

Representation of Information

ECE 598 LV – Lecture 17

Lav R. Varshney

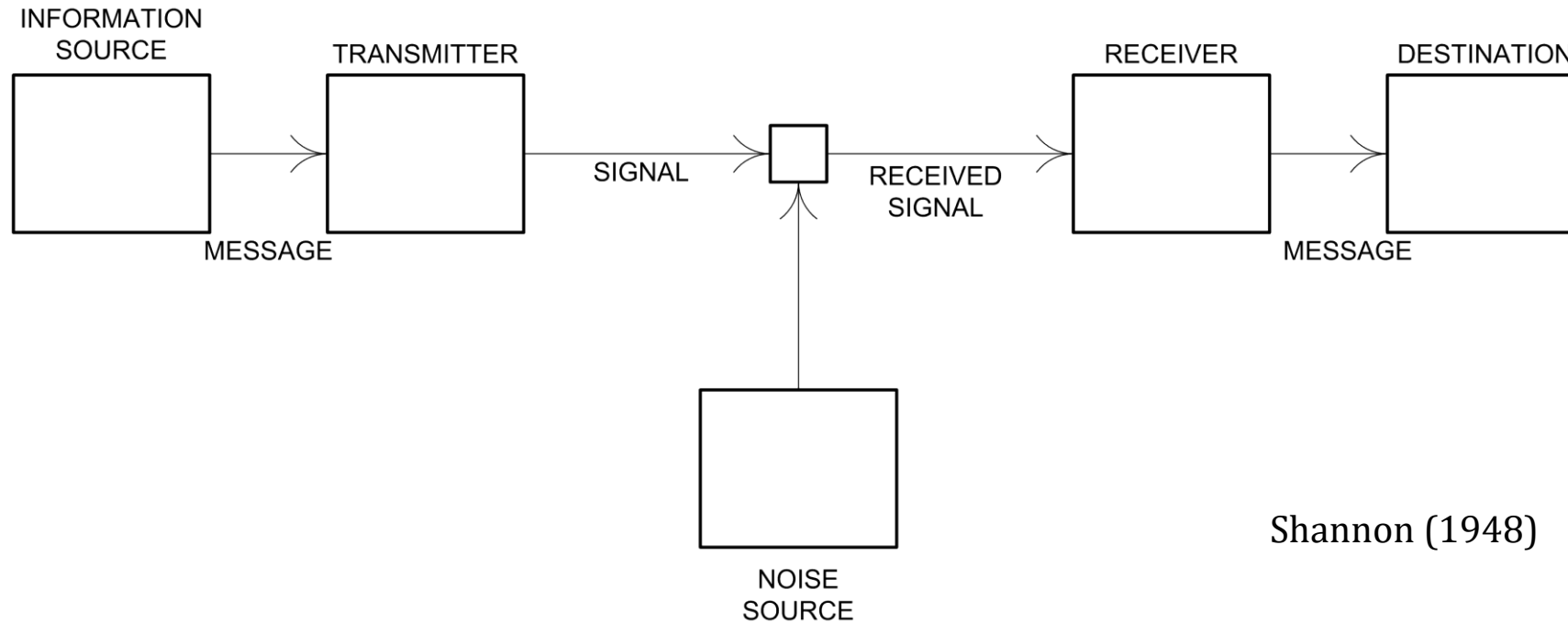
21 March 2024

**Noise-enhanced associative memory,
creativity, and other problems in faulty
information processing**

Motivations

- Engineering domains
 - Nanoscale information fabrics
 - Computational creativity (for culinary recipes)
- Scientific understanding
 - Hippocampus, piriform cortex (for culinary recipes?)
 - Variability is the name of the game in biology: are there functional benefits?

Problem of reliable communication

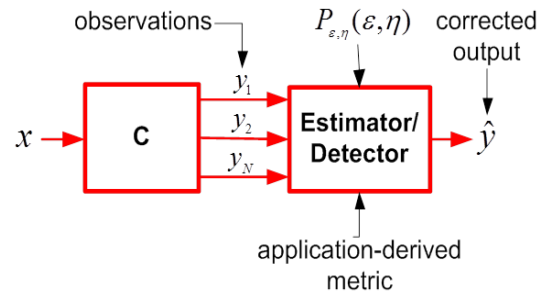


Shannon (1948)

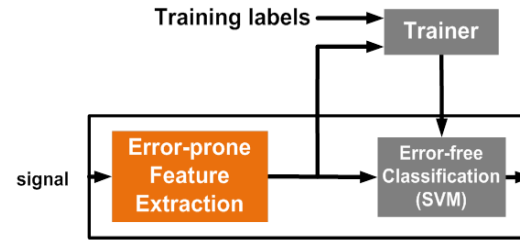
Arbitrarily reliable information transmission is possible at information rates below channel capacity

- An exponential number of possible messages
- A chosen subset of possible signals

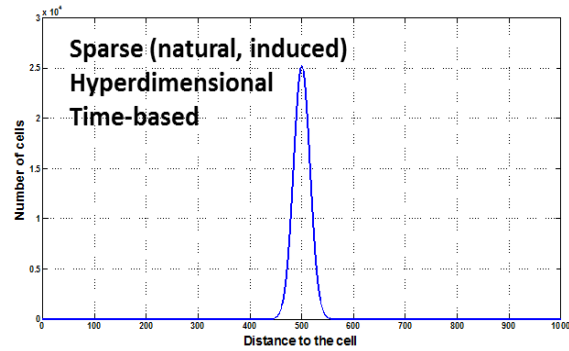
Statistical error compensation (SEC)



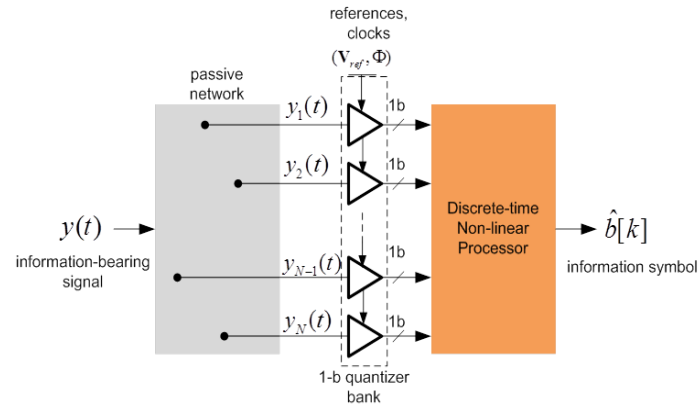
Data driven hardware resilience (DDHR)



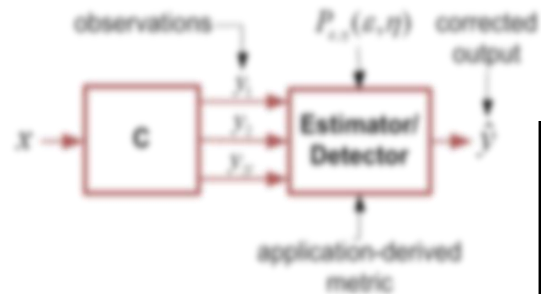
Unconventional information representations



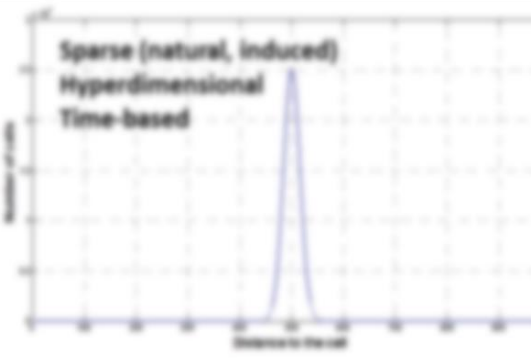
Information-maximizing analog front-ends



Statistical error compensation (SEC)



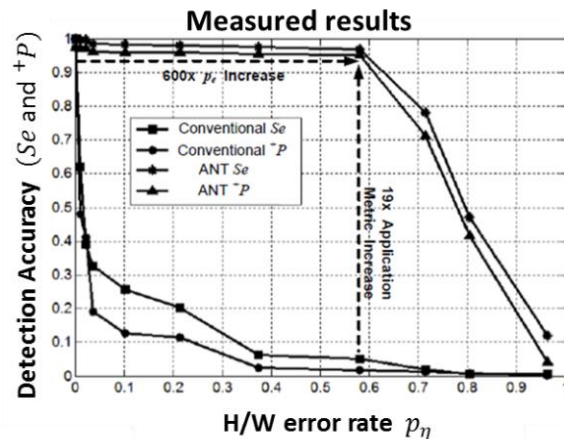
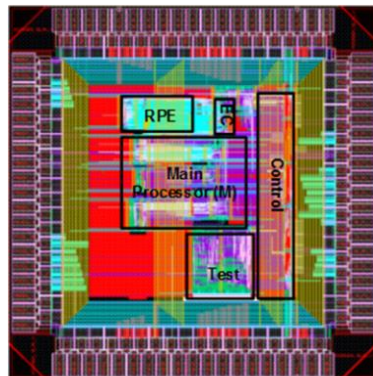
Unconventional information representations



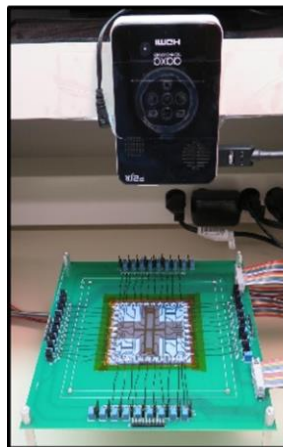
Data driven hardware resilience (DDHR)



SEC-based Subthreshold ECG Processor in 45nm CMOS

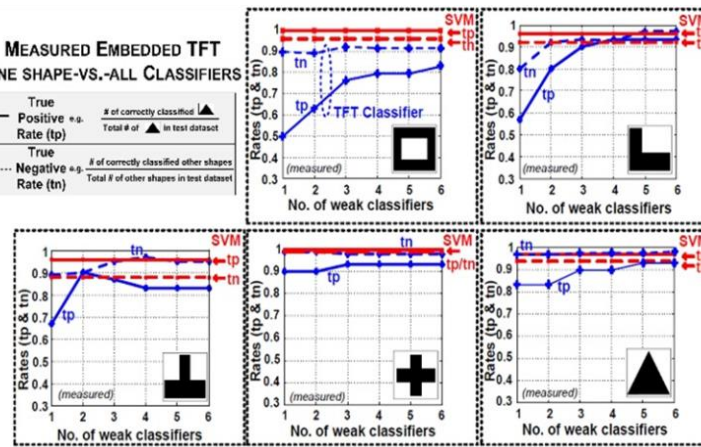


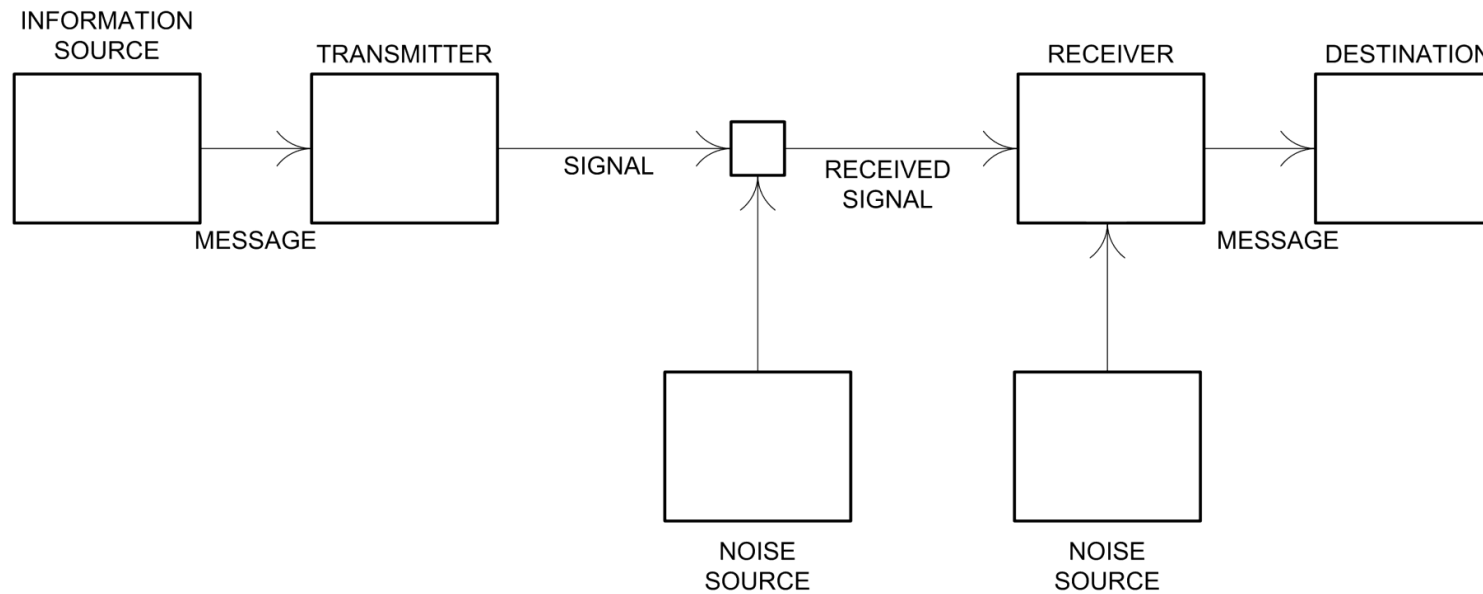
DDHR-based Thin-film image sensing and classification system



