Name: ______________
Group members: ______________ ______________ ______________

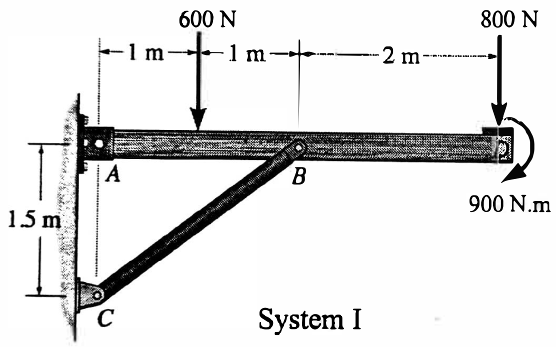
TAM 210/211 - Worksheet 6

Objectives:

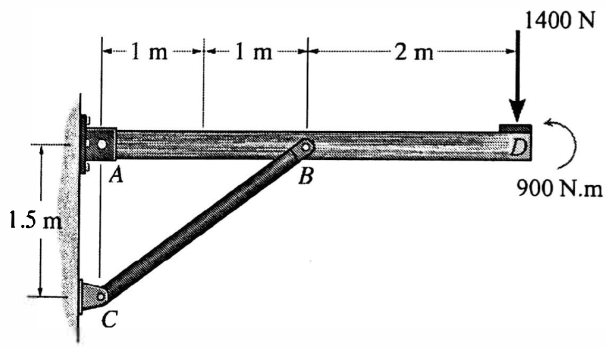
- Obtain resultant forces and moments for equivalent systems.
- Evaluate distributed loadings intensities.

Equivalent systems

1) The overhanging beam is supported by a pin at A and the strut BC. Show that the loading conditions below are equivalent by replacing the loadings by a single resultant force and a moment at A.



System I



$\Sigma F_R = -600\hat{j} - 800\hat{j}$

$= (-1400\hat{j})$ N

$\Sigma M_R = -900\hat{k} - 600(1)\hat{k}$

$-800(4)\hat{k}$

$= (-4700\hat{k})$ N-m

$\vec{M}_{RA} = d(-1400)\hat{k} = -4700\hat{k}$

$\Rightarrow d = 3.36$m

$$\boxed{\begin{array}{l} F_R = -1400\hat{j} \text{ N @ } d = 3.36\text{m} \\ M_R = (-4700\hat{k}) \text{ N-m} \end{array}}$$

$\Sigma F_R = -1400\hat{j}$ N

$\Sigma M_R = -1400(4) + 900$

$= -4700$ N-m

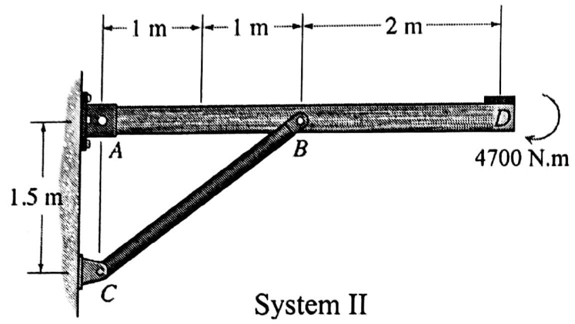
$M_{RA} \Rightarrow d(-1400) = -4700\hat{k}$

$d = 3.36$m

$$\boxed{\begin{array}{l} F_R = -1400\hat{j} \text{ N @ } d = 3.36\text{m} \\ M_R = (-4700)\hat{k} \text{ N-m} \end{array}}$$

As both F_R & M_R are equal, the systems are equivalent.

2) Is the loading condition in System II equivalent to the ones above? Explain.

