Midterm Review
Midterm will be held here in class this Thursday (9th March) during regular class hours 9:30am-10:50am
Exam Logistics

- Closed book exam; no books, notes, laptops, phones, etc.
- Calculators (not on smartphones) can be used
- You can prepare one 2-sided cheat sheet
- You will be provided with printouts for:
  - The Z-score table
  - A list of discrete and continuous probability distributions
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<td><strong>Discrete</strong></td>
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<tr>
<td>Uniform</td>
<td>[ \frac{1}{n}, a \leq b ]</td>
<td>( \frac{b + a}{2} )</td>
<td>( \frac{(b - a + 1)^2 - 1}{12} )</td>
<td>3-5</td>
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<td>Binomial</td>
<td>( \left( \begin{array}{c} n \ x \end{array} \right) p^x (1 - p)^{n-x}, ) ( x = 0, 1, \ldots, n, 0 \leq p \leq 1 )</td>
<td>( np )</td>
<td>( np(1 - p) )</td>
<td>3-6</td>
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<tr>
<td>Geometric</td>
<td>( (1 - p)^{x-1} p, ) ( x = 1, 2, \ldots, 0 \leq p \leq 1 )</td>
<td>( \frac{1}{p} )</td>
<td>( \frac{(1 - p)}{p^2} )</td>
<td>3-7.1</td>
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<tr>
<td>Negative binomial</td>
<td>( \binom{x - 1}{r - 1} (1 - p)^{r-1} p^x ) ( x = r, r + 1, r + 2, \ldots, 0 \leq p \leq 1 )</td>
<td>( \frac{r}{p} )</td>
<td>( \frac{r(1 - p)}{p^2} )</td>
<td>3-7.2</td>
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<td>Hypergeometric</td>
<td>( \frac{\binom{K}{x} \binom{N - K}{n - x}}{\binom{N}{n}} ) ( x = \text{max}(0, n - N + K), 1, \ldots ), ( x = \text{min}(K, n), K \leq N, n \leq N )</td>
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<tr>
<td>Poisson</td>
<td>( \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \ldots, 0 &lt; \lambda )</td>
<td>( \lambda )</td>
<td>( \lambda )</td>
<td>3-9</td>
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<td><strong>Continuous</strong></td>
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<td>Uniform</td>
<td>( \frac{1}{b - a}, a \leq x \leq b )</td>
<td>( \frac{b + a}{2} )</td>
<td>( \frac{(b - a)^2}{12} )</td>
<td>4-5</td>
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<td>Normal</td>
<td>( \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} )</td>
<td>( \mu )</td>
<td>( \sigma^2 )</td>
<td>4-6</td>
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<td>Exponential</td>
<td>( \lambda e^{-\lambda x}, 0 \leq x, 0 &lt; \lambda )</td>
<td>( \frac{1}{\lambda} )</td>
<td>( \frac{1}{\lambda^2} )</td>
<td>4-8</td>
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<tr>
<td>Erlang</td>
<td>( \frac{\lambda x^{r-1} e^{-\lambda x}}{(r - 1)!}, 0 &lt; x, r = 1, 2, \ldots )</td>
<td>( \frac{r}{\lambda} )</td>
<td>( \frac{r^2}{\lambda^2} )</td>
<td>4-9.1</td>
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<tr>
<td>Gamma</td>
<td>( \frac{\lambda^x \Gamma(r)}{\Gamma(r)}, 0 &lt; x, 0 &lt; r, 0 &lt; \lambda )</td>
<td>( \frac{r}{\lambda} )</td>
<td>( \frac{r^2}{\lambda^2} )</td>
<td>4-9.2</td>
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What could be on the exam?

- Probability of events and set operations
- Multiplication rules. Combinatorics
- Conditional probability and Bayes Theorem
- Discrete Random Variables
- Continuous Random Variables
- Other topics covered

Problems will be similar in style to homework problems.
Probability, Multiplication Rules, Combinatorics
Mr. Jones has 6 different books that he is going to put on his bookshelf. Of these, 3 are chemistry books, 2 are physics books, and 1 is a mathematics book. Jones wants to arrange his books so that two conditions are met:

- all the books dealing with the same subject are together on the shelf AND
- all chemistry books are on the leftmost side.

How many such different arrangements are possible?
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And there are are 3! ways to arrange the chemistry books, 2! ways to arrange physics books and 1! ways to arrange math books.
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- all the books dealing with the same subject are together on the shelf AND
- all chemistry books are on the leftmost side.

How many such different arrangements are possible?

And there are are 3! ways to arrange the chemistry books, 2! ways to arrange physics books and 1! ways to arrange math books.