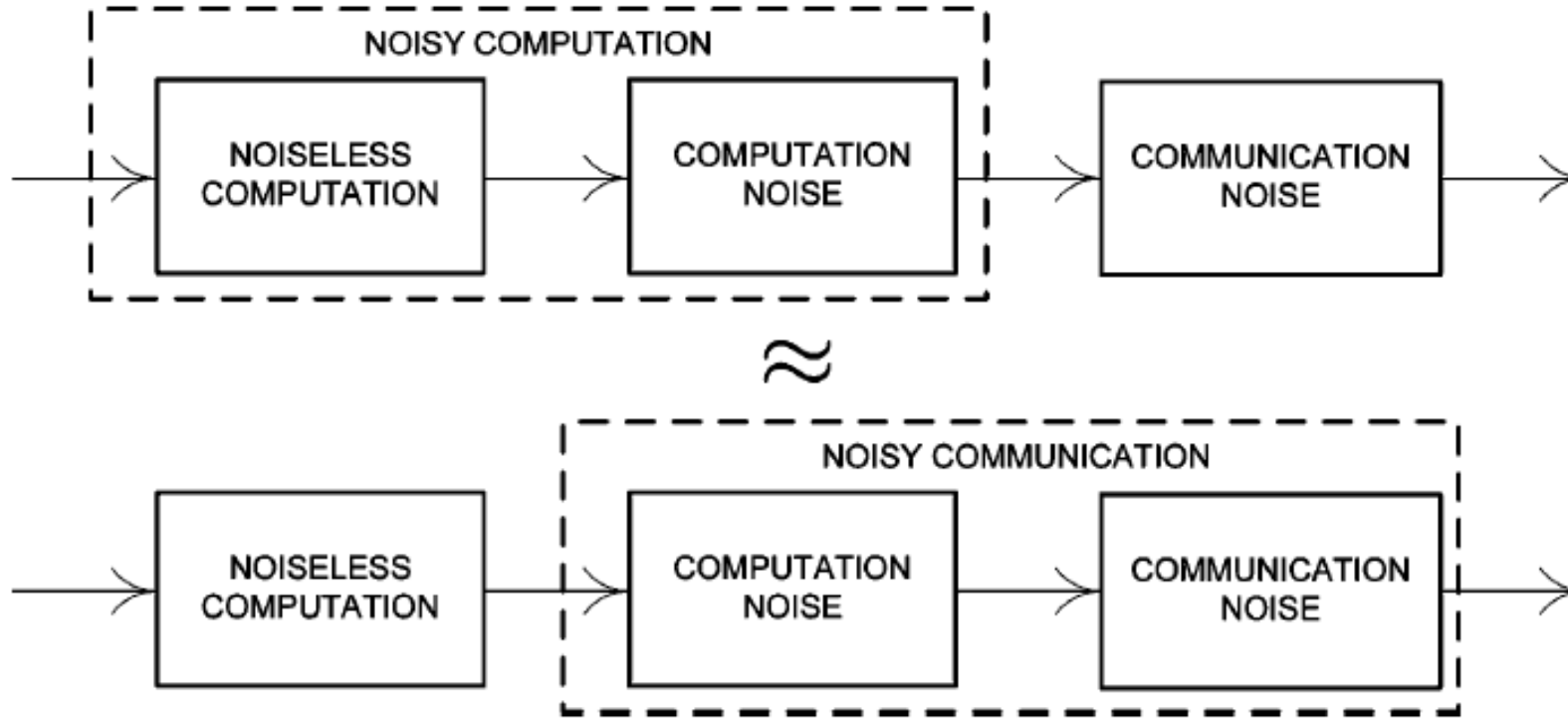
MEASURED EMBEDDED TFT ONE SHAPE-VS.-ALL CLASSIFIERS

— True Positive Rate (tp) $\frac{\# \text{ of correctly classified}}{\text{Total \# of } \blacktriangle \text{ in test dataset}}$
 - - - True Negative Rate (tn) $\frac{\# \text{ of correctly classified other shapes}}{\text{Total \# of other shapes in test dataset}}$



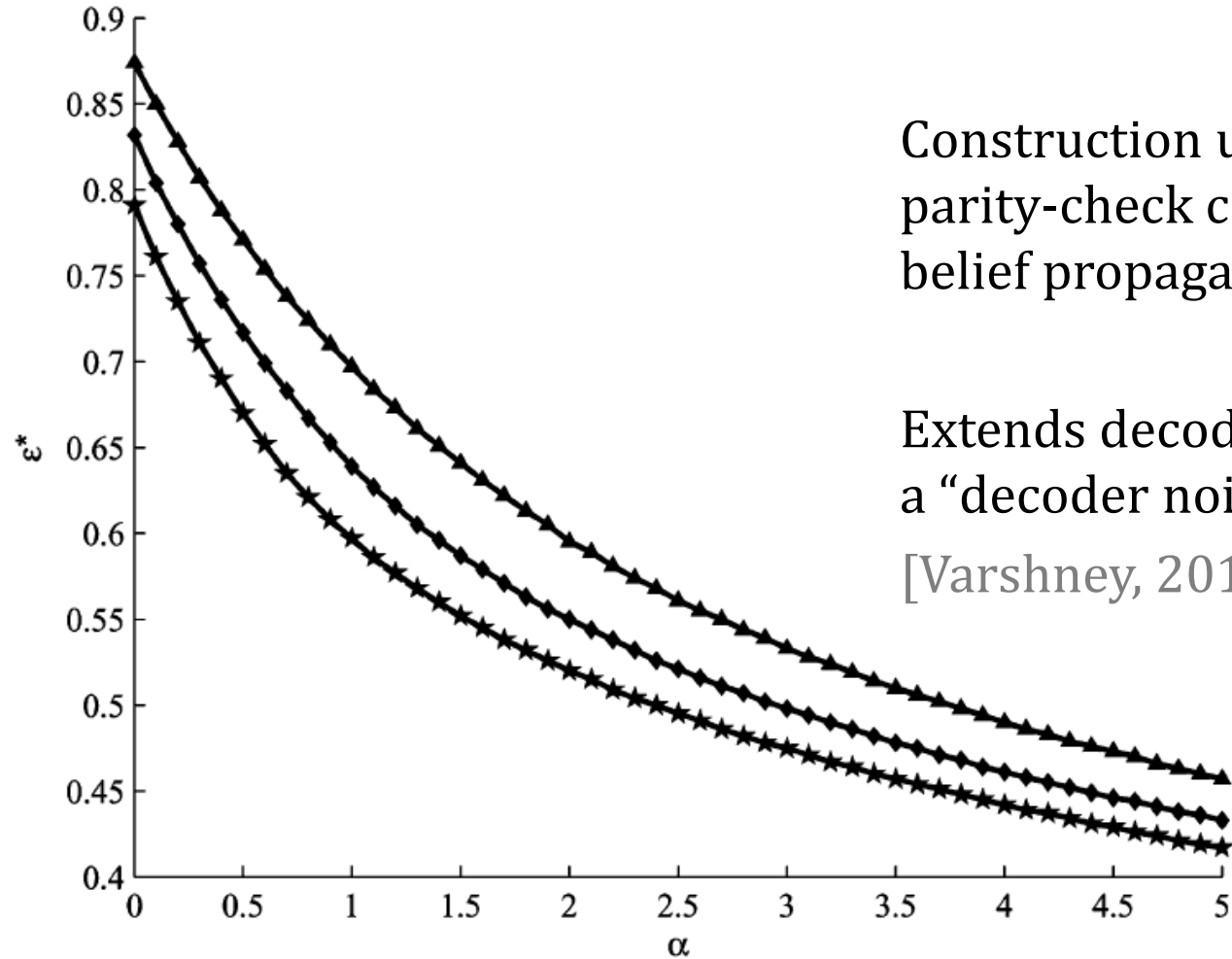


Basic Question What is possible and what is impossible in processing unreliable signals with unreliable circuits?



- Overall system: think of encoder noise as more channel noise
- Within decoder: combine noises, without loss of generality

Communication system with noisy channel and noisy message-passing decoder achieves arbitrarily reliable communication

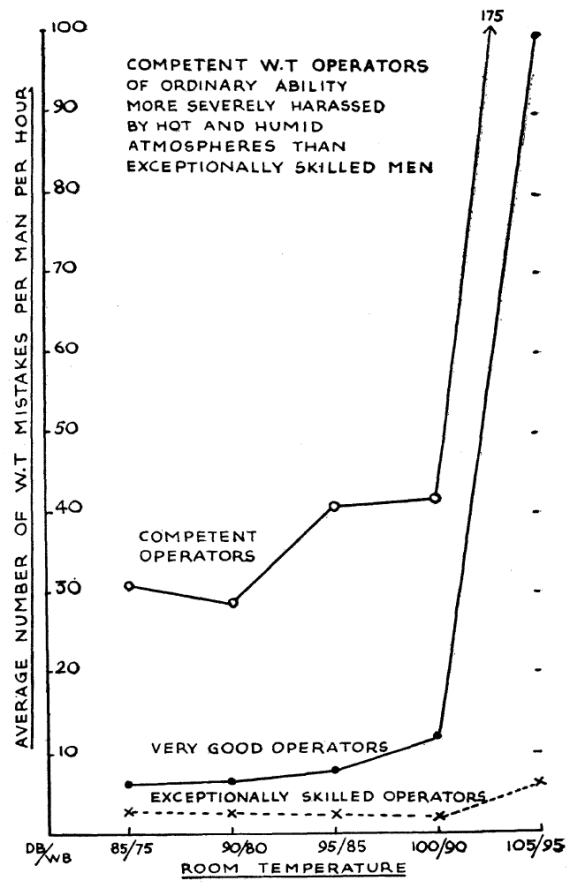


Construction using low-density parity-check codes with noisy belief propagation decoder

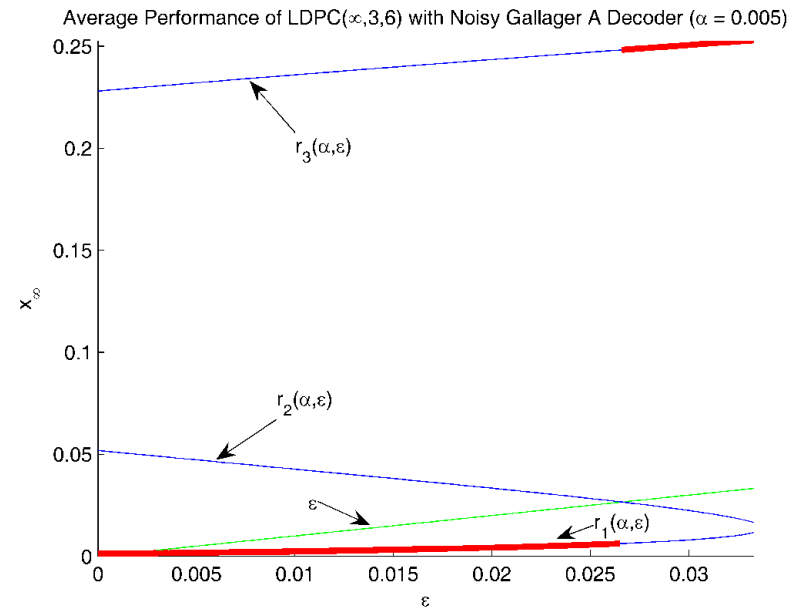
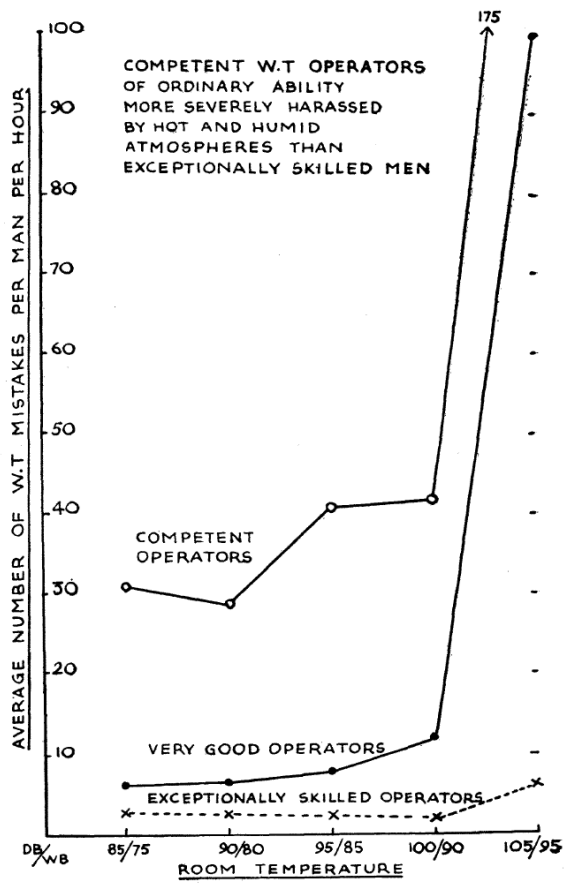
Extends decoding threshold to a “decoder noise” axis

[Varshney, 2011]

An aside: Heat stress on telegraphers



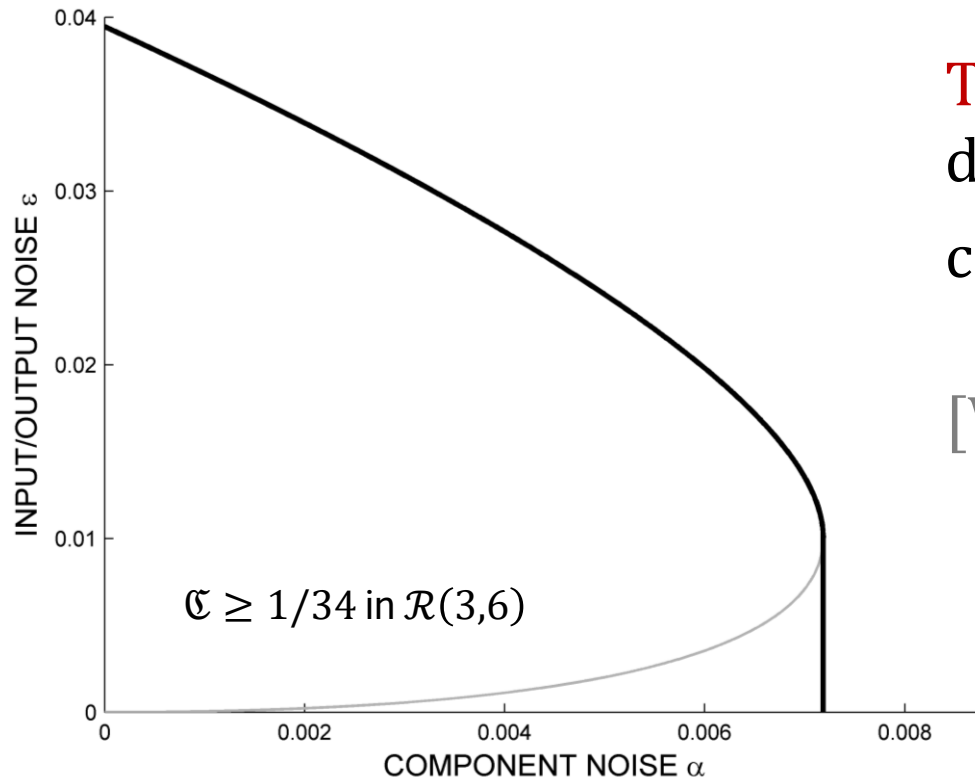
[Mackworth, 1946]



[L. R. Varshney, "Performance of LDPC codes under faulty iterative decoding," *IEEE Trans. Inf. Theory*, vol. 57, pp. 4427-4444, July 2011.]