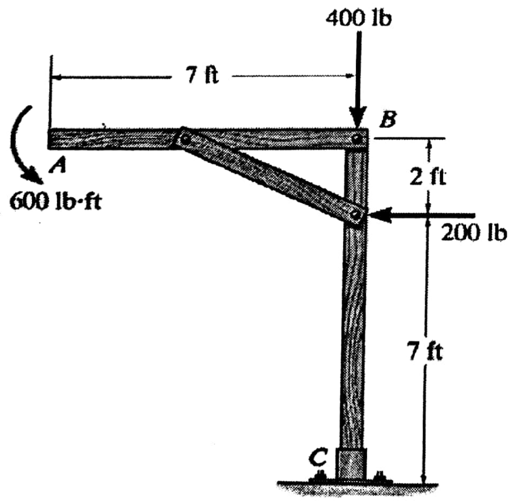


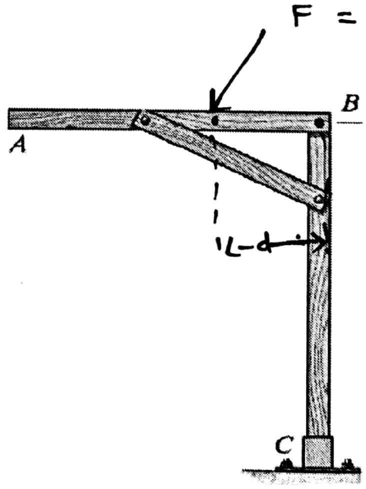
System II

$\boxed{\text{NO}}$

Reason : though moment about A is the same for both the case F_R is not.

3) Replace the force system acting on the left frame below by a single resultant force acting on member AB. Sketch your equivalent system on the right frame.