2! (3!*2!*1!)
Mr. Jones has 6 different books that he is going to put on his bookshelf. Of these, 3 are chemistry books, 2 are physics books, and 1 is a mathematics book. Jones wants to arrange his books so that two conditions are met:

- all the books dealing with the same subject are together on the shelf AND
- all chemistry books are on the leftmost side.

How many such different arrangements are possible?

And there are are 3! ways to arrange the chemistry books, 2! ways to arrange physics books and 1! ways to arrange math books.
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?

\[0.3 \times 0.9 \times 0.7 = 0.19\]
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?
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\[ 1 - (1 - 0.8)(1-0.1) = 0.82 \]
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?
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\[ 0.82 \times 0.5 = 0.41 \]
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?
(4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?

\[
1 - (1 - 0.41)(1 - 0.19) = 0.52
\]
In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let 1/3 be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

Let K be the event that he knows the answer and C be the event that he answered correctly.

What are the probabilities that are given to us?
In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let $1/3$ be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

Let $K$ be the event that he knows the answer and $C$ be the event that he answered correctly

$P(K) = 1/3, \quad P(K') = 2/3, \quad P(C|K) = 1, \quad P(C|K') = 1/4$
In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let $1/3$ be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

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What is the $P(K|C)$?
In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let 1/3 be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

Let K be the event that he knows the answer and C be the event that he answered correctly

\[ P(K) = 1/3, \quad P(K') = 2/3, \quad P(C|K) = 1, \quad P(C|K') = 1/4 \]

\[ P(K|C) = \frac{P(C|K)P(K)}{P(C)} \]

where

\[ P(C) = P(C|K)P(K) + P(C|K')P(K') \]

\[ = \frac{1 \times (1/3)}{1 \times (1/3) + (1/4) \times (2/3)} \]

\[ = 2/3 = 0.666... \]
Discrete Probability Distributions
Strategy: What is X in this problem?

- What is the random variable? Look for keywords:
  - Find the probability that....
  - What is the mean (or variance) of...
- What are parameters? Look for keywords:
  - Given that...
  - Assuming that...

3. Find x.
A guide to discrete distributions

- Binomial: Given a fixed number of trials, $n$, the number of successes, $x$ is the random variable.
- Geometric: The random variable is the number of trials to get 1 success.
- Negative binomial: The random variable is the number of trials to get a fixed number of $r$ successes.
- Poisson: The random variable is the number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate.
Poisson distribution in genomics

- $G$ - genome length (in bp)
- $L$ - short read average length
- $N$ – number of short read sequenced
- $\lambda$ – sequencing redundancy = $LN/G$
- $x$- number of short reads covering a given site on the genome

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Ewens, Grant, Chapter 5.1

Poisson as a limit of Binomial. For a given site on the genome for each short read Prob(site covered): $p=L/G$ is very small. Number of attempts (short reads): $N$ is very large. Their product (sequencing redundancy): $\lambda = NL/G$ is $O(1)$. 
Probability that a base pair in the genome is not covered by any short reads is 0.1. One randomly selects base pairs until exactly 5 uncovered base pairs are found. Which discrete probability distribution describes the number of attempts?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Poisson</td>
<td>$\frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, ..., 0 &lt; \lambda$</td>
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<tr>
<td>Binomial</td>
<td>( \binom{n}{x} p^x (1-p)^{n-x} )</td>
</tr>
<tr>
<td>Geometric</td>
<td>( (1-p)^{x-1} p )</td>
</tr>
<tr>
<td>Negative Binomial</td>
<td>( \binom{x-1}{r-1} (1-p)^{x-r} p^r )</td>
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</table>
Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

Which discrete probability distribution describes the number of attempts?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea
Probability that a base pair in the genome is not covered by any short reads is 0.1. One randomly selects base pairs until exactly 5 uncovered base pairs are found. What are the values of $p$, $r$?

A. $p=0.5$, $r=5$
B. $p=0.1$, $r=0.5$
C. $p=0.1$, $r=5$
D. $p=0.5$, $r=0.1$
E. I have no idea
Probability that a base pair in the genome is not covered by any short reads is 0.1.

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

What are the values of p, r?

A. p=0.5, r=5
B. p=0.1, r=0.5
C. p=0.1, r=5
D. p=0.5, r=0.1
E. I have no idea
Cancer happens when the gene p53 mutates. Probability of p53 to mutate per year is 5%.

How many years before a patient gets disease? Which discrete probability distribution would you use to answer?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea
Cancer happens when the gene p53 mutates. Probability of p53 to mutate per year is 5%.

How many years before a patient gets disease?

Which discrete probability distribution would you use to answer?

