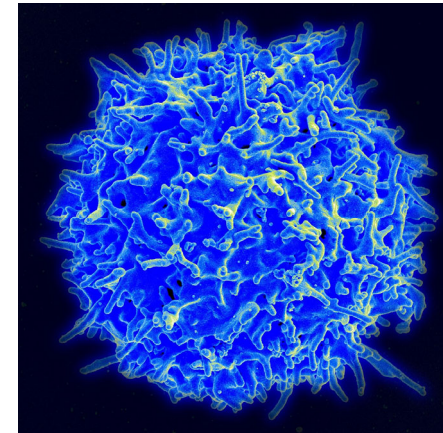


# Clustering analysis of gene expression data

Chapter 11 in  
Jonathan Pevsner,  
Bioinformatics and Functional Genomics,  
3<sup>rd</sup> edition  
(Chapter 9 in 2<sup>nd</sup> edition)

# Human T cell expression data

- The matrix contains **47 expression samples** from Lukk et al, Nature Biotechnology 2010
- All samples are **from T cells in different individuals**
- Only the **top 3000 genes** with the largest variability **were used**
- The value is **log2 of gene's expression level** in a given sample as measured by the microarray technology



## A global map of human gene expression

Margus Lukk, Misha Kapushesky, Janne Nikkilä, Helen Parkinson, Angela Goncalves, Wolfgang Huber, Esko Ukkonen & Alvis Brazma

Affiliations | Corresponding author

*Nature Biotechnology* **28**, 322–324 (2010) | doi:10.1038/nbt0410-322

Although there is only one human genome sequence, different genes are expressed in many different cell types and tissues, as well as in different developmental stages or diseases. The structure of this 'expression space' is still largely unknown, as most transcriptomics experiments focus on sampling small regions. We have constructed a global gene expression map by integrating microarray data from 5,372 human samples representing 369 different cell and tissue types, disease states and cell lines. These have been compiled in an online resource (<http://www.ebi.ac.uk/gxa/array/U133A>) that allows the user to search for a gene of interest and



**WHEEL OF FORTUNE**

**Correlated pairs  
plausible biological connection based  
on short description**

**g1=1994; g2=188; group 1**

**g1=2872; g2=1269; group 2**

**g1=1321; g2=10; group 3**

**g1= 886; g2=819; group 4**

**g1=2138; g2=1364; group 5**

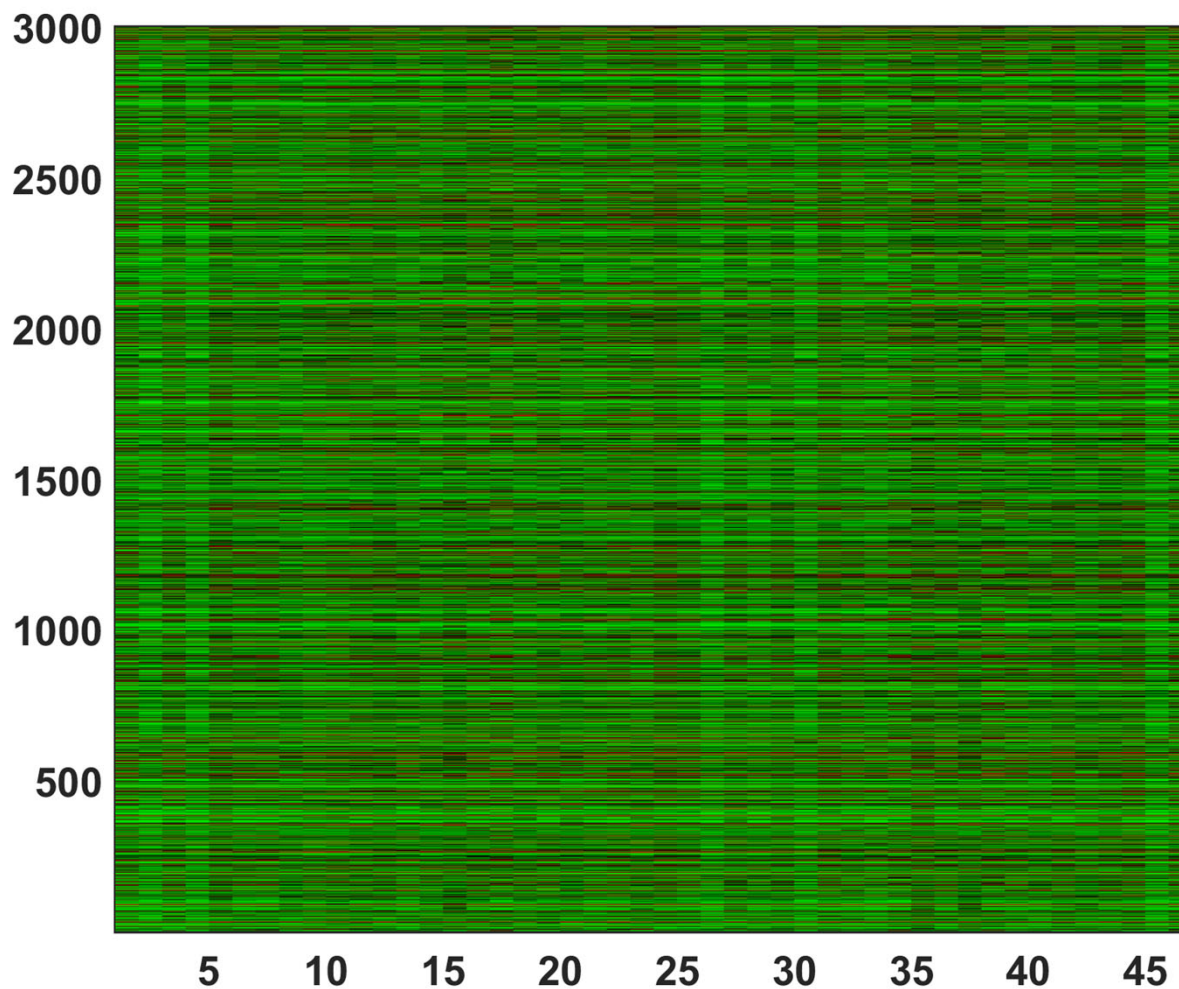
**no obvious biological common function**

```
g1=1+floor(rand.*3000); g2=1+floor(rand.*3000);  
disp([g1, g2])
```