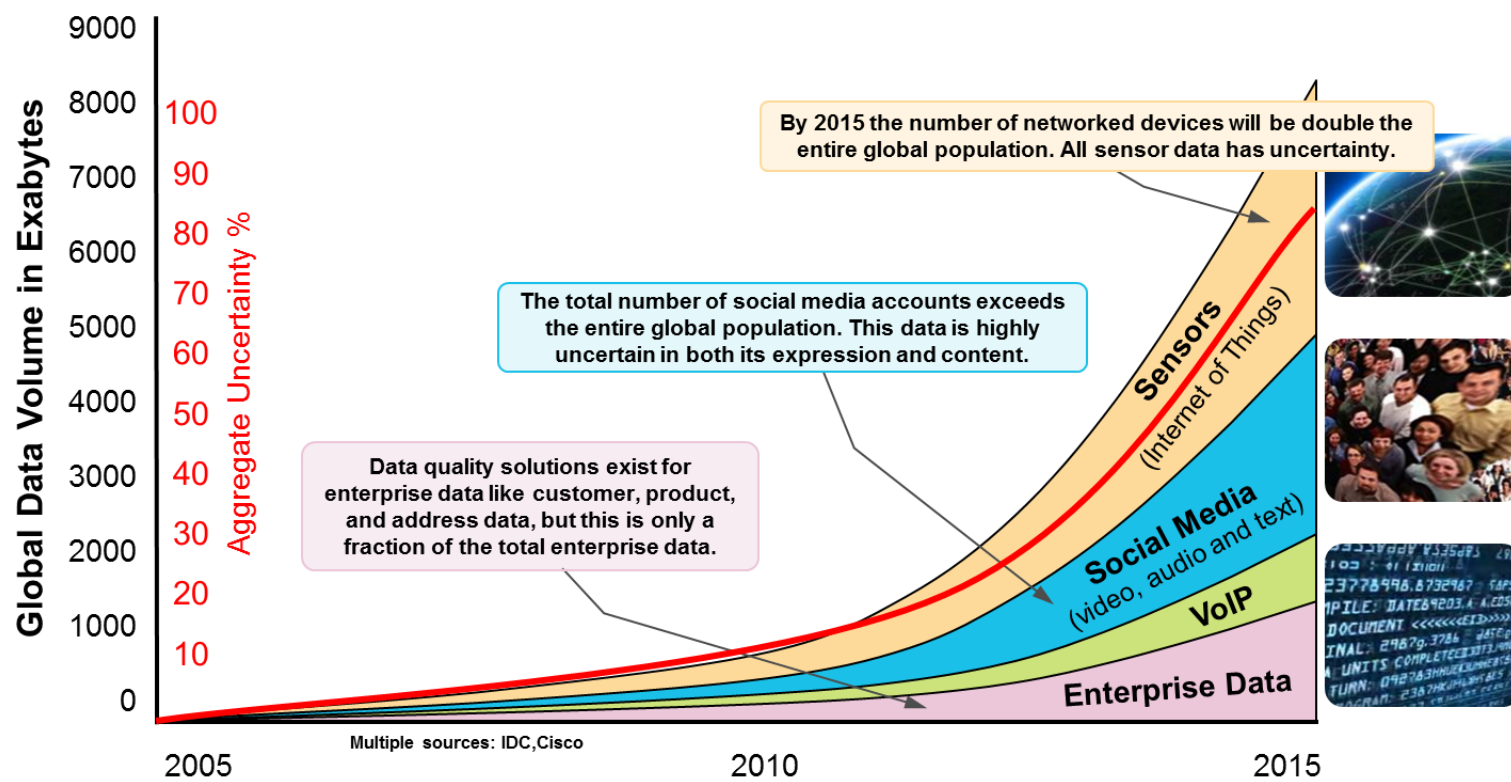
Constructing reliable memories from unreliable components possible with linear circuit complexity

Theorem For memories constructed from components with noise levels within the region $\mathcal{R}(d_v, d_c)$, achievable storage capacity is $\mathfrak{C} \geq \left(1 - \frac{d_v}{d_c}\right) / (d_v d_c - 1)$ [Varshney, 2011]



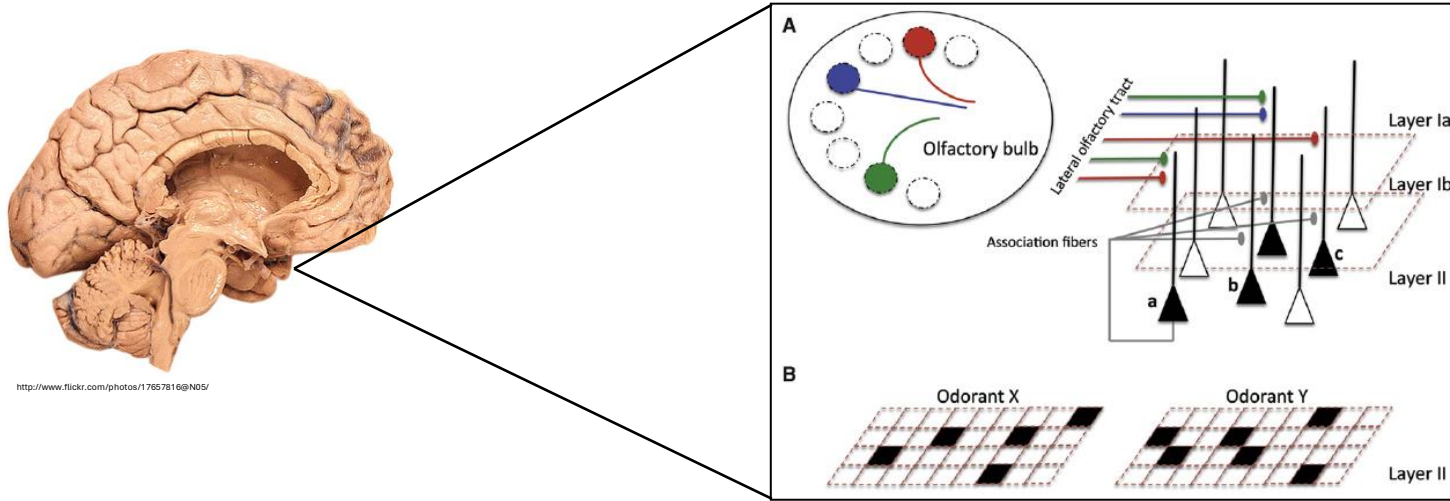
Theorem An entropy-dissipation argument gives a converse: $\mathfrak{C}(\alpha) \leq \frac{C(\alpha)}{1 + \frac{h_2(\alpha)}{2 - h_2(\frac{\alpha}{2} + \frac{1}{4})}}$

[Varshney, 2015]



- In the information overload regime, it is not enough to reliably store information forever
- We need to determine whether we have relevant data and how to retrieve it
- Are there similar limit theorems for content-addressable memory?

Humans do fine with natural stimuli



[Wilson and Sullivan, 2011]

Olfactory cortex and hippocampus are thought to act as content-addressable memory to allow nearest-neighbor search, etc.

Associative memory

- Wish to store a desired set of states—the memories—as fixed points of a network such that errors in input representation of a memory are corrected and the memory retrieved
 - Nearest-neighbor search
- Many problems faced by integrated circuit designers are those that biology has overcome to deliver reliable, real-time computation in neural circuits
 - Not only explain certain features of the mammalian brain, but also impact theory and practice of nanoscale memory system design in big data era
- Modern coding theory: matrix of synaptic weights are like a code matrix

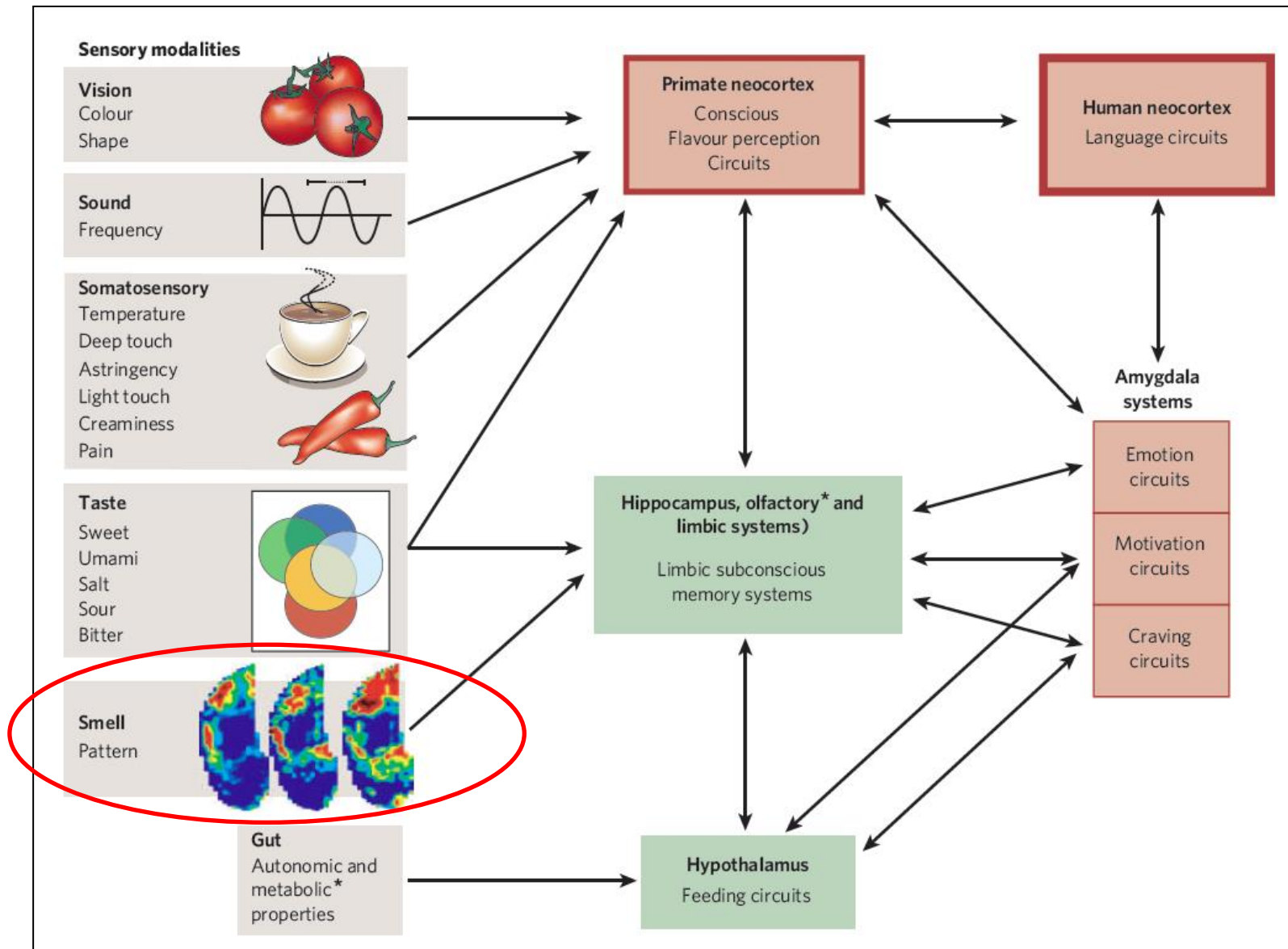
Olfactory cortex

Orthonasal/retronasal olfaction key to human flavor perception

- [C. Bushdid, M. O. Magnasco, L. B. Vosshall, and A. Keller, “Humans can discriminate more than 1 trillion olfactory stimuli,” *Science*, 2014.]
- [L. Secundo, K. Snitz, and N. Sobel, “The perceptual logic of smell,” *Curr. Opin. Neurobiol.*, 2014.]

