

## 6.3 SIGNAL PROCESSING

### Sources of noise

- skin motion artifact
- human error
- electronic noise (60 Hz)
- improper sampling

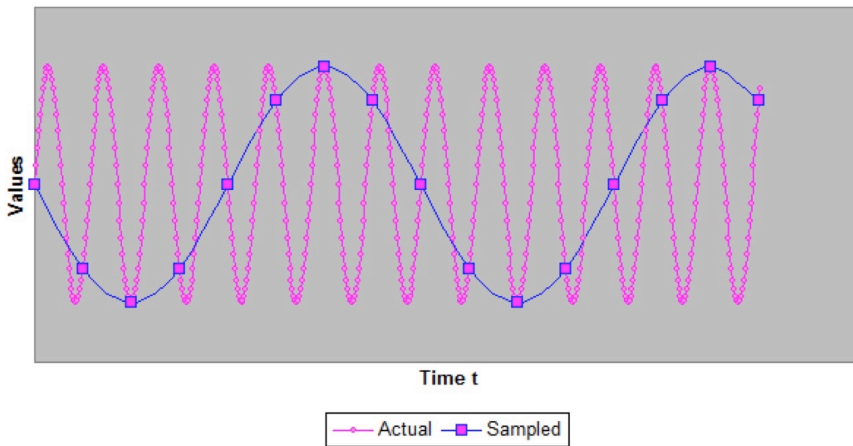
### Angular velocities & accelerations

$$\text{velocity} = \omega = \frac{\Delta\theta_{\text{hip}}}{\Delta t}$$

$$\text{Acceleration} = \alpha = \frac{\Delta\omega}{\Delta t}$$

Noise becomes a problem!

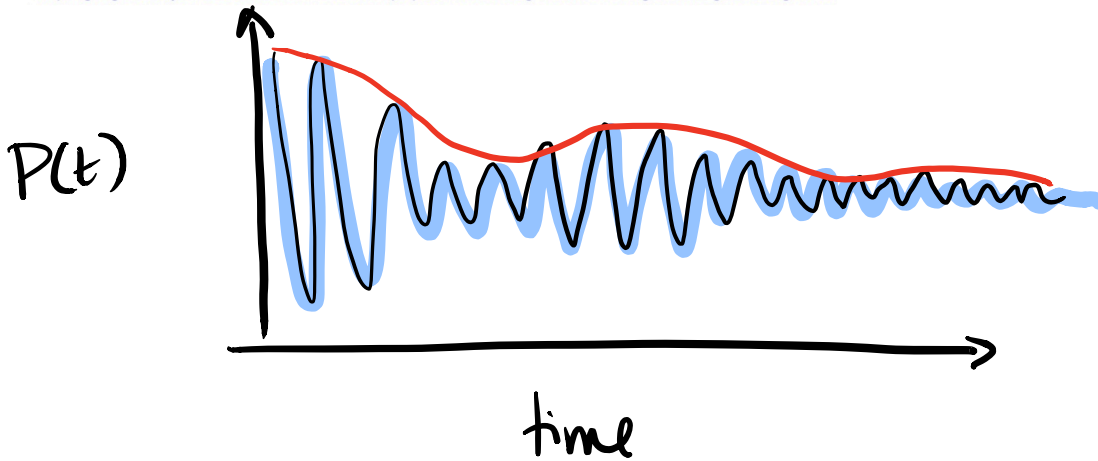
### Aliasing example



Nyquist sampling theorem:

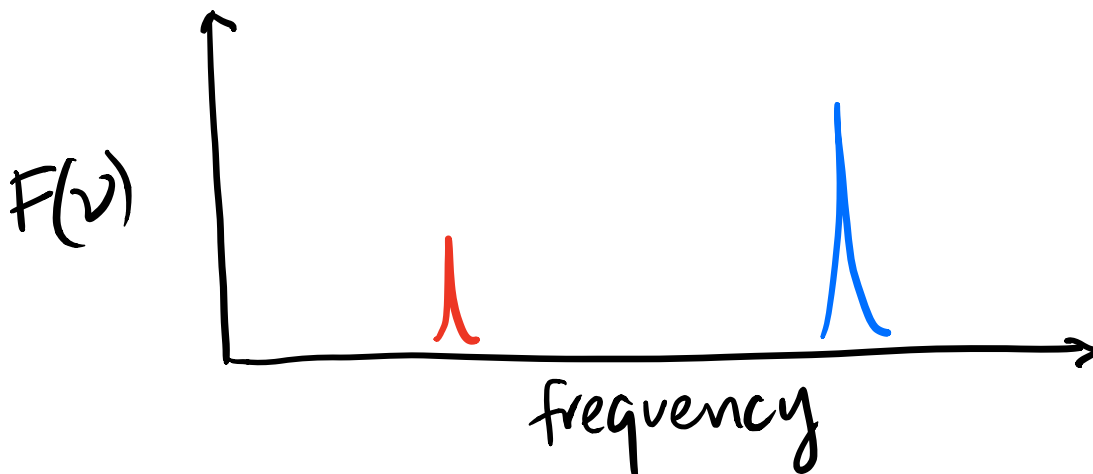
Sampling freq  $\geq 2 * \text{highest freq of actual signal of interest}$