# Matlab exercise

- Every group works with  
g0=2907; g1=1527; g2=2629; g3=2881;  
g4=1144; g5=1066;
- Compute **Multiple Linear Regression (MLR)**,  
where  $y = \text{exp\_t}(g0)$ ;  
 $x1 = \text{exp\_t}(g1)$ ;  $x2 = \text{exp\_t}(g2)$ ;
- **How much better** the MLR did compared to the  
Single Linear Regression (SLR)?
- **Continue increasing** the number of genes in x  
until  $R_{\text{adj}}$  starts to decrease

How to find the entire groups of mutually correlated genes if you have **many genes** and **many samples**?



Clustering to the rescue!



# Clustering is a part of Machine Learning

- ***Supervised Learning:***

A machine learning technique whereby a system uses a set of human-labelled training examples to learn how to correctly perform a task

**Example:** a sample of cancer expression profiles each annotated with cancer type

**Goal:** predict cancer type based on expression pattern

- ***Unsupervised Learning (including clustering):***

In machine learning, unsupervised learning is a class of problems in which one seeks to determine how the data are organized. One only has unlabeled examples.

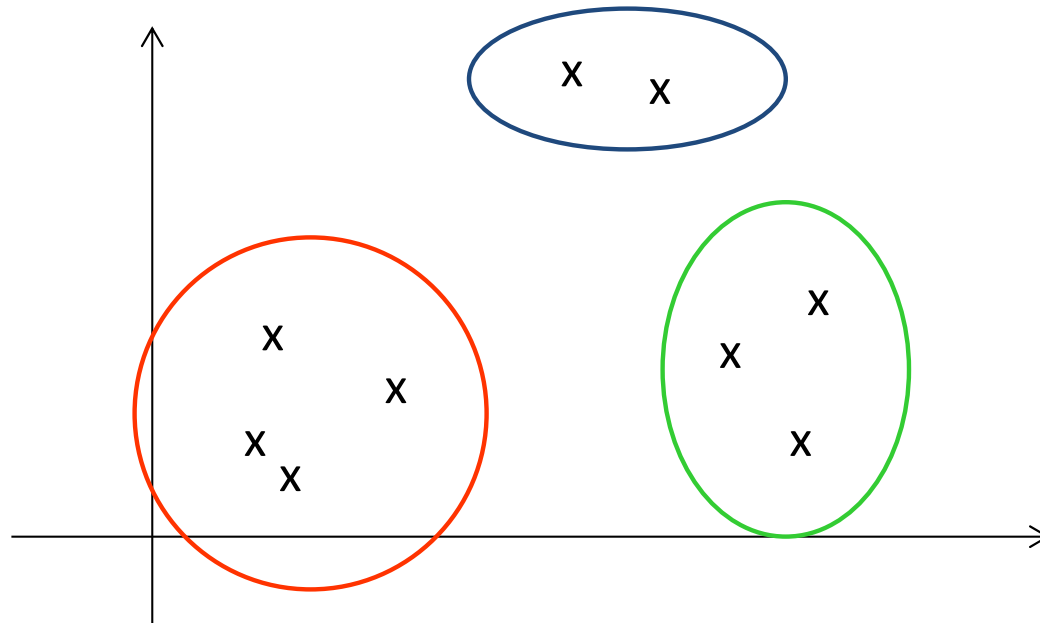
**Example:** a sample of breast cancer expression profiles.