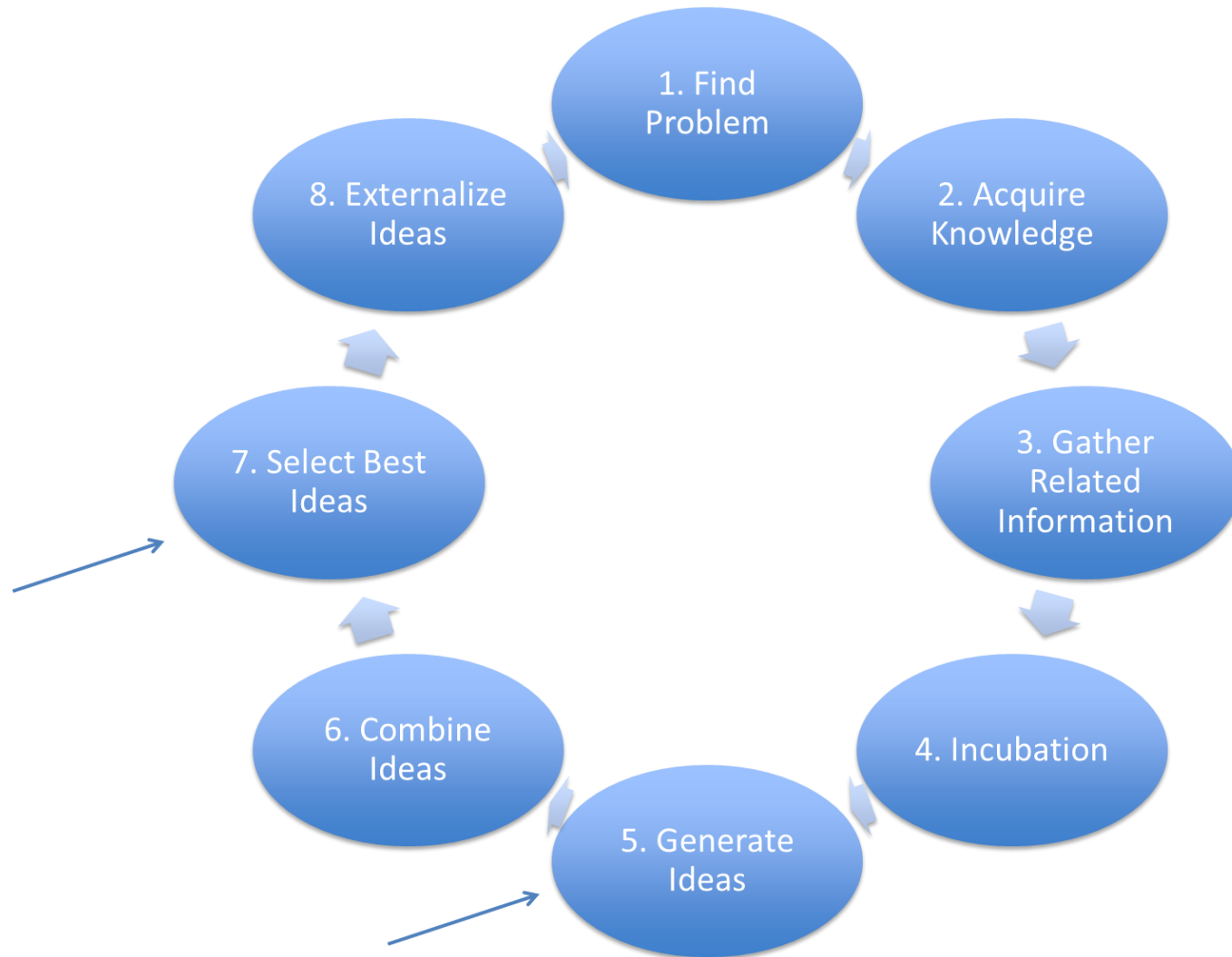
A computational model that reproduces certain aspects of associative olfactory memory

- Large capacity [T. L. White, “Olfactory Memory: the Long and Short of It,” *Chem. Senses*, 1998.]
- Noisy information processing circuitry



[Shepherd,
2006]

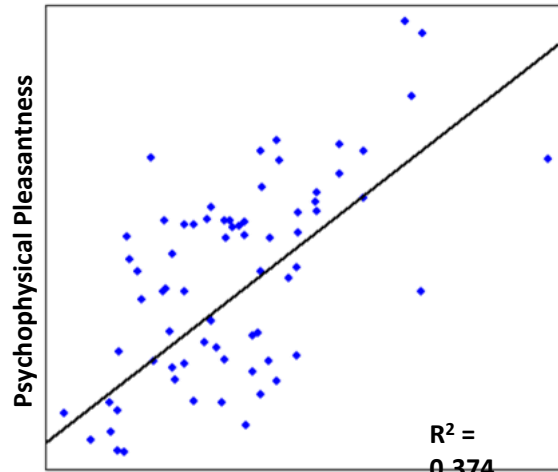
“to create consists of making new combinations of associative elements which are useful” – Henri Poincaré



[Sawyer, 2012]

Computational creativity for culinary recipes

- Combine food chemistry and human flavor hedonic psychophysics data to predict the most pleasant foods
- Build computational creativity system that produces *surprising* and flavorful culinary recipes automatically



Chemistry [TPSA, heavy atom count, complexity, rotatable bond count, hydrogen bond acceptor count]

Black Tea
Bantu Beer
Beer
Strawberry
White Wine
Cooked Apple





Cognitive Cooking with Chef Watson

Recipes for Innovation from IBM & the Institute of Culinary Education



GET HELP FROM WATSON

borsch fricassee brine
pasta salad potato salad
fettuccine enchilada bu

SHOWING: DISHES MEALS CO

Q search

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brownie
bruschetta
burger
burrito
caesar salad
cake
carpaccio


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Yard To Table Plantain Borsch

HERE'S A STARTING POINT '...'

6 servings

DAIRY 3/4 cup sour cream	HERB 1 1/2 tbsp chopped thyme 1/2 cup chopped cilantro	MEAT 2 cup extra-firm tofu	STOCK/SOUP 3 quart canned vegetable stock
VEGETABLE 4 cup chopped sugar snap peas 1 1/2 bulb trimmed fennel 3 cup sliced, peeled plantain 1 cup sections with juice, chopped banana blossom 4 peeled, firm		VINEGAR 3 tbsp balsamic vinegar	



Based On Borscht
From Bon Appétit

1. Bring the vegetable stock, extra-firm tofu, and fennel to boil in large pot.
2. Reduce heat, cover, and simmer about 1 hour 30 minutes.
3. Transfer extra-firm tofu to work surface; trim fat, sinew and bone and discard.
4. Chop extra-firm tofu; cover and chill.
5. Cool vegetable stock slightly.
6. Chill uncovered in pot at least 4 hours and up to 1 day.

Based on: Borscht from Bon Appétit

thyme, cilantro, vegetable stock, sour cream

IBM Chef Watson with bon appétit

TUTORIAL PRIVACY TERMS OF USE



Former IBM Research scientist Lav Varshney presents a demo of an early version of the cognitive cooking technology at IBM Research. CREDIT: COURTESY IBM

How IBM's Chef Watson Actually Works

 47  Comment

WRITTEN BY ROCHELLE BILOW

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1. Sample from state space, using culturally well-chosen sampling distribution
2. Rank according to psychophysical predictors of surprise and flavor
3. Select either automatically or semi-automatically depending on human-computer interaction model

And Now, From I.B.M., Chef Watson



Robert Caplin for TI

I.B.M. plans to serve a breakfast pastry devised by Watson and the chef James I. meeting on Thursday.

By STEVE LOHR

Published: February 27, 2013

[I.B.M.'s Watson](#) beat "Jeopardy" champions two years ago. But can it whip up something tasty in the kitchen?

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That is just one of the questions that I.B.M. is asking as it tries to expand its artificial intelligence technology and turn Watson into something that

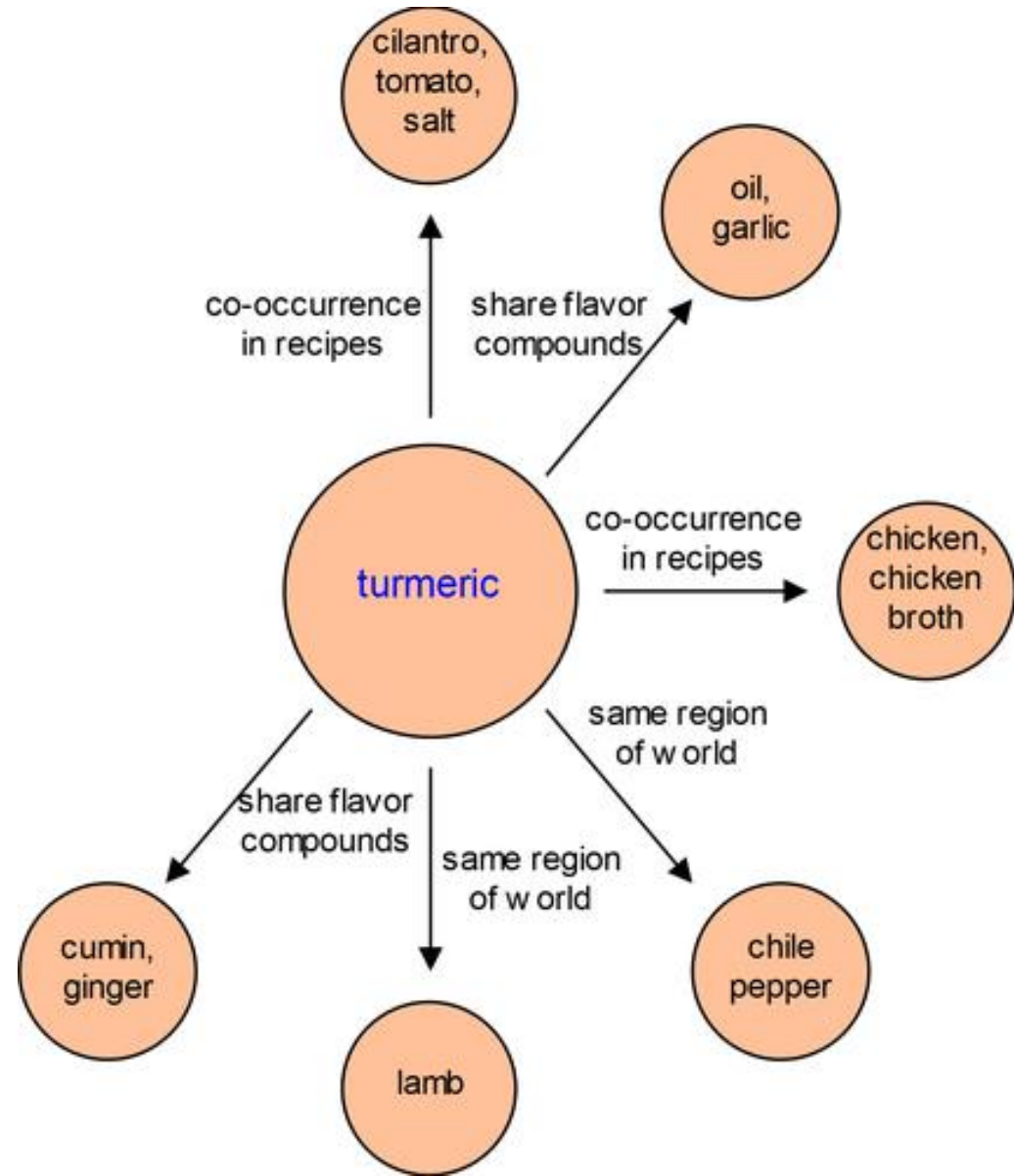
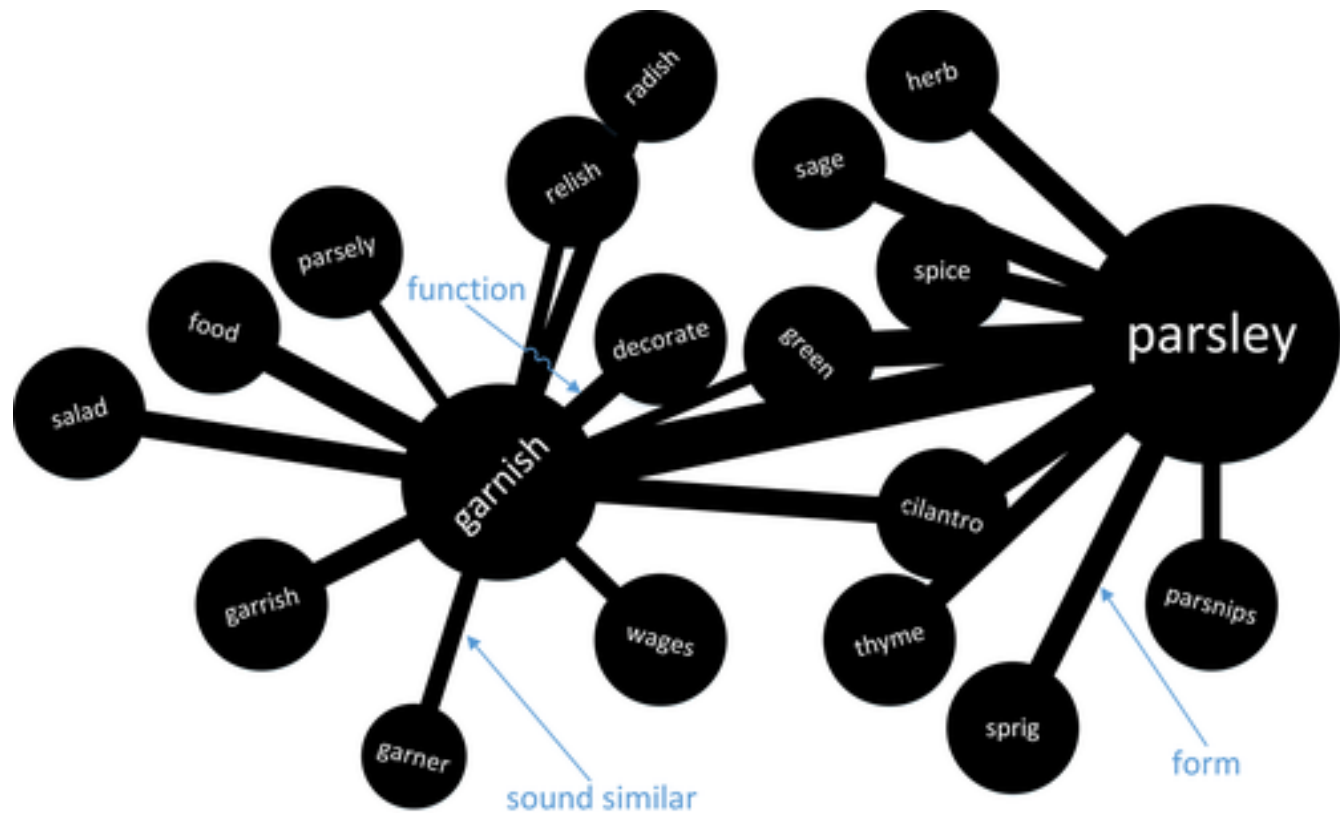
actually makes commercial sense.