<http://gregstanleyandassociates.com/whitepapers/FaultDiagnosis/Filtering/Aliasing/aliasing.htm>



Convert to frequency domain using fast fourier transform (FFT)

How many frequencies are in this signal?



Which is the signal?

Walking  $\rightarrow$  120 steps/minute

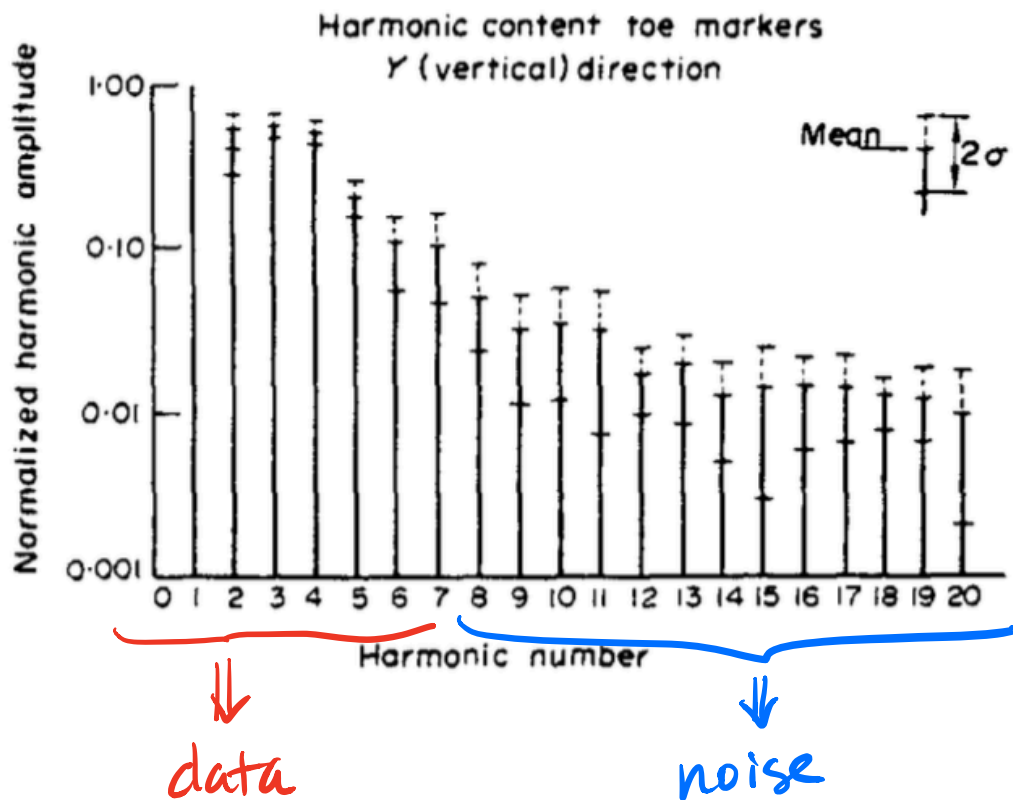
Step frequency = 2 Hz

Stride frequency = 1 Hz

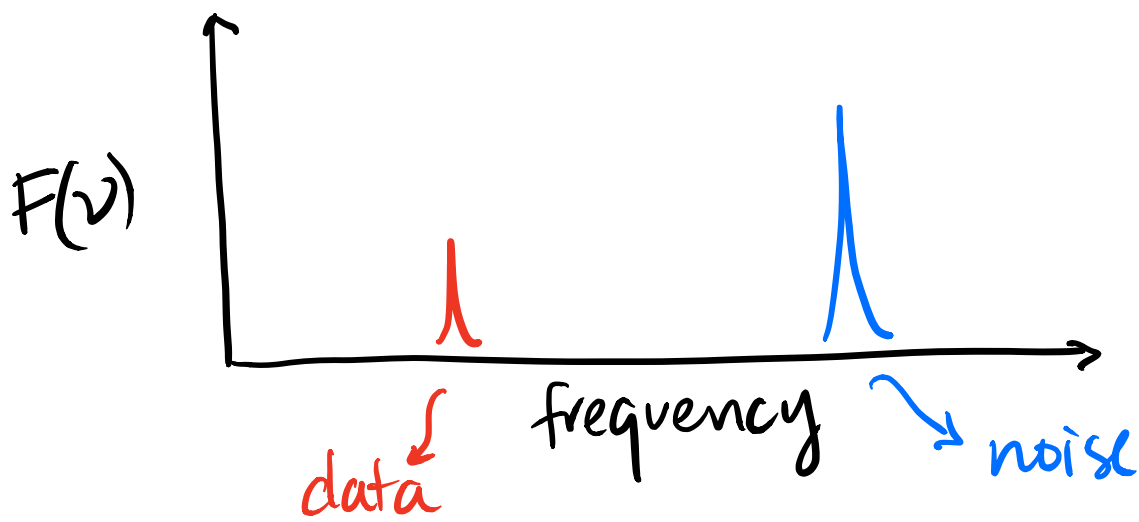
Which marker  
will we  
track to  
get this?

In repetitive movements  $\rightarrow$  frequencies will  
be at harmonics of stride frequency

- Most of the data is below 6 Hz