**Goal:** Identify several different (yet unknown) subtypes with potentially different treatments



# What is clustering?

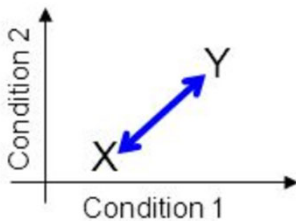
- The goal of **clustering** is to
  - group data points that are close (or **similar**) to each other
  - Usually, one needs to identify such groups (or clusters) in an **unsupervised** manner
  - Sometimes one takes into account **prior information** (Bayesian methods)
- Need to define some **distance  $d_{ij}$**  between **objects  $i$  and  $j$**
- Clustering is easy in **2 dimensions** but **hard in 3000 dimensions** -> need to somehow **reduce dimensionality**



# How to define the distance?

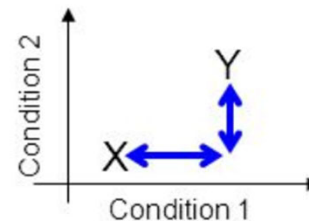
- Euclidean distance:
  - Most commonly used distance
  - Sphere shaped cluster
  - Corresponds to the geometric distance into the multidimensional space

$$d(X, Y) = \sqrt{\sum_i (x_i - y_i)^2}$$



- City Block (Manhattan) distance:
  - Sum of differences across dimensions
  - Less sensitive to outliers
  - Diamond shaped clusters

$$d(X, Y) = \sum_i |x_i - y_i|$$



The Canberra distance metric is calculated in R by

$$\sum \left( \frac{|x_i - y_i|}{|x_i + y_i|} \right).$$

Correlation coefficient distance

$$d(X, Y) = 1 - \rho(X, Y) = 1 - \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}}$$

# Common types of clustering algorithms

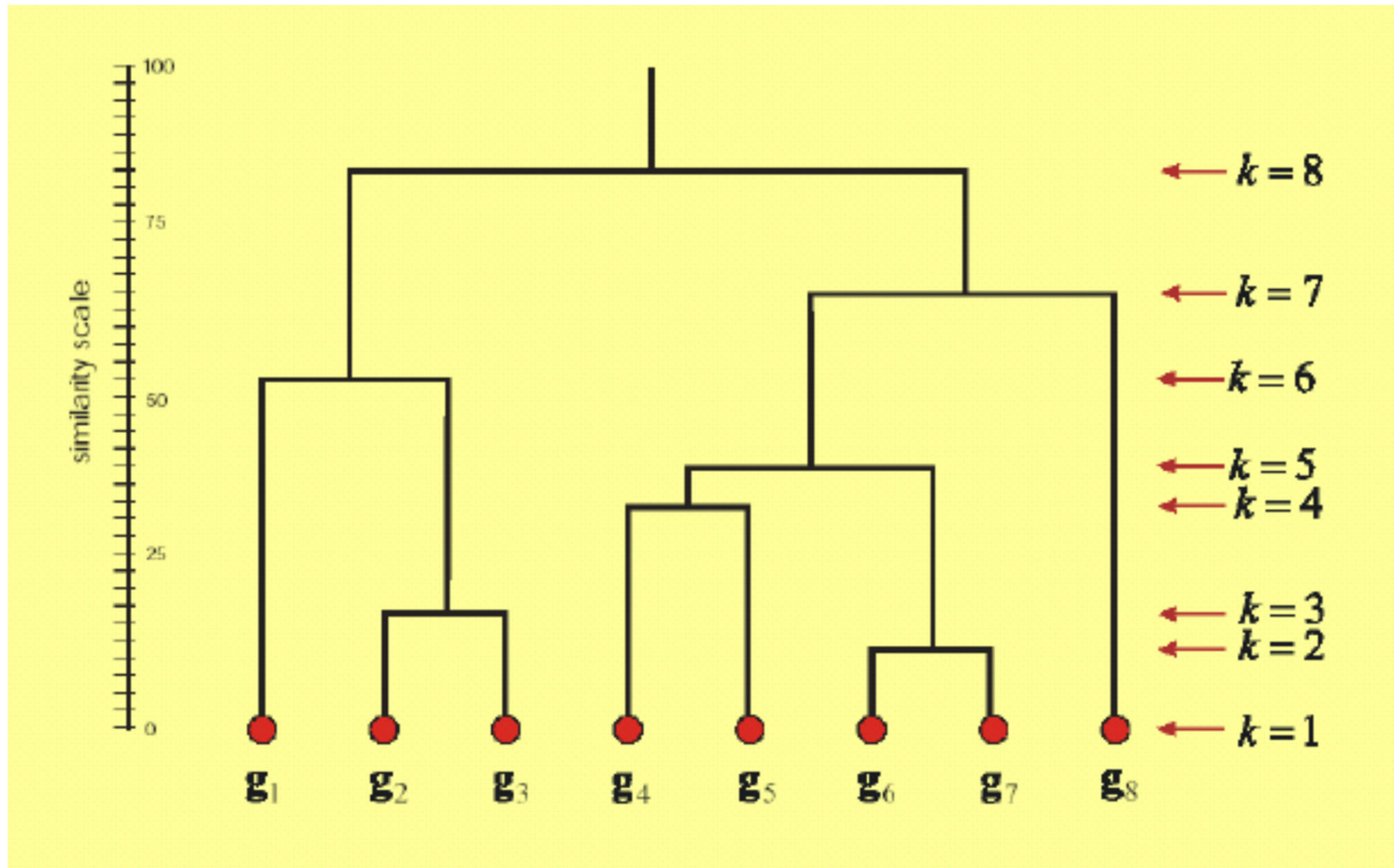
- Hierarchical if one doesn't know in advance the # of clusters
  - Agglomerative: start with N clusters and gradually merge them into 1 cluster
  - Divisive: start with 1 cluster and gradually break it up into N clusters
- Non-hierarchical algorithms
  - K-means clustering:
    - Iteratively apply the following two steps:
    - Calculate the centroid (center of mass) of each cluster
    - Assign each to the cluster to the nearest centroid
  - Principal Component Analysis (PCA)
    - plot pairs of top eigenvectors of the covariance matrix  $\text{Cov}(X_i, X_j)$  and uses visual information to group