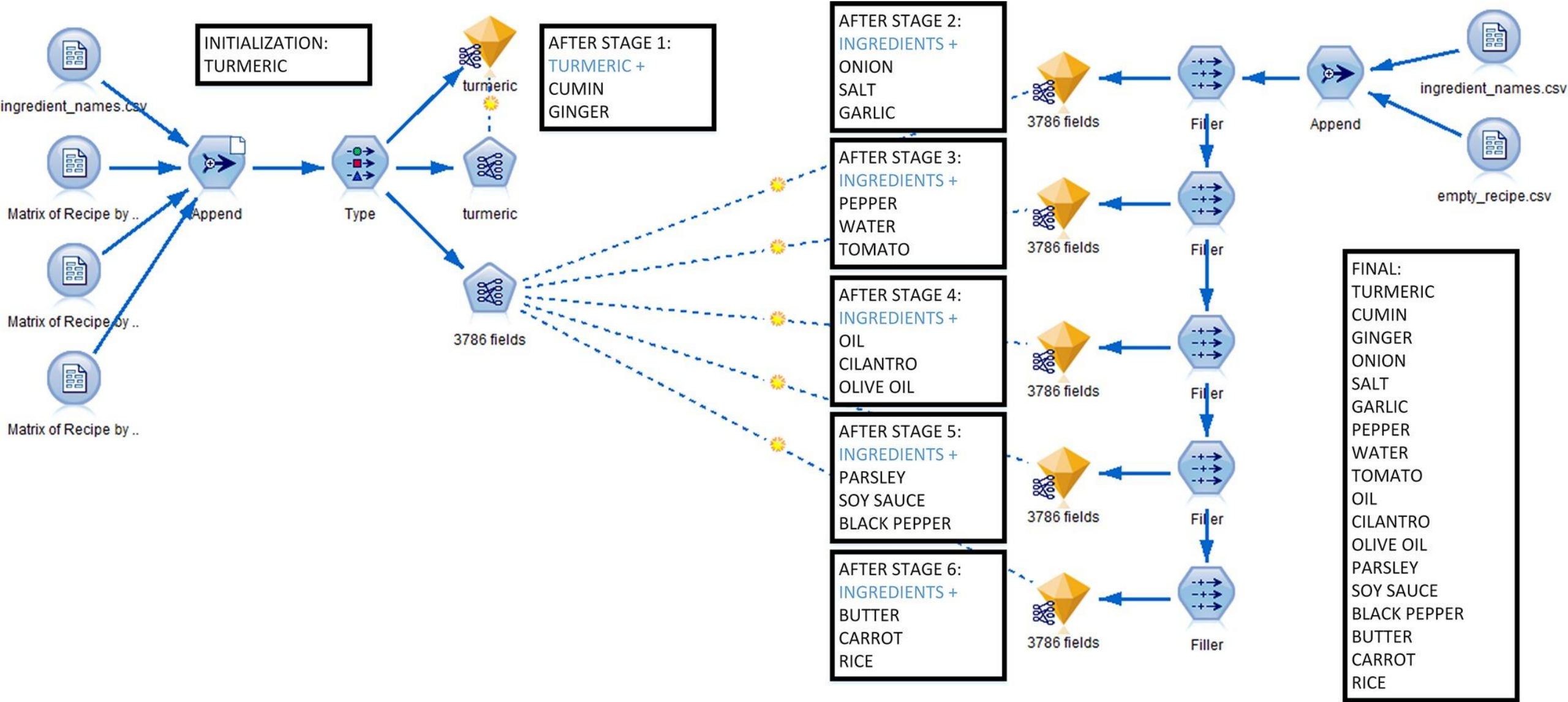
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“I always start with one ingredient, I have to identify that main item first and you know, what are we going to build. You want to start with the really good looking zucchini that was at the market that day, you want to start with these cool little mushrooms you’ve never seen before. You want to start with that ingredient, begin pulling in all the flavors.”

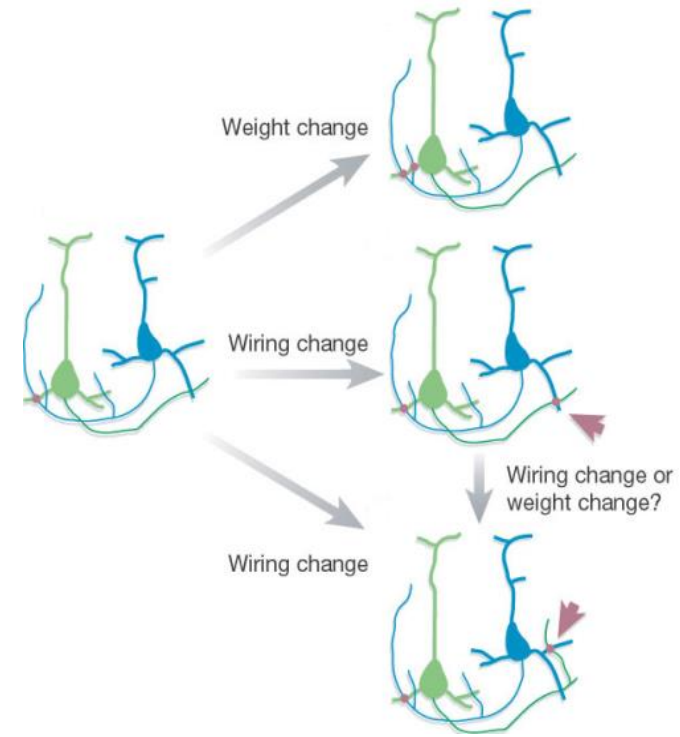
“For me its memory, its all on this taste memory, you don’t have to be a chef: anyone with a lot of experience and who focuses on those kind of things—you start to build out that memory and you start accessing and grabbing from these things that you’ve seen or tasted or smelled before and start putting them into little pairs and its one of those things that evolves.”

“This ingredient grabs that one, and that ingredient grabs another one that you wouldn’t necessarily have thought of with the first one but you start building this chain and that’s where the really interesting things start to happen.”





Information storage in the brain



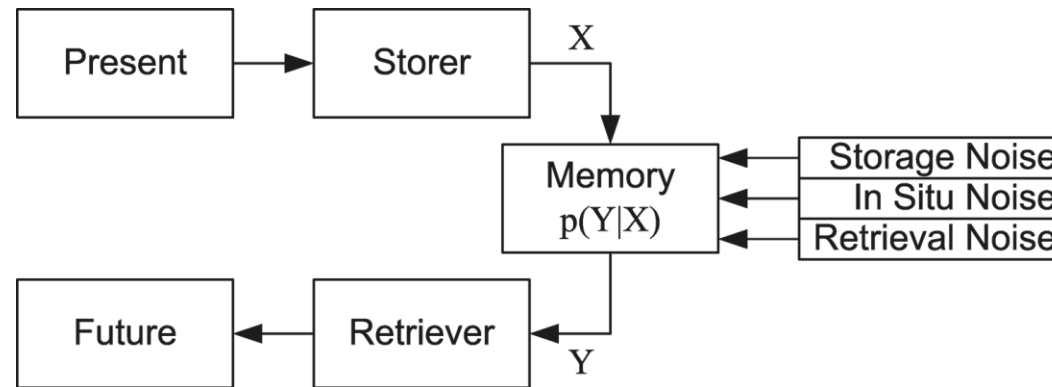
Does the information-theoretic viewpoint provide insight into neural information storage and retrieval?

Model of Information Storage

Optimal Information Storage in Noisy Synapses under Resource Constraints

Lav R. Varshney,^{1,2} Per Jesper Sjöström,³
and Dmitri B. Chklovskii^{2,*}

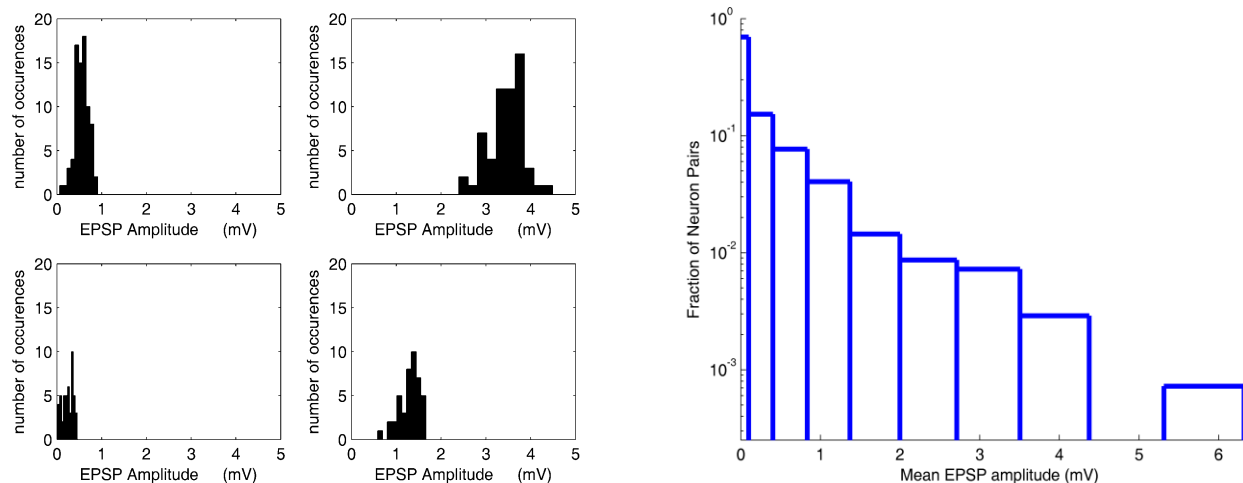
- Treat the brain as a noisy storage channel:



- Each potential synapse strength is a channel symbol: x when stored and y when retrieved
- Note that channel symbols are separated in space, not time

Properties of Synapses

- Synapses are small and noisy on average
- Heavy-tailed synaptic strength distribution



Neocortical L5 pyramidal neurons in young rats

- Synaptic connectivity is sparse
 - Filling fraction between 0.1 and 0.3 for various brain regions in various mammals [Stepanyants, et al., 2002]
- Synapses may be discrete-valued

Use optimization approach to biology for a unified theory

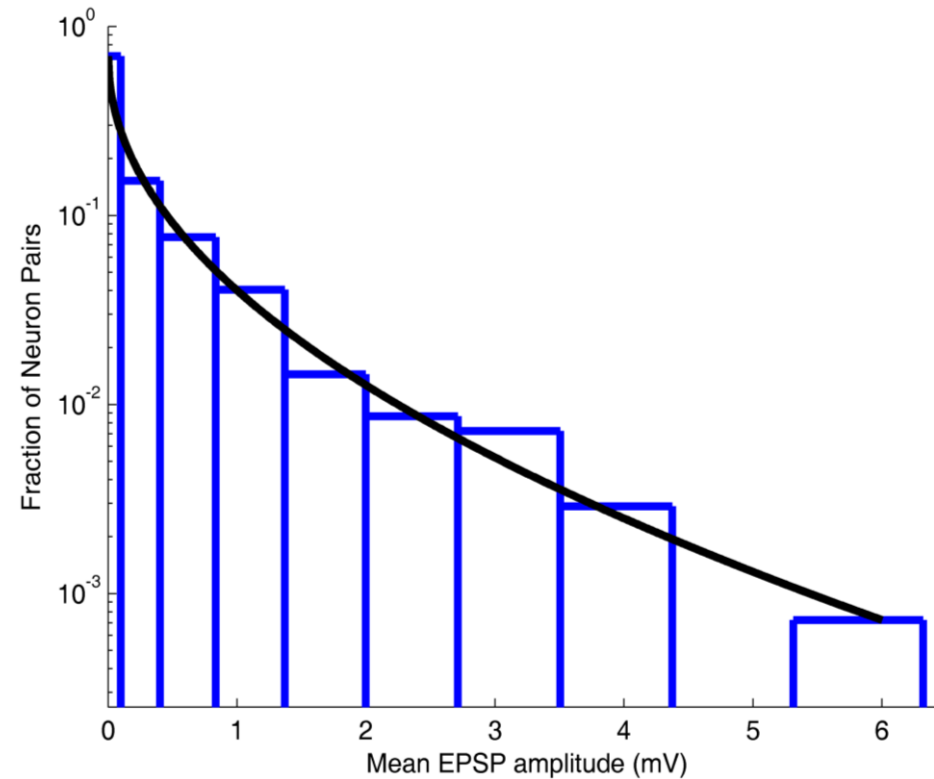
Optimization Approach to Synapses

- Think of cortex as information storage device
- Beneficial to have large storage capacity
- Volume is a costly resource [Cajal, 1899]:
 - material, metabolic energy, head size, etc.