Noise tends to be random  $\Rightarrow$  tends to  
be high frequency  $\Rightarrow$  show Winter text



For biomechanics  $\Rightarrow$  tend to use low pass filter  $\Rightarrow$  Butterworth

CAREFUL! may be activity dependent!

Butterworth not so good for step/impulse inputs

- show some videos

Must choose appropriate cut off frequency

CAREFUL! may be activity dependent!

- Many options

- We will use residual analysis

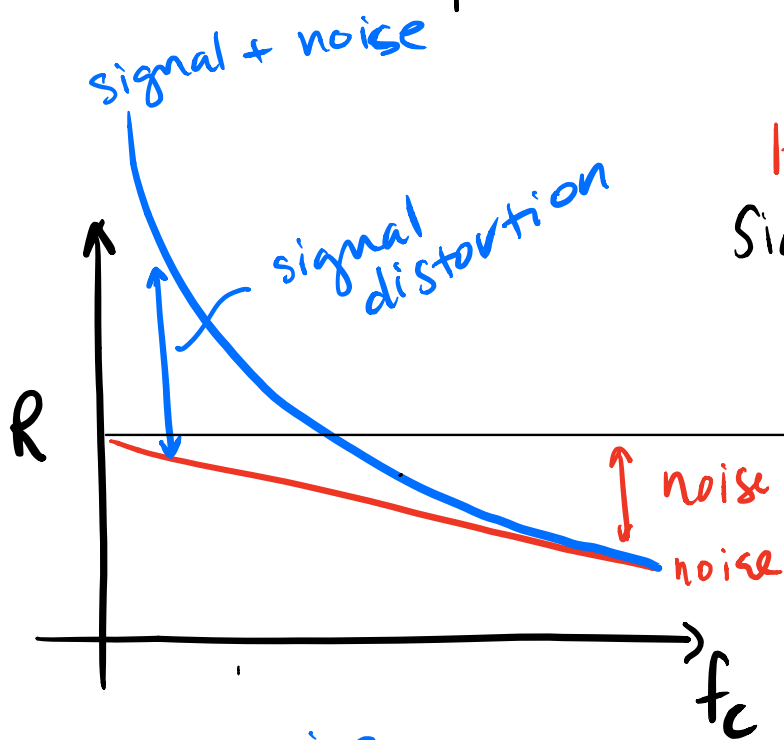
For each cut off frequency ( $f_c$ ); the residual ( $R$ ) is difference between raw data ( $X$ ) and filtered data ( $\hat{X}$ ) of all datapoints ( $N$ )