# Hierarchical clustering

# UPGMA algorithm

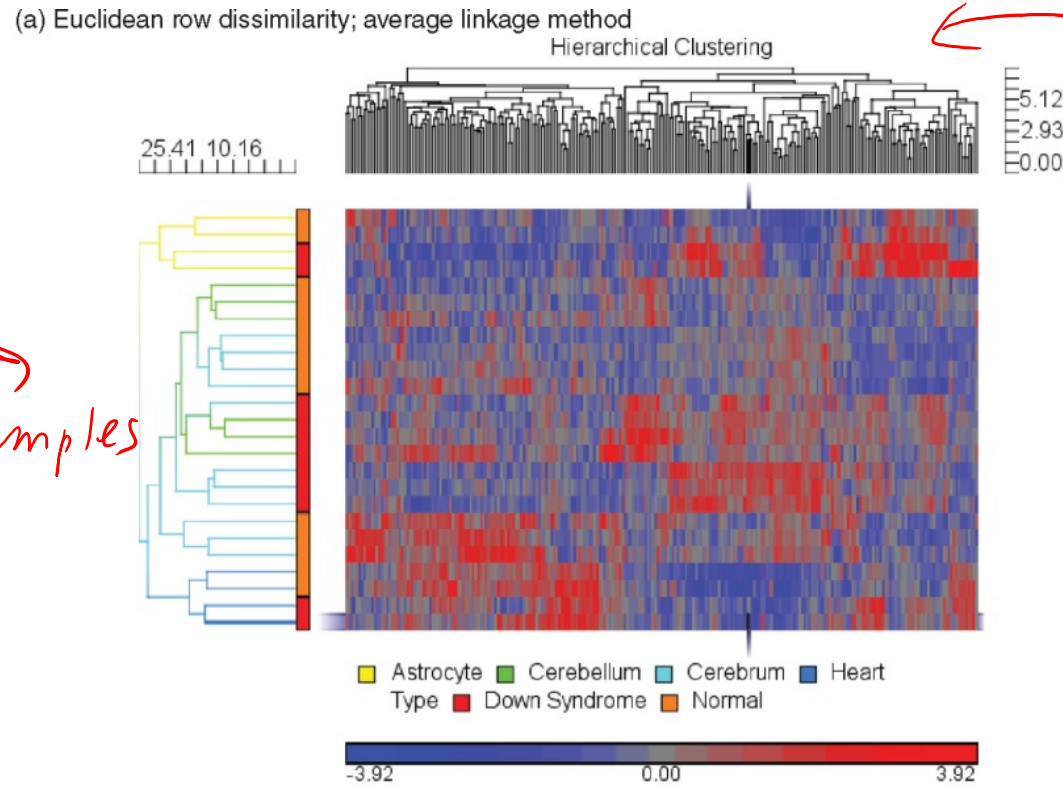
- Hierarchical agglomerative clustering algorithm
- **UPGMA** = **U**nweighted **P**air **G**roup **M**ethod with **A**rithmetic mean
- **Iterative** algorithm:
- Start with a **pair with the smallest  $d(X,Y)$**
- **Cluster these two together** and replace it with their arithmetic mean  $(X+Y)/2$
- **Recalculate all distances to this new “cluster node”**
- **Repeat** until all nodes are merged