$F_R = \langle -200, 400 \rangle$ lb

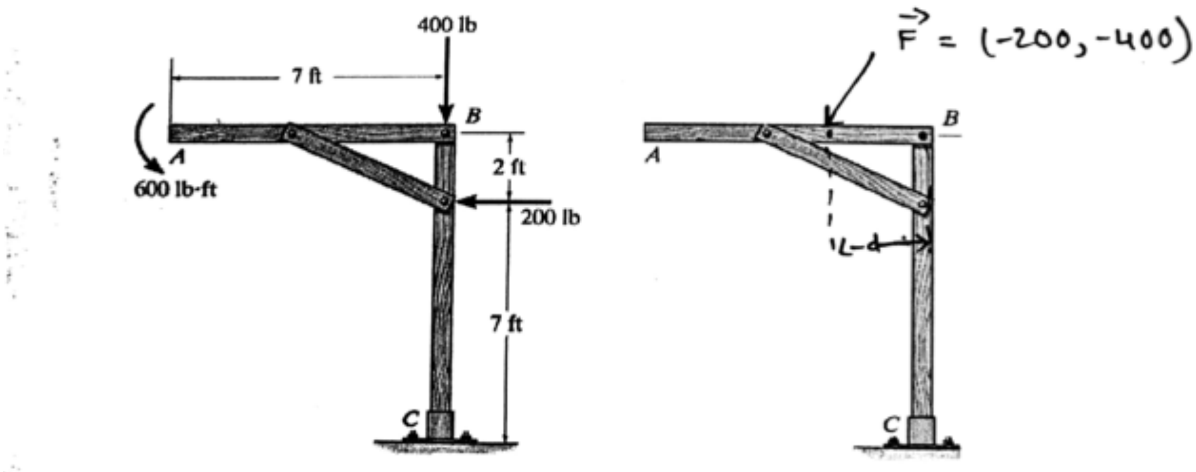
$M_R = 600 + (200)(7)$

$= 2000$ lbft

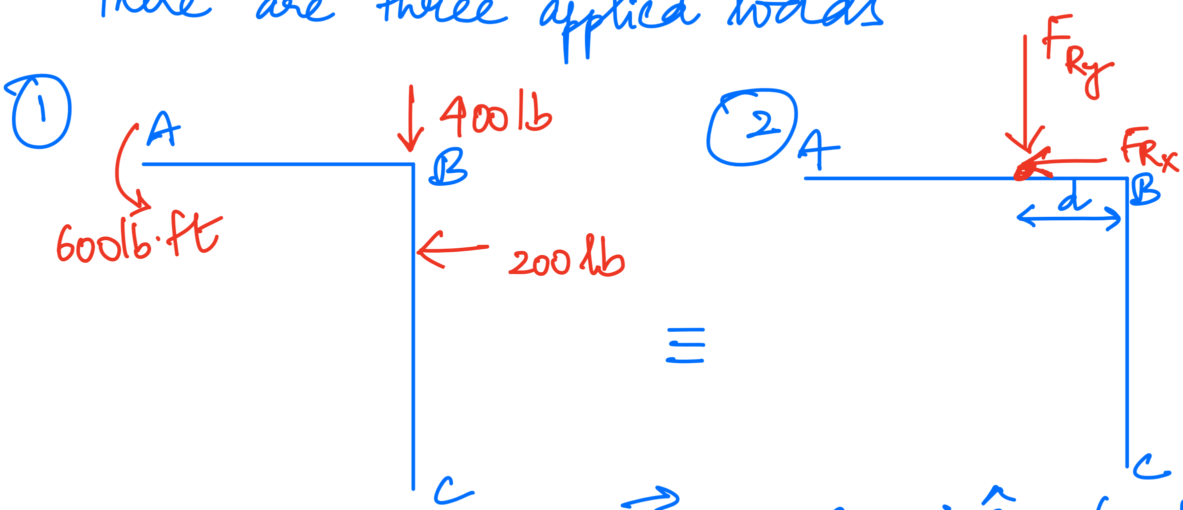
$M_C = 200(9) + 400(d) = 2000$ lb-ft

$d = 0.5$ ft

3) Replace the force system acting on the left frame below by a single resultant force acting on member AB. Sketch your equivalent system on the right frame.



There are three applied loads



The resultant force $\vec{F}_R = -(200\,lb)\,\hat{i} - (400\,lb)\,\hat{j}$

If the above two systems are equivalent, then the moment (about any point!) due to forces in 1 should be equal to the moment due to those in 2.

Let us pick point C to compute the moment

$$\left(\sum M_C\right)_{1} = (200\ \text{lb})(7\ \text{ft}) + 600\ \text{lb}\cdot\text{ft} = 2000\ \text{lb}\cdot\text{ft}$$

$$\left(\sum M_C\right)_{2} = (F_{R_x})(9\ \text{ft}) + (F_{R_y})\,d$$

$$= (200\ \text{lb})(9\ \text{ft}) + (400\ \text{lb})\,d$$

Equating the above two :

$$d = 0.5\ \text{ft}$$

Instead of point C, let us pick point B –

our answer $d = 0.5$ ft should not change

$$\left(\sum M_B\right)_{2} = (F_{R_y})\,d = (400\ \text{lb})\,d$$

$$\left(\sum M_B\right)_{1} = -(200\ \text{lb})(2\ \text{ft}) + 600\ \text{lb}\cdot\text{ft} = 200\ \text{lb}\cdot\text{ft}$$

Equating the above two :

$$d = 0.5\ \text{ft}$$

Summary: It does not matter which point you measure the moment about as long as you $\boxed{\text{use the same point}}$ for measuring moment in the two systems.

Here is an example of an incorrect solution if you used different point to calculate moment in the two systems:

Incorrect solution

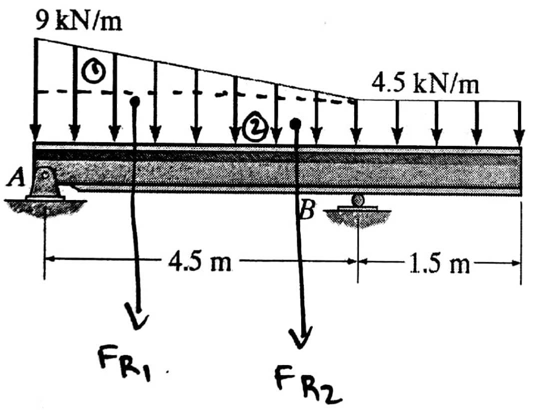
$$\underbrace{\left(\sum M_C\right)_1}_{2000\ \text{lb-ft}} = \underbrace{\left(\sum M_B\right)_2}_{(400\ \text{lb})\ d}$$

$$\Rightarrow d = 5\ \text{ft}$$ WRONG!

Work space for Problem 3.

Reduction of distributed loads

4) Determine the resultant force and specify where it acts on the beam measured from end A.



$$\vec{F_R} = \vec{F_{R_1}} + \vec{F_{R_2}} = -\frac{1}{2}(4.5)(4.5) - 6(4.5) = -37.125\,\hat{j}\text{ KN}$$

$$\vec{M_A} = -F_{R_1}\left(\frac{4.5}{3}\right) - F_{R_2}\left(\frac{6}{2}\right) = -10.125(1.5) - 27(3)$$

$$\vec{M_A} = -96.1875\text{ KN-m} = d(F_R) = d(-37.125)$$

d = distance from (A).

$$\Rightarrow d = 2.59\text{ m}$$

$$\vec{F_R} = -37.125\,\hat{j}\text{ KN} \quad @ \quad d = 2.59\text{ m}$$