

## ECE 563: Problem Set 1

### Source Coding

**Released:** Monday, January 22

**Due:** Monday, January 29

1. **[AEP]**

State and prove a version of the asymptotic equipartition property that you like.

2. **[Coding]**

Suppose there is an i.i.d. source that is generating the symbol *llama* with probability  $1/2$ , the symbol *gnu* with probability  $1/4$ , the symbol *sheep* with probability  $1/8$ , and the symbol *giraffe* with probability  $1/8$ .

- (a) Find upper and lower bounds on the average codeword length for an optimal lossless source code for this source, as tight as you can.
- (b) Find an optimal prefix-free code for this source. What is its average codeword length?
- (c) (Optional) Find a Tunstall code for this source.

3. **[Balance]**

Suppose you have a pan balance and 12 coins. One of the two following cases is true: (1) exactly two of the coins are too light by the same amount, e.g. 1%; or (2) exactly two of them are 1% heavy. You need to discover the identity of the two faulty coins and determine which of the two cases holds. The scale reports results accurately.

- (a) Derive the smallest upper bound you can for the number of weighings needed for you to accomplish the task.
- (b) Exhibit a way of performing your task using the balance no more than five times.

4. **[Axiomatics]**

Yilun Xu, Shengjia Zhao, Jiaming Song, Russell Stewart, and Stefano Ermon wrote a paper, “A Theory of Usable Information under Computational Constraints” in ICLR 2020 and proposed an information measure called predictive  $\mathcal{V}$ -information.

- (a) Can you come up with an axiomatic derivation of this measure?
- (b) (Optional) Cryptographers often discuss a quantity called accessible entropy. See e.g. a book chapter by Salil Vadhan entitled “Computational Entropy”. Discuss the relationship between accessible entropy and usable information, formally, if possible.

5. **[Texture]**

Implement a version of the Lempel-Ziv-based texture synthesis approach discussed in Lecture 1 and show some novel examples. If you use any existing code libraries, please note them. Do submit a printout of your code in addition to the outputs.