# Output of UPGMA algorithm



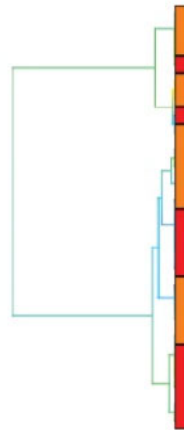
UPGMA  
algorithm

25 samples

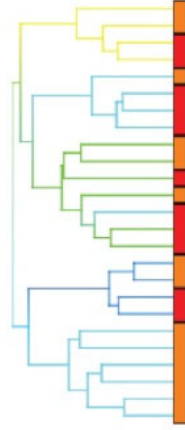


250 genes  
on  
Chromosome  
21

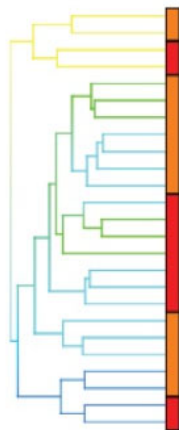
(b) Canberra dissimilarity



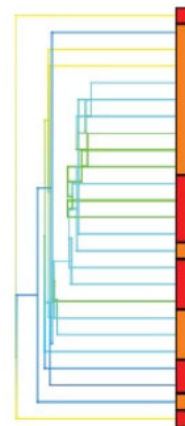
(c) Pearson's Dissimilarity



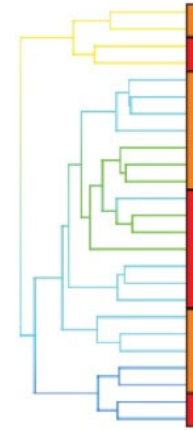
(d) City Block



(e) Euclidean, centroid linkage



(f) Euclidean, complete-linkage



**FIGURE 11.16** Hierarchical clustering of 250 chromosome 21 transcripts in 25 samples using Partek software. (a) Hierarchical clustering of microarray data using the default settings of Euclidean dissimilarity for rows (samples) and columns (transcripts). Colors correspond to expression intensity values.



Credit: XKCD  
comics

# WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS  
WHY ARE AMERICANS AFRAID OF DRAGONS

WHY IS HTTPS CROSSED OUT IN RED  
WHY IS THERE A LINE THROUGH HTTPS  
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK  
WHY IS HTTPS IMPORTANT

# QUESTIONS FOUND IN GOOGLE AUTOCOMPLETE



WHY ARE THERE WEEKS  
WHY DO I FEEL DIZZY

WHY DO WHALES JUMP  
WHY ARE WITCHES GREEN  
WHY ARE THERE MIRRORS ABOVE BEDS  
WHY DO I SAY UH  
WHY IS SEA SALT BETTER  
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS  
WHY IS THERE NOT A POKEMON MMO  
WHY IS THERE LAUGHING IN TV SHOWS  
WHY ARE THERE DOORS ON THE FREEWAY  
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING  
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA  
WHY ARE THERE SCARY SOUNDS IN MINECRAFT  
WHY IS THERE KICKING IN MY STOMACH  
WHY ARE THERE TWO SLASHES AFTER HTTP  
WHY ARE THERE CELEBRITIES  
WHY DO SNAKES EXIST  
WHY DO OYSTERS HAVE PEARLS  
WHY ARE DUCKS CALLED DUCKS  
WHY DO THEY CALL IT THE CLAP  
WHY ARE KYLE AND CARTMAN FRIENDS  
WHY IS THERE AN ARROW ON AANG'S HEAD  
WHY ARE TEXT MESSAGES BLUE  
WHY ARE THERE MUSTACHES ON CLOTHES  
WHY ARE THERE MUSTACHES ON CARS  
WHY ARE THERE MUSTACHES EVERYWHERE  
WHY ARE THERE SO MANY BIRDS IN OHIO  
WHY IS THERE SO MUCH RAIN IN OHIO  
WHY IS OHIO WEATHER SO WEIRD

WHY DO IGUANAS DIE  
WHY AREN'T THERE DINOSAUR GHOSTS

WHY AREN'T ECONOMISTS RICH  
WHY DO AMERICANS CALL IT SOCCER  
WHY ARE MY EARS RINGING  
WHY ARE THERE SO MANY AVENGERS  
WHY ARE THE AVENGERS FIGHTING THE X MEN  
WHY IS WOLVERINE NOT IN THE AVENGERS

WHY ARE THERE SWARMS OF GNATS  
WHY IS THERE PHLEGM  
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN  
WHY IS PSYCHIC WEAK TO BUG  
WHY DO CHILDREN GET CANCER  
WHY IS POSEIDON ANGRY WITH ODYSSEUS  
WHY IS THERE ICE IN SPACE

# WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED  
WHY IS SPACE BLACK  
WHY IS OUTER SPACE SO COLD  
WHY ARE THERE PYRAMIDS ON THE MOON  
WHY IS NASA SHUTTING DOWN

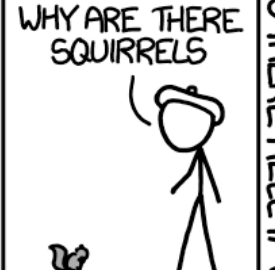


WHY IS THERE AN OWL IN MY BACKYARD  
WHY IS THERE AN OWL OUTSIDE MY WINDOW  
WHY IS THERE AN OWL ON THE DOLLAR BILL  
WHY DO OWLS ATTACK PEOPLE  
WHY ARE AK 47s SO EXPENSIVE  
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE  
WHY ARE THERE GODS  
WHY ARE THERE TWO SPOCKS

WHY ARE DOGS AFRAID OF FIREWORKS  
WHY IS THERE NO KING IN ENGLAND

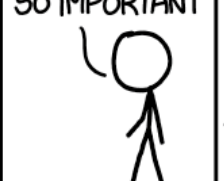
WHY ARE THERE BRIDESMAIDS  
WHY DO DYING PEOPLE REACH UP  
WHY AREN'T THERE VARICOSE ARTERIES  
WHY ARE OLD KUNGONS DIFFERENT