- Optimize information storage capacity per unit volume
- Make predictions about physically measurable properties of synapses using information-theoretic optimization

Experimental Test: Strength Distribution

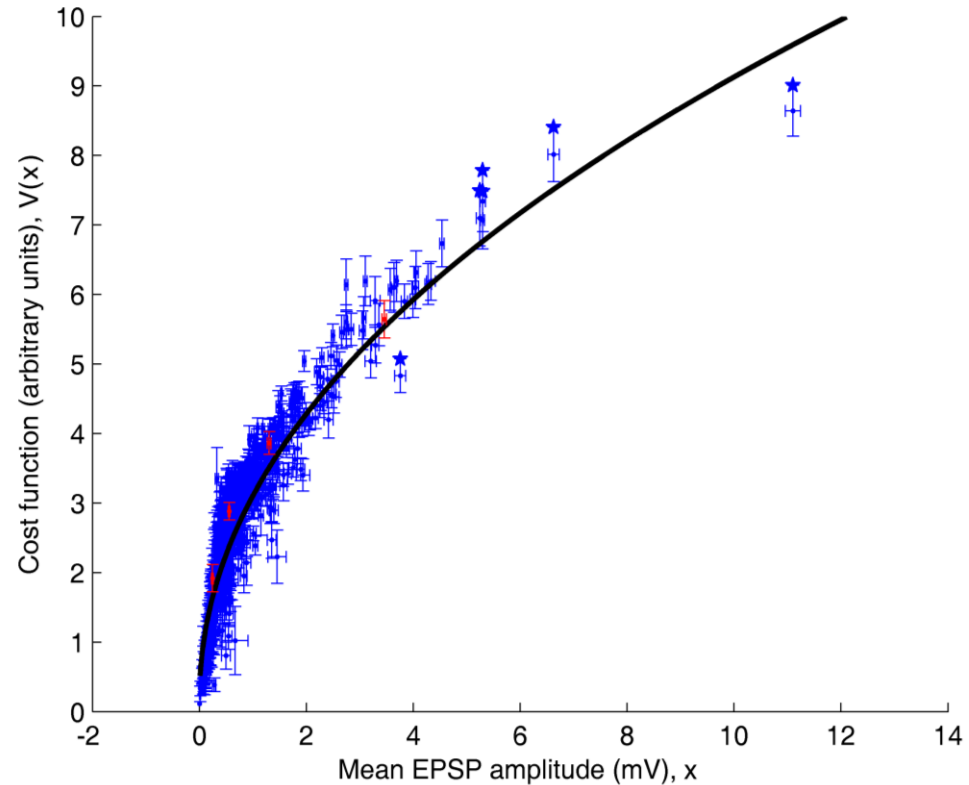
- Capacity achieving input distribution is stretched exponential
- Data for 637 L5 neurons [Varshney, et al., 2006]



Stretched exponential fit
gives prediction of $\alpha =$
0.79

Optimizing Cost Function

- Data for 637 L5 neurons [Varshney, et al., 2006]



Power law fit gives prediction of optimizing cost function with $\alpha = 0.77$

- Joint imaging and electrophysiology experiments to confirm

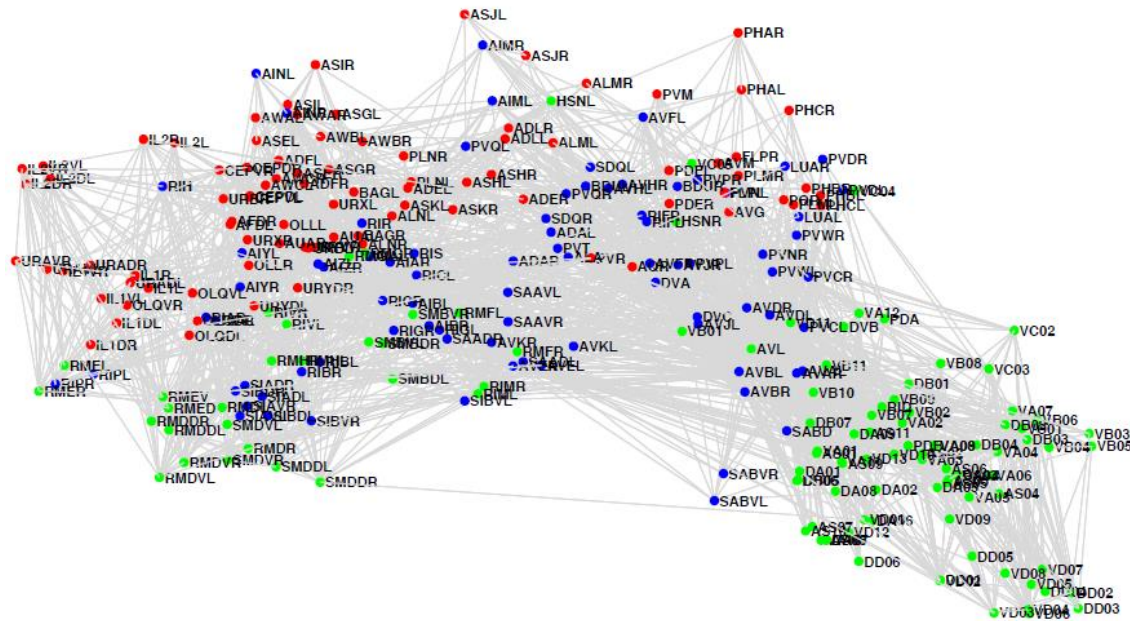
Optimal Information Storage and the Distribution of Synaptic Weights: Perceptron versus Purkinje Cell

Nicolas Brunel,¹ Vincent Hakim,²
Philippe Isope,³ Jean-Pierre Nadal,²
and Boris Barbour^{4,*}

- Rather than information-theoretic limits, consider specific learning rule
- Obtained optimal synaptic weight distribution for classical perceptron (single-layer feedforward network) with excitatory synapses
- Contains more than 50% silent synapses, and this fraction increases with storage reliability
- Well-matched to data from cerebellar Purkinje cells
- Perceptrons have fairly low storage capacity

Neuronal Circuits (for Coding/Decoding)

- Specific perceptron learning rule did not achieve much storage capacity
- Information-theoretic approach abstracted away encoding and decoding in neural circuits (cf. connectomics)



**Neural circuits
have variability!**

Functional benefits of variability?

Table 1 | Representative experimental and modelling studies of stochastic resonance (in chronological order)

Approach	System, or level of organization	Technique, or level of detail *	Signal and noise	Result [‡]	Function in vivo or proposed computation
Experimental studies	Shark multimodal sensory cell	Extracellular recording	Ramped temperature, and electrical current changes and intrinsic noise in neurons	Information-transmitting spikes generated, allowing dual coding of temperature and electrical fields	Water temperature and depth sensing, and prey detection
	Cricket cercal receptor — innervating interneurons	Intracellular recording	23-Hz sinusoidal and 5–400-Hz broadband modulation of air current, and 5–400-Hz white noise-modulated air currents	SNR (23-Hz signal) and mutual information (broadband signal) enhanced by noise	Predator avoidance
	Human muscle spindle afferents in arm	Extracellular recording	0.5-Hz sinusoidal rotation of arm and random stretching of tendon	SNR of afferent firing at signal frequency enhanced by noise	Movement sensation
	Whole human brain	EEG	5-dB sensation level, 1000-Hz and 500-Hz pure tones and broadband acoustic noise	Neural synchrony within (40-Hz transient response) and between (θ , α and γ frequency bands) brain regions enhanced by noise	Auditory processing

[McDonnell and Ward, 2011]

Motivations

- Engineering domains
 - Nanoscale information fabrics
 - Computational creativity (for culinary recipes)
- Scientific understanding
 - Hippocampus, piriform cortex (for culinary recipes?)
 - Variability is the name of the game in biology: are there functional benefits?

Motivations

A computational model that reproduces certain aspects of associative memory

- Large capacity [T. L. White, “Olfactory Memory: the Long and Short of It,” *Chem. Senses*, 1998.]
- Noisy information processing circuitry

Are there fundamental limits of such computational systems?

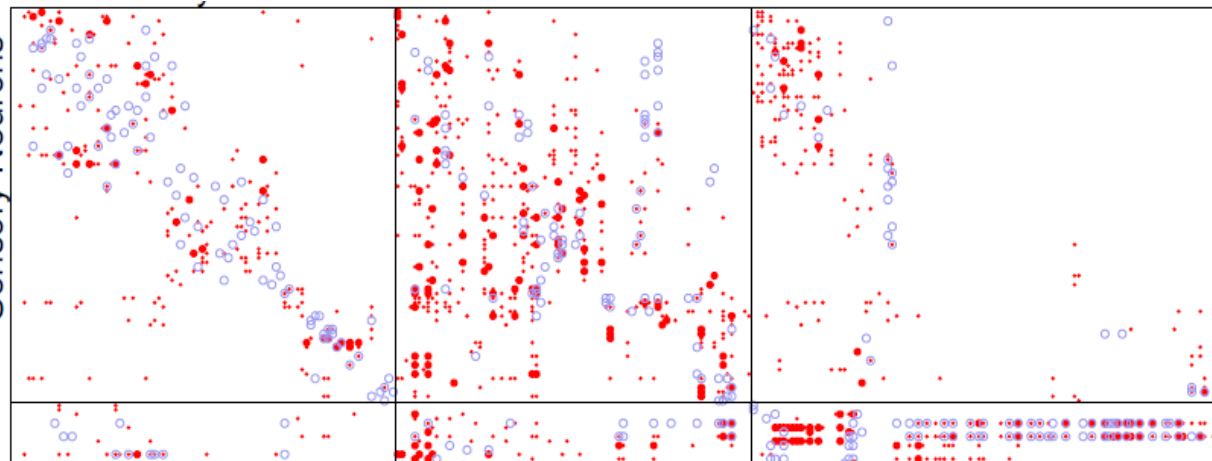
- Nanoscale information fabrics
- Computational creativity systems

Do the models explain experimentally measurable properties of neural systems?

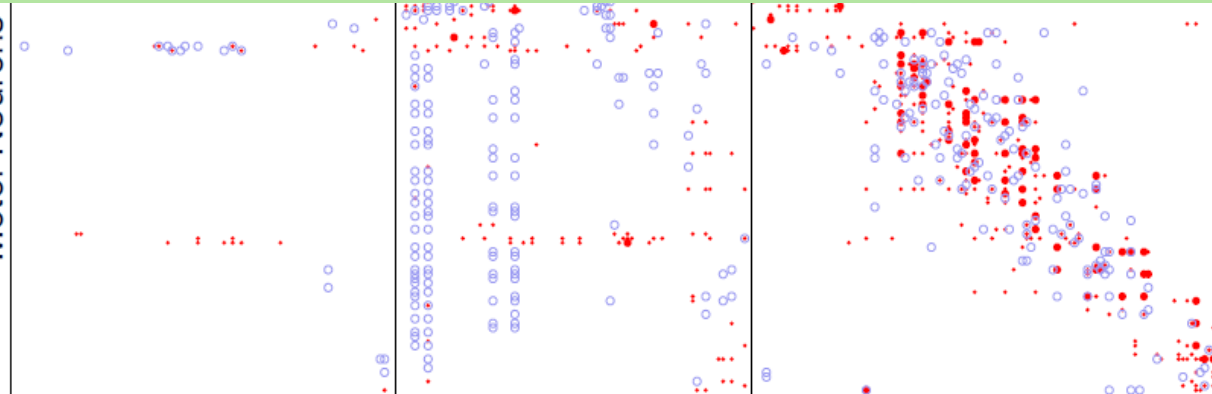
- Synaptic microarchitecture

Associative memory

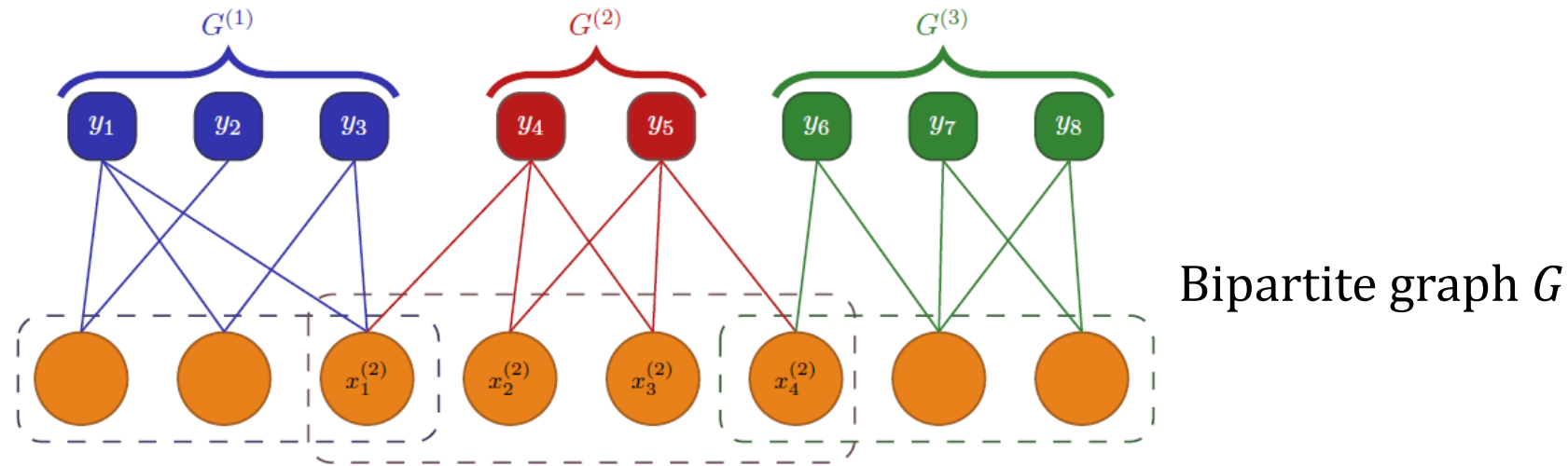
- Wish to store a desired set of states—the memories—as fixed points of a network such that errors in input representation of a memory are corrected and the memory retrieved
 - Nearest-neighbor search
- Modern coding theory: matrix of synaptic weights are like a code matrix



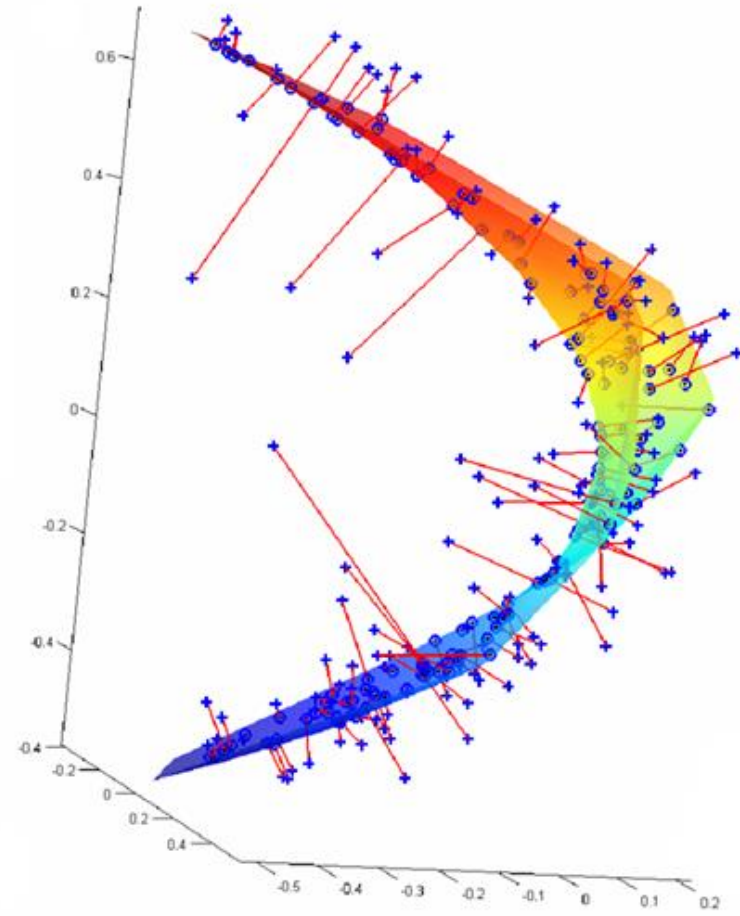
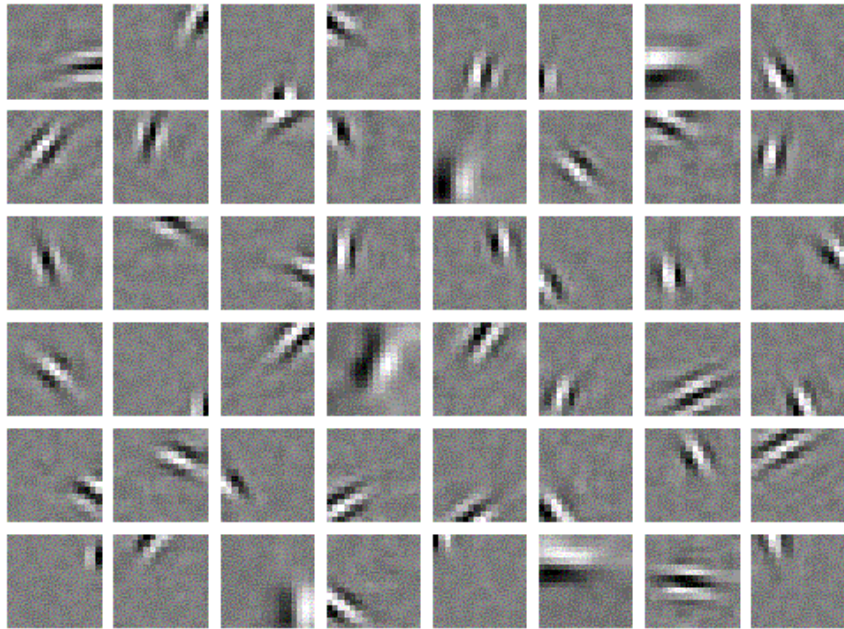
- Consider a specific learning and recall algorithm, inspired by Shannon-theoretic limits and modern codes
 - Associative memory with large capacity
 - No requirement of memorizing arbitrary patterns
 - Noisy recall circuits



Architecture with overlapping clusters



- Convolutional, graph code-based, associative memory model
- Memorize patterns with strong local correlation, with linear constraints within clusters, rather than arbitrary patterns
 - Natural stimuli
- Clustered structure matches cortical column structure



[Koulakov, et al., 2011]

Learning algorithm

- Subspace learning algorithm tries to ensure weight vectors within clusters are orthogonal to all presented patterns and are sparse
- \mathcal{C} is the pattern retrieval capacity, *exponential* in the number of neurons
 - Much better than, say, traditional Hopfield network

INPUT: Set of memories \mathcal{X} with $|\mathcal{X}| = \mathcal{C}$, and stopping point ε

OUTPUT: weights for ℓ th cluster $w^{(\ell)}$

Theorem Algorithm converges to a local optimum (in terms of weight of synapses) such that orthogonality requirement is met

Pattern retrieval capacity

- Capacity *exponential* in the number of neurons
 - Much better than Hopfield network which has capacity linear in number of neurons [McEliece et al., 1987]
- Restricting pattern set for Hopfield network utilizing neural cliques increased capacity of Hopfield networks to quadratic in number of neurons [Gripon and Berrou, 2011]
- Can bring it to exponential via probability flow argument [Hillar and Tran, 2014]
- (Idea of restricting pattern sets first introduced by Venkatesh and by Biswas in 1980s)

Recall algorithm

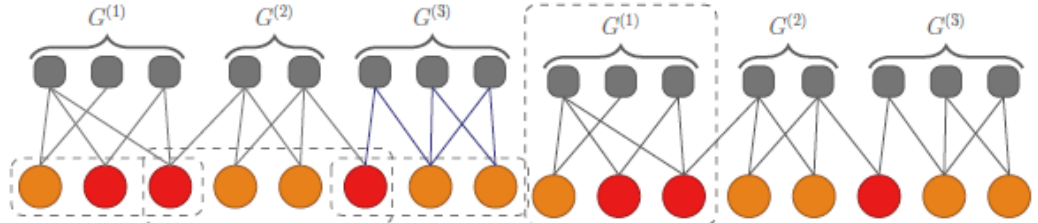
- Presented with noisy query, and want to recover a pattern that was actually memorized
- Locally based on iterative message-passing within cluster (which hopefully have linear constraints)

Theorem Intra-module error correction algorithm can correct at least one error in the query [Karbasi, et al., 2013]

- Globally use sequential peeling to transfer information in one cluster to help an overlapping cluster

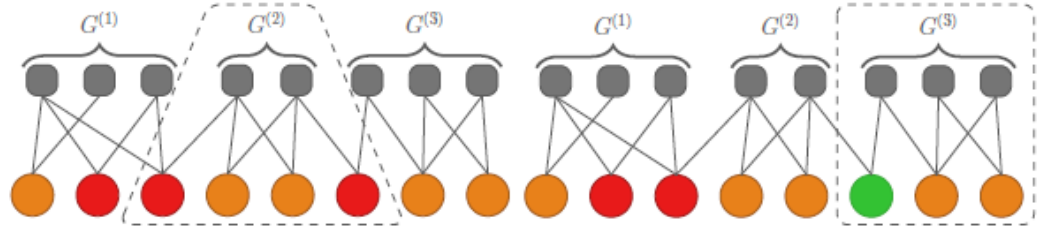
Theorem In the limit of large graphs that meet suitable degree distribution constraints, the decoding algorithm will be successful when below a specified external error probability threshold [Karbasi, et al., 2013]

Introduce circuit noise



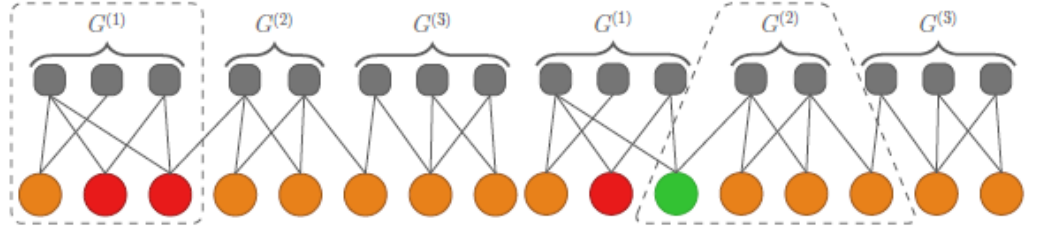
(a) Initial step

(b) Step 1: cluster 1 fails.



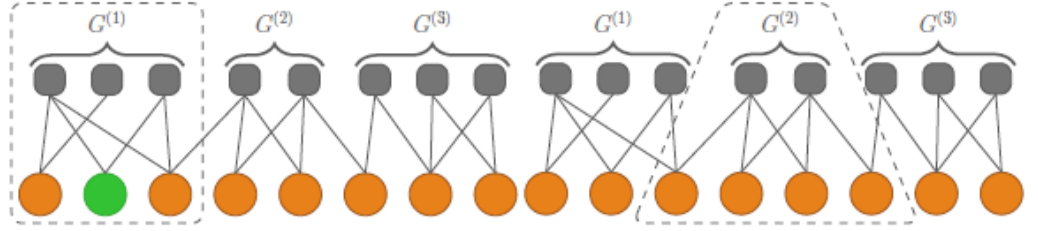
(c) Step 2: cluster 2 fails.

(d) Step 3: cluster 3 succeeds.



(e) Step 4: cluster 1 fails again.

(f) Step 5: cluster 2 succeeds.



(g) Step 7: cluster 1 succeeds.

(h) Step 8: Algorithm finishes successfully.

Mathematical model for recall

- A neuron can assume an integer-valued state from the set $Q = \{0, \dots, Q - 1\}$, interpreted as short term firing rate
- A neuron updates its state based on the states of its neighbors $\{s_i\}_{i=1}^n$ as follows:
 - First compute weighted sum $h = \sum_{i=1}^n w_i s_i + \zeta$, where w_i is weight of link from s_i and ζ is internal noise
 - Then apply to h a nonlinear function $f: \mathbb{R} \rightarrow Q$
- Associative memory is represented by weighted bipartite graph, G , with pattern neurons and constraint neurons
- Each pattern $x = (x_1, x_2, \dots, x_n)$ is vector of length n , $x_i \in Q$
- Divide entries of each x into L overlapping subpatterns of lengths n_1, n_2, \dots, n_L

Mathematical model for recall

- Let $x^{(i)}$ be the i th subpattern, and due to subspace learning, the corresponding synaptic weights $W^{(i)}$ in the bipartite graph satisfy $W^{(i)} \cdot x^{(i)} = 0$ for all patterns x in the dataset \mathcal{X} that has been memorized

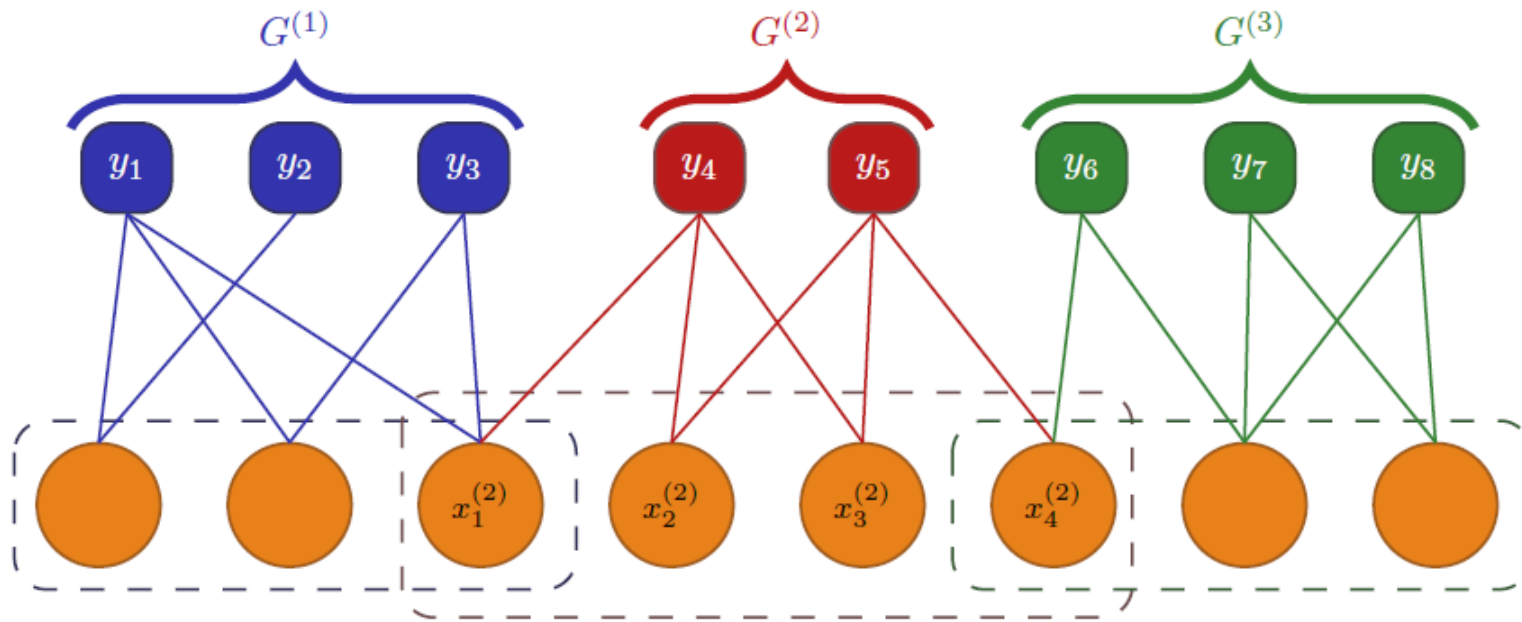
External Errors

- Goal is to retrieve memorized pattern \hat{x} from its corrupted version x due to external errors, where external error is additive vector of size n denoted by z , satisfying $x = \hat{x} + z$

Internal Noise

- Random numbers uniformly distributed in intervals $[-\nu, \nu]$ and $[-\nu, \nu]$ additively affect local computations for pattern and constraint neurons, respectively

Two iterative algorithms



- Locally based on iterative message-passing within cluster (which hopefully have linear constraints)
- Globally use sequential peeling to transfer information in one cluster to help an overlapping cluster

Algorithm 1 Intra-Module Error Correction

Input: Training set \mathcal{X} , thresholds φ, ψ , iteration t_{\max}

Output: $x_1^{(\ell)}, x_2^{(\ell)}, \dots, x_{n_\ell}^{(\ell)}$

1: **for** $t = 1 \rightarrow t_{\max}$ **do**

2: *Forward iteration:* Calculate the input $h_i^{(\ell)} = \sum_{j=1}^{n_\ell} W_{ij}^{(\ell)} x_j^{(\ell)} + v_i$, for each neuron $y_i^{(\ell)}$ and set $y_i^{(\ell)} = f(h_i^{(\ell)}, \psi)$.

3: *Backward iteration:* Each neuron $x_j^{(\ell)}$ computes

$$g_j^{(\ell)} = \frac{\sum_{i=1}^{m_\ell} \text{sign}(W_{ij}^{(\ell)}) y_i^{(\ell)}}{\sum_{i=1}^{m_\ell} \text{sign}(|W_{ij}^{(\ell)}|)} + u_i.$$

4: Update state of each pattern neuron j according to $x_j^{(\ell)} = x_j^{(\ell)} - \text{sign}(g_j^{(\ell)})$ only if $|g_j^{(\ell)}| > \varphi$.

5: **end for**

(Note biologically plausible)

Algorithm 2 Sequential Peeling Algorithm

Input: $\tilde{G}, G^{(1)}, G^{(2)}, \dots, G^{(L)}$.

Output: x_1, x_2, \dots, x_n

- 1: **while** there is an unsatisfied $v^{(\ell)}$ **do**
 - 2: **for** $\ell = 1 \rightarrow L$ **do**
 - 3: If $v^{(\ell)}$ is unsatisfied, apply Alg. 1 to cluster $G^{(\ell)}$.
 - 4: If $v^{(\ell)}$ remained unsatisfied, revert state of pattern neurons connected to $v^{(\ell)}$ to their initial state. Otherwise, keep their current states.
 - 5: **end for**
 - 6: **end while**
 - 7: Declare x_1, x_2, \dots, x_n if all $v^{(\ell)}$'s are satisfied. Otherwise, declare failure.
-

(Note state reversion is somewhat difficult to implement biologically)

Recall performance analysis

- If local message-passing thresholds ψ and ϕ are chosen properly, then in the absence of external errors the constraints remain satisfied and internal noise cannot result in violations
 - Key for making sequential peeling work

Lemma In the absence of external errors, the probability that a constraint neuron in cluster ℓ makes a wrong decision due to its internal noise is $\max\left(0, \frac{v-\psi}{v}\right)$ and for a pattern neuron, the probability is $\max\left(0, \frac{v-\phi}{v}\right)$

Proof From the thresholding operation in the algorithm

Recall performance analysis

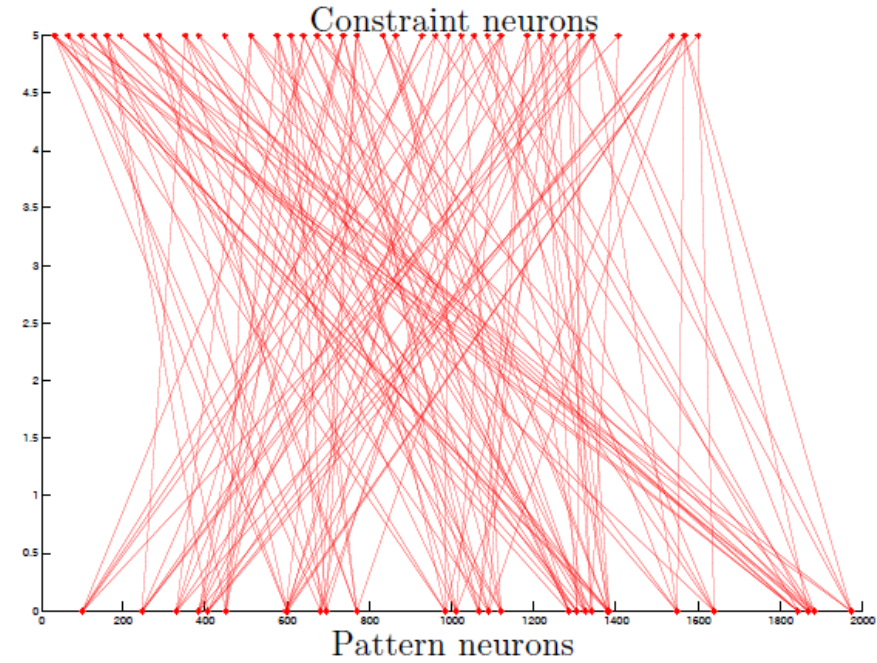
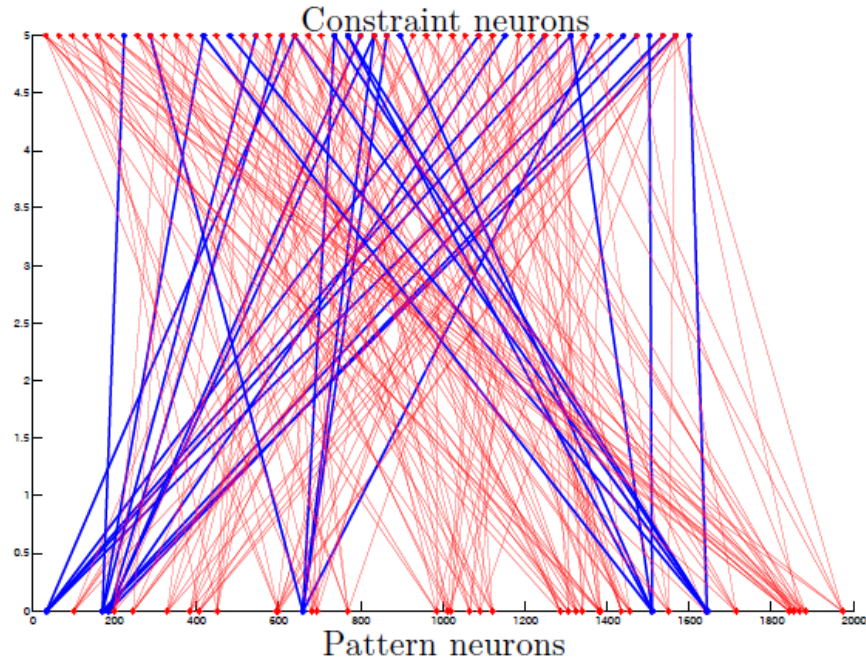
- A neural network with internal noise outperforms one without (while maintaining the same capacity)
- Let the fraction of external errors corrected by a noiseless recall algorithm after T iterations be $\Lambda(T)$ and that of a recall algorithm with internal noise be $\Lambda_{\nu,\nu}(T)$.
- Further let the $T \rightarrow \infty$ values be Λ^* and $\Lambda_{\nu,\nu}^*$

Theorem For appropriately chosen thresholds and the same capacity \mathcal{C} , for the same realizations of external errors,

$$\Lambda_{\nu,\nu}^* \geq \Lambda^*$$

Proof The noisy network can correct any external error pattern that the noiseless counterpart can correct (stopping set argument)

Stopping sets



Internal noise pushes the recall algorithm out of local minimums where it may be stuck

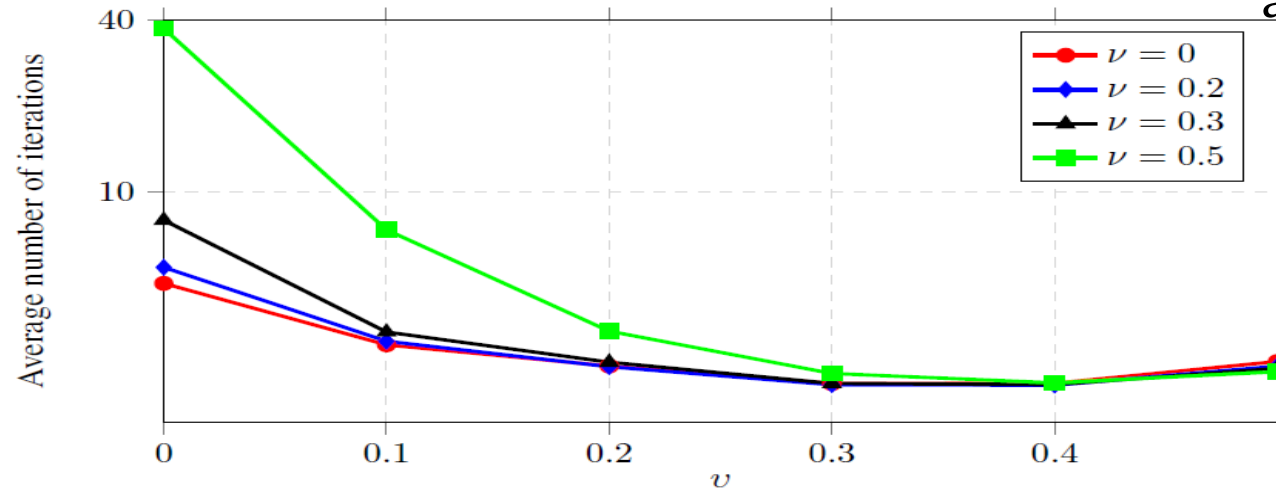
Recall performance analysis: recovery speed

Theorem suggests only possible downside to using a noisy network is its possible running time in eliminating external errors: the noisy neural network may need more iterations to achieve the same error correction performance.

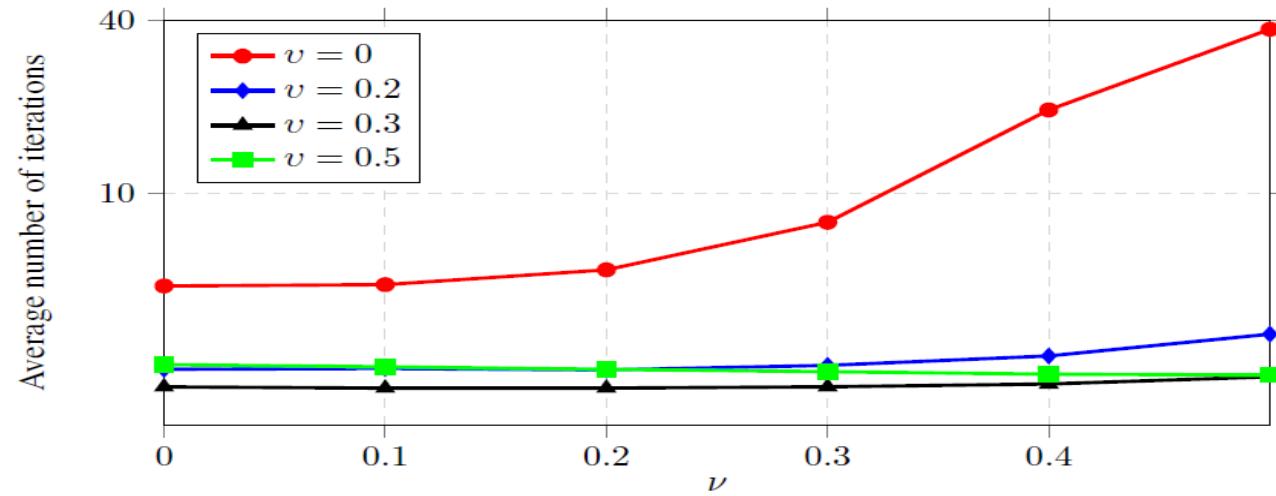
Empirical experiments show that in certain scenarios, even the running time improves when using a noisy network

Recall performance analysis: recovery speed

Setting where all algorithms succeed



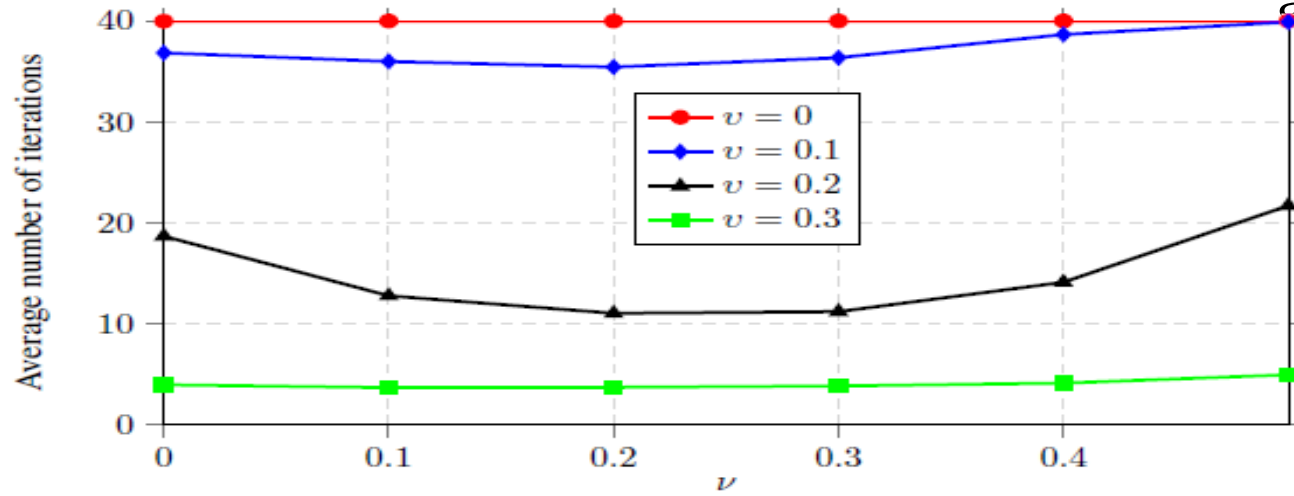
(a) Effect of internal noise at pattern neurons side.



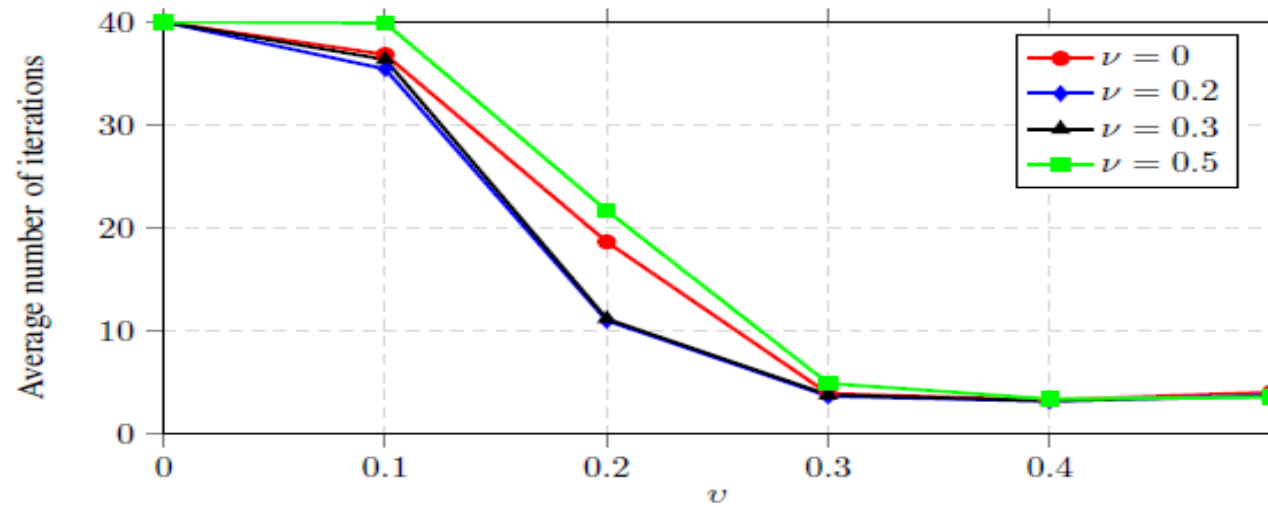
(b) Effect of internal noise at constraint neurons side.

Recall performance analysis: recovery speed

Setting where algorithms may fail