In Math-ese:  $(x - \hat{x})$   
 all data  $\sum_{i=1}^N (x_i - \hat{x}_i)$

repeat for  
 different  
 $f_c$

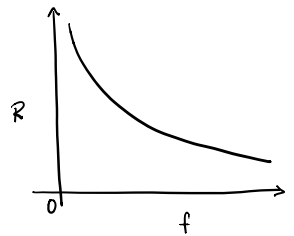
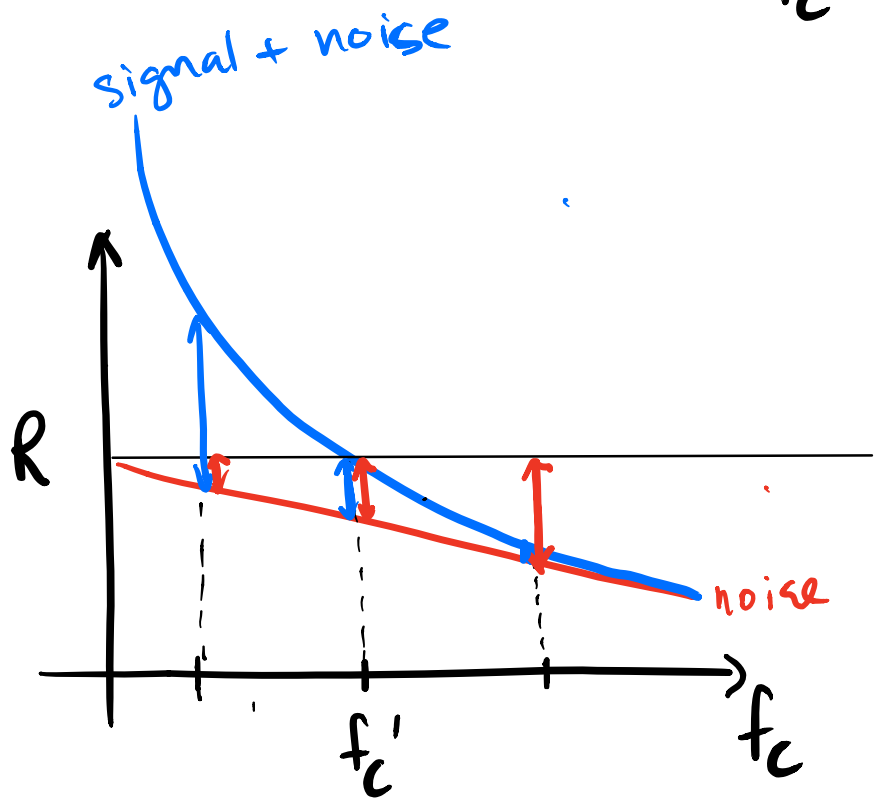
avoid mean = 0  $\sqrt{\sum_{i=1}^N (x_i - \hat{x}_i)^2}$

$R(f_c) = \sqrt{\sum_{i=1}^N (x_i - \hat{x}_i)^2}$



If all noise  
 Signal + noise  $\Rightarrow$  residual  
 increases as  $f_c \downarrow$