WHY ARE THERE TINY SPIDERS IN MY HOUSE  
WHY DO SPIDERS COME INSIDE  
WHY ARE THERE HUGE SPIDERS IN MY HOUSE  
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE  
WHY ARE THERE SPIDERS IN MY ROOM  
WHY ARE THERE SO MANY SPIDERS IN MY ROOM  
WHY DO SPIDER BITES ITCH  
WHY IS DYING SO SCARY



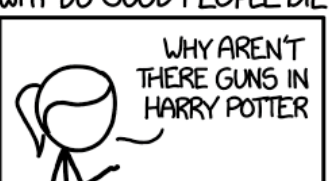
WHY IS THERE NO GPS IN LAPTOPS  
WHY DO KNEES CLICK  
WHY AREN'T THERE E GRADES  
WHY IS ISOLATION BAD  
WHY DO BOYS LIKE ME  
WHY DON'T BOYS LIKE ME  
WHY IS THERE ALWAYS A JAVA UPDATE  
WHY ARE THERE RED DOTS ON MY THIGHS  
WHY IS LYING GOOD

WHY IS SEX SO IMPORTANT



WHY IS MT VESUVIUS THERE  
WHY DO THEY SAY T MINUS  
WHY ARE THERE OBELISKS  
WHY ARE WRESTLERS ALWAYS WET  
WHY ARE OCEANS BECOMING MORE ACIDIC  
WHY IS ARWEN DYING  
WHY AREN'T MY QUAIL LAYING EGGS  
WHY AREN'T MY QUAIL EGGS HATCHING  
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA

WHY IS LIFE SO BORING



WHY ARE ULTRASOUNDS IMPORTANT  
WHY ARE ULTRASOUND MACHINES EXPENSIVE  
WHY IS STEALING WRONG

WHY IS PROGRAMMING SO HARD  
WHY IS THERE A 0 OHM RESISTOR  
WHY DO AMERICANS HATE SOCCER  
WHY DO RHYMES SOUND GOOD  
WHY DO TREES DIE  
WHY IS THERE NO SOUND ON CNN  
WHY AREN'T POKEMON REAL  
WHY AREN'T BULLETS SHARP  
WHY DO DREAMS SEEM SO REAL

# Clustering

## Matlab demo

# Choices of distance metrics in `clustergram(... 'RowPDistValue' ..., 'ColumnPDistValue' ...)`

Metric	Description
'euclidean'	Euclidean distance (default).
'seuclidean'	Standardized Euclidean distance. Each coordinate difference between rows in X is scaled by dividing by the corresponding element of the standard deviation <code>S=nansd(X)</code> . To specify another value for S, use <code>D=pdist(X, 'seuclidean', S)</code> .
'cityblock'	City block metric.
'minkowski'	Minkowski distance. The default exponent is 2. To specify a different exponent, use <code>D = pdist(X, 'minkowski', P)</code> , where P is a scalar positive value of the exponent.
'chebychev'	Chebychev distance (maximum coordinate difference).
'mahalanobis'	Mahalanobis distance, using the sample covariance of X as computed by <code>nancov</code> . To compute the distance with a different covariance, use <code>D = pdist(X, 'mahalanobis', C)</code> , where the matrix C is symmetric and positive definite.
'cosine'	One minus the cosine of the included angle between points (treated as vectors).
'correlation'	One minus the sample correlation between points (treated as sequences of values).
'spearman'	One minus the sample Spearman's rank correlation between observations (treated as sequences of values).
'hamming'	Hamming distance, which is the percentage of coordinates that differ.
'jaccard'	One minus the Jaccard coefficient, which is the percentage of nonzero coordinates that differ.
custom distance function	A distance function specified using <code>@</code> : <code>D = pdist(X, @distfun)</code>  A distance function must be of form  <code>d2 = distfun(XI, XJ)</code>  taking as arguments a 1-by-n vector XI, corresponding to a single row of X, and an m2-by-n matrix XJ, corresponding to multiple rows of X. <code>distfun</code> must accept a matrix XJ with an arbitrary number of rows. <code>distfun</code> must return an m2-by-1 vector of distances d2, whose kth element is the distance between XI and XJ(k, :).



# Choices of hierarchical clustering algorithm in `clustergram( ...'linkage',...)`

X	Matrix with two or more rows. The rows represent observations, the columns represent categories or dimensions.																
method	<p>Algorithm for computing distance between clusters.</p> <table border="1"><thead><tr><th>Method</th><th>Description</th></tr></thead><tbody><tr><td>'average'</td><td>Unweighted average distance (UPGMA)</td></tr><tr><td>'centroid'</td><td>Centroid distance (UPGMC), appropriate for Euclidean distances only</td></tr><tr><td>'complete'</td><td>Furthest distance</td></tr><tr><td>'median'</td><td>Weighted center of mass distance (WPGMC), appropriate for Euclidean distances only</td></tr><tr><td>'single'</td><td>Shortest distance</td></tr><tr><td>'ward'</td><td>Inner squared distance (minimum variance algorithm), appropriate for Euclidean distances only</td></tr><tr><td>'weighted'</td><td>Weighted average distance (WPGMA)</td></tr></tbody></table> <p><b>Default:</b> 'single'</p>	Method	Description	'average'	Unweighted average distance (UPGMA)	'centroid'	Centroid distance (UPGMC), appropriate for Euclidean distances only	'complete'	Furthest distance	'median'	Weighted center of mass distance (WPGMC), appropriate for Euclidean distances only	'single'	Shortest distance	'ward'	Inner squared distance (minimum variance algorithm), appropriate for Euclidean distances only	'weighted'	Weighted average distance (WPGMA)
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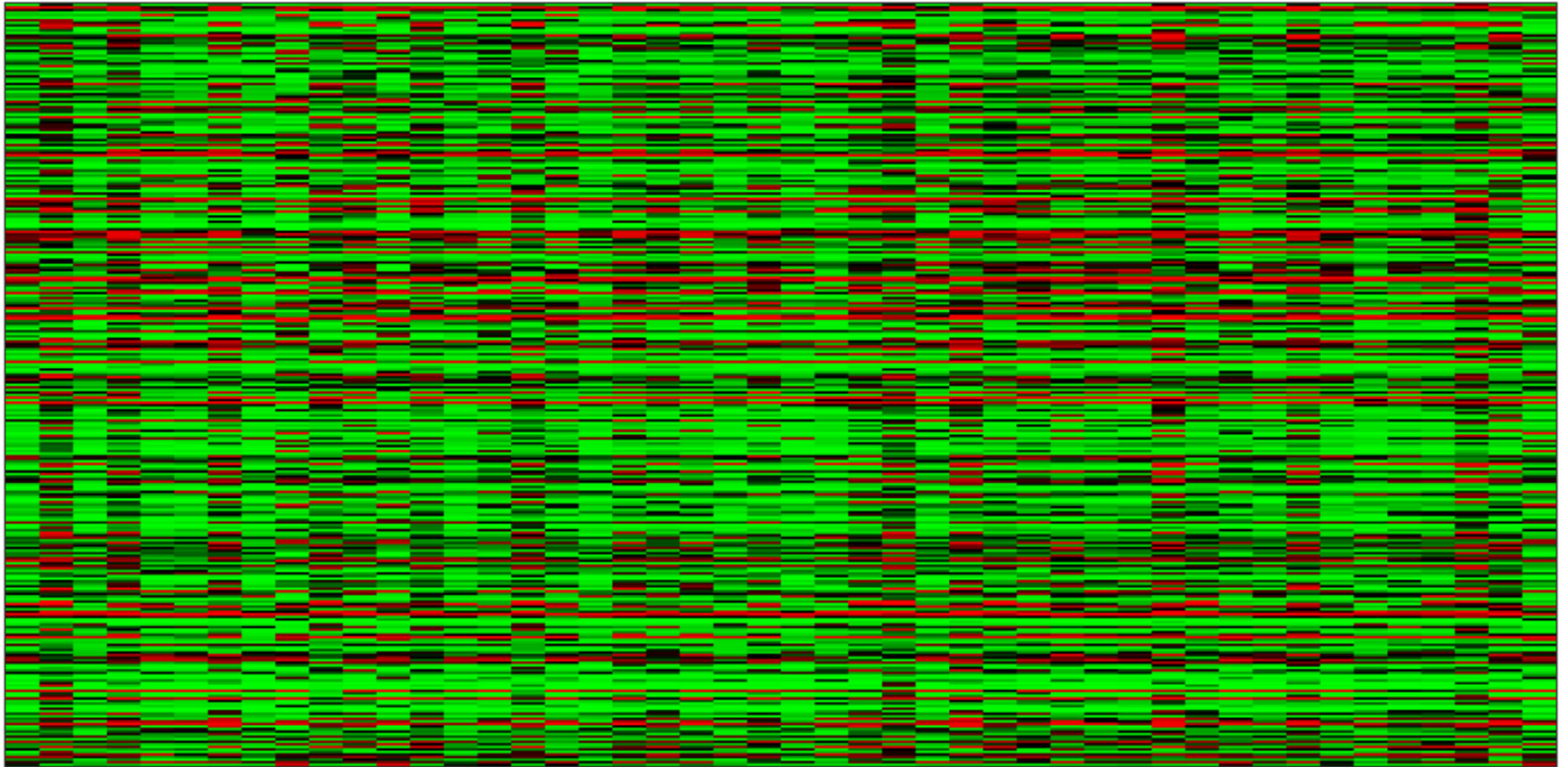
# Clustering group exercise

- Each group will analyze a **cluster of genes** identified in the T cell expression table
- Analyze the table of **top 100 genes by variance** in 47 samples
- Cluster them using:
  - Group 1: UPGMA = 'linkage', 'average', 'RowPDistValue', 'euclidean',
  - Group 2: 'linkage', 'single', 'RowPDistValue', 'cityblock',
  - Group 3: 'linkage', 'average', 'RowPDistValue', 'correlation',
  - Group 4: UPGMA = 'linkage', 'single', 'RowPDistValue', 'euclidean',
  - Group 5: UPGMA = 'linkage', 'weighted', 'RowPDistValue', 'correlation',
- Use clustergram(..., **'Standardize','Row'**,  
**'linkage', as specified for your group**,  
**'RowPDistValue' as specified for your group**,  
'RowLabels',gene\_names1,'ColumnLabels', array\_names)

# Matlab code

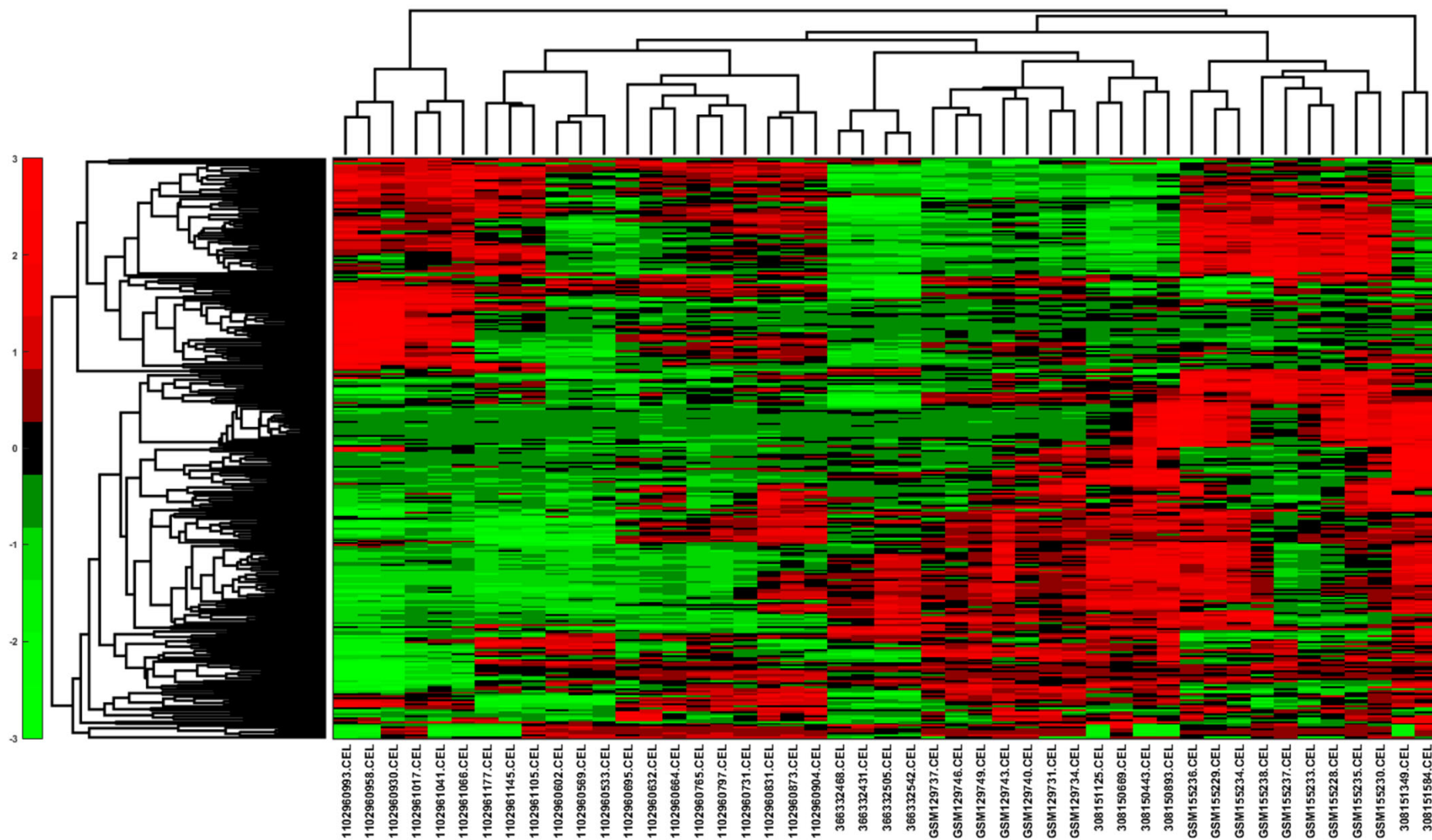
- `load expression_table.mat`
- `gene_variation=std(exp_t)';`
- `[a,b]=sort(gene_variation,'descend');`
- `ngenes=100;`
- `exp_t1=exp_t(b(1:ngenes),:);`
- `gene_names1=gene_names(b(1:ngenes));`
- **%%% for group 1**
- `CGobj1 = clustergram(exp_t1, 'Standardize','Row',  
'RowLabels',  
gene_names1,'ColumnLabels',array_names)`
- `set(CGobj1,'RowLabels',gene_names1,'ColumnLabels',  
array_names,'linkage',  
'average','RowPDist','euclidean');`

Before clustering





# UPGMA hierarchical clustering, Euclidian distance



# UPGMA hierarchical clustering, correlation distance

