in the general population is  $10^{-4}$ . A suspected cancer patient with likelihood of cancer 50% takes the test, and the result is positive.

What is the probability that he/she has cancer?

A. 99%

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C. 30%

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Get your i-clickers