A. Poisson
B. Binomial
C. Geometric
D. Negative Binomial
E. I have no idea
Continuous Probability Distributions
(8 points) The expression level of a TP53 tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A) (4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

(B) (4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” TP53 expression?
(8 points) The expression level of a TP53 tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A) (4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

Let $X \sim \text{Norm}(\mu = 20, \sigma = 8)$

$P(16 < X < 24) = P\left(\frac{16 - 20}{8} < Z < \frac{24 - 20}{8}\right) = P(-0.5 < Z < 0.5)$

(B) (4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” TP53 expression?
(8 points) The expression level of a TP53 tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A)(4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

Let $X \sim \text{Norm}(\mu=20,\sigma=8)$

$P(16 < X < 24) = P\left(\frac{(16-20)}{8} < Z < \frac{(24-20)}{8}\right) = P(-0.5 < Z < 0.5)$

(B)(4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” TP53 expression?
Let $X \sim \text{Norm}(\mu=20, \sigma=8)$

$P(16<X<24) = P\left(\frac{16-20}{8} < Z < \frac{24-20}{8}\right) = P(-0.5 < Z < 0.5)$

$= P(Z<0.5) - P(Z<-0.5)$

(A) (4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

Let $X \sim \text{Norm}(\mu=20, \sigma=8)$

$P(16<X<24) = P\left(\frac{16-20}{8} < Z < \frac{24-20}{8}\right) = P(-0.5 < Z < 0.5)$

$= P(Z<0.5) - P(Z<-0.5)$

(B) (4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” $TP53$ expression?
$P(Z \leq -0.5)$
P(Z< -0.5)
$P(Z<-0.5)$

$P(Z>0.5)$
$P(Z<-0.5)$

$1 - P(Z<0.5)$
(8 points) The expression level of a TP53 tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A)(4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

Let $X \sim \text{Norm}(\mu=20, \sigma=8)$

$P(16<X<24) = P\left((16-20)/8 < Z < (24-20)/8\right) = P(-0.5 < Z < 0.5)$

$= P(Z<0.5) - P(Z<-0.5)$

$= P(Z<0.5) - (1 - P(Z<0.5))$

$= 0.6914 - (1-0.6914) = 0.3829$

(B)(4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” TP53 expression?
The expression level of a TP53 tumor suppressor gene in a randomly selected cell is normally distributed with mean $\mu = 20$, and standard deviation $\sigma = 8$.

(A) (4 points) What is the probability that the expression level in a given cell will be between 24 and 16?

Let $X \sim \text{Norm}(\mu = 20, \sigma = 8)$

$$P(16 < X < 24) = P\left(\frac{(16-20)}{8} < Z < \frac{(24-20)}{8}\right) = P(-0.5 < Z < 0.5)$$

$$= P(Z < 0.5) - P(Z < -0.5)$$

$$= P(Z < 0.5) - (1 - P(Z < 0.5))$$

$$= 0.6914 - (1 - 0.6914) = 0.3829$$

(B) (4 points) How many cells does one have to sample (on average) until there will be exactly 2 cells with such “close to average” TP53 expression?

Using the negative binomial distribution one gets

$$\frac{2}{0.3829} = 5.22$$
Exponential Distribution

Assume: expected number of successes produced in x seconds is $r \cdot x$

Exponential random variable $X$ describes interval between two successes of a constant rate (Poisson) random process with success rate $r$ per unit interval.
You have been calling a call center multiple times to solve a problem with your utility bill. Suppose the waiting time on a phone call to a call centre is an exponential distribution with a mean rate of one of your calls answered every 10 minutes.

What is the probability that you have to wait longer than 20 mins for your next call to be answered?
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\[ \lambda = 1 \text{ call} / 10 \text{ minutes} = 0.1 \text{ calls/minute}. \]

Exponential distribution

\[ X \sim \text{Exp}(\lambda = 0.1) \]

\[ P(X > 20) = \ldots \]
You have been calling a call center multiple times to solve a problem with your utility bill. Suppose the waiting time on a phone call to a call center is an exponential distribution with a mean rate of one of your calls answered every 10 minutes.

What is your expected wait time to have 6 calls answered?

\[ \lambda = \text{1 call/10 minutes} = 0.1 \text{ calls/minute.} \]

\[ r = 6 \]
You have been calling a call center multiple times to solve a problem with your utility bill. Suppose the waiting time on a phone call to a call centre is an exponential distribution with a mean rate of one of your calls answered every 10 minutes.

What is your expected wait time to have 6 calls answered?

\[
\lambda = \frac{1 \text{ call}}{10 \text{ minutes}} = 0.1 \text{ calls/minute.}
\]

\[ r = 6 \]

Erlang distribution

\[ X \sim \text{Erlang}(\lambda = 0.1, r = 6) \]

\[
E[X] = \frac{6}{0.1} = 60 \text{ minutes}
\]