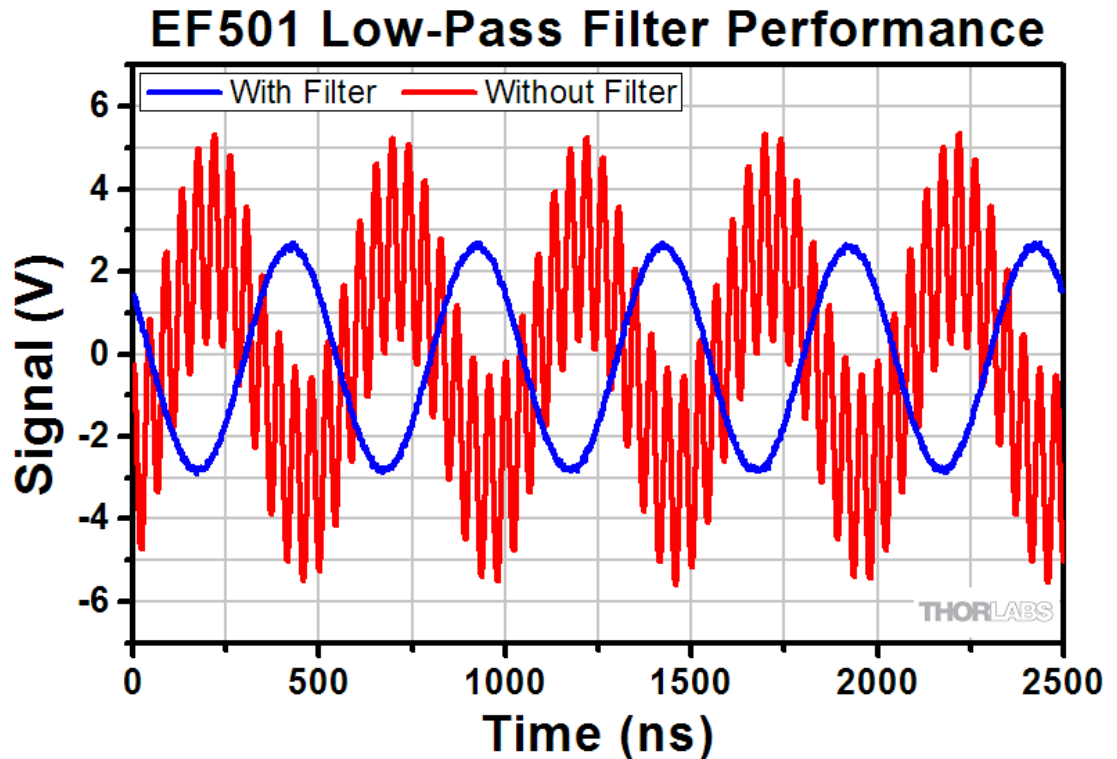
noise passed through filter  
 For distortion = noise,



## + Lag due to filtering

Filtering causes a phase shift in the signal → seen as a “lag”

Use recursive or forward-backward filtering to get zero-lag  
(Matlab: `filtfilt`)



# + MATLAB Commands

```
[b,a]=butter(n,wn);
```

```
y=filtfilt(b,a,x);
```

$N_f =$  Nyquist frequency


$$N_f = \frac{1}{2} f_s$$

a,b = coefficients for Butterworth filter

n = n<sup>th</sup> order filter,

wn = cutoff freq must be btw.  $0.0 < wn < 1.0$ , where 1.0 corresponds to  $\frac{1}{2}$  sampling rate

$\frac{f_c \text{ (cut off freq)}}{N_f}$



filtfilt = gives zero-lag recursive filtering

y = filtered version of signal x

# + MATLAB Commands

---

Example for data collected at 100 Hz:

```
% filter the data
```

```
% 4th order Butterworth with 6Hz cut-off frequency
```

```
% use zero-phase forward-backward filtering
```

```
[b,a]=butter(n,wn);
```

```
y=filtfilt(b,a,x);
```

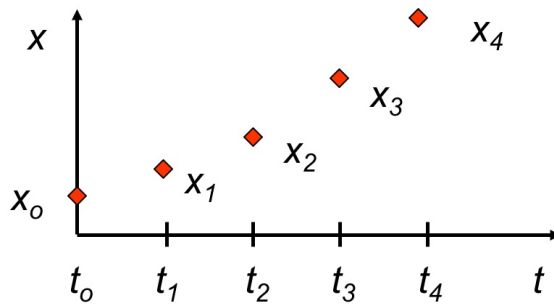
wn = cutoff freq must be btw.  $0.0 < wn < 1.0$ ,  
where 1.0 corresponds to  $\frac{1}{2}$  sampling rate

What should be the values for n and wn?

```
[b,a]=butter(4,0.12);
```



## + How to calculate velocity if we know position?



## + Three options:

Euler's Method  
(Forward Difference Method)

$$\frac{d(x_n)}{dt} = \frac{x_{n+1} - x_n}{\Delta t}$$

Backward Difference Method

$$\frac{d(x_n)}{dt} = \frac{x_n - x_{n-1}}{\Delta t}$$

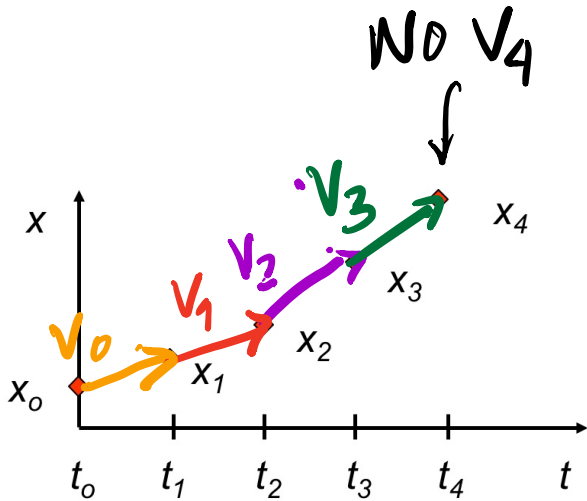
Three-Point Formula  
(Centered-Difference  
Formula)

$$\frac{d(x_n)}{dt} = \frac{1}{2} \left[ \frac{x_{n+1} - x_{n-1}}{\Delta t} \right]$$

## + Euler's Method (Forward Difference Method)

Simplest!  
velocity

$$\frac{dx_n}{dt} = \frac{x_{n+1} - x_n}{\Delta t}$$



If  $x: 1 \rightarrow n$  datapoints

$$\frac{dx}{dt} : 1 \rightarrow (n-1)$$

Error  $\sim f(\Delta t)$

$\downarrow \Delta t$ , more accurate

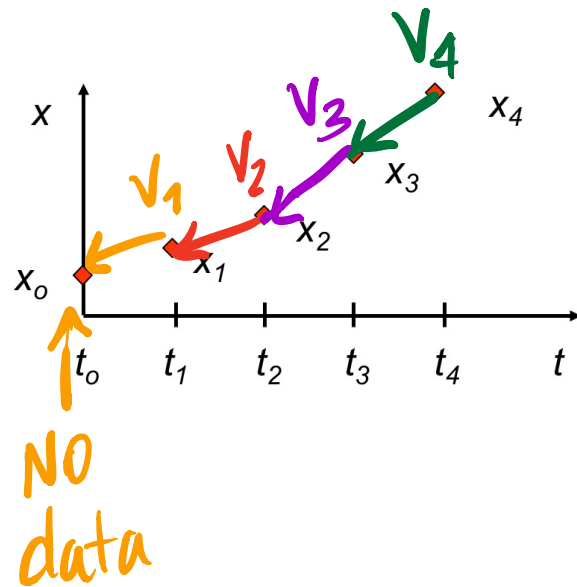
Matlab function diff

diff =  $[x_n - x_{(n-1)}]$  vector

$\Delta t \rightarrow$  constant

## + Backward Difference Method

Just like  
forward,  
but backward

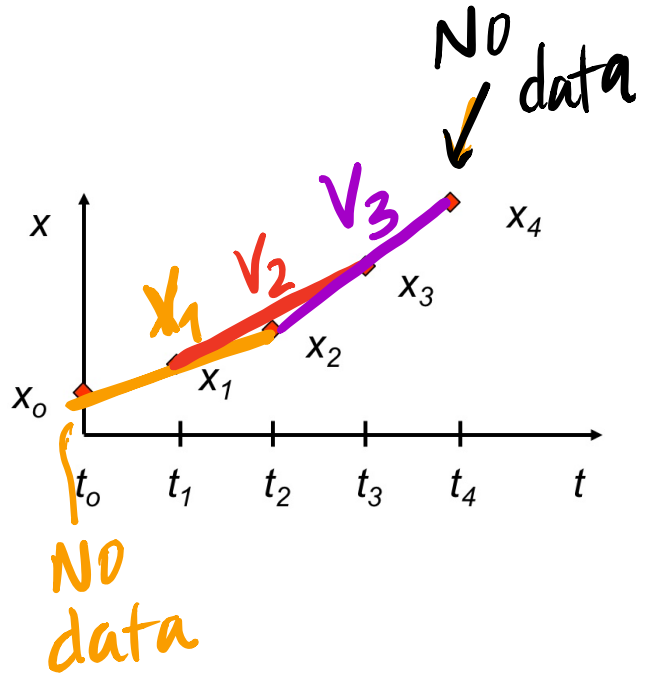


$$\frac{dx_n}{dt} = \frac{x_n - x_{n-1}}{\Delta t}$$

$$\text{error} \sim (\Delta t)$$

## + Three-Point Formula (Centered-Difference Formula)

- Average  
of  
forward &  
backward



$$\frac{dx_n}{dt} = \frac{1}{2} \left( \frac{x_{n+1} - x_{n-1}}{\Delta t} \right)$$

error  $\sim \Delta t^2$  ☺

Comp. more expensive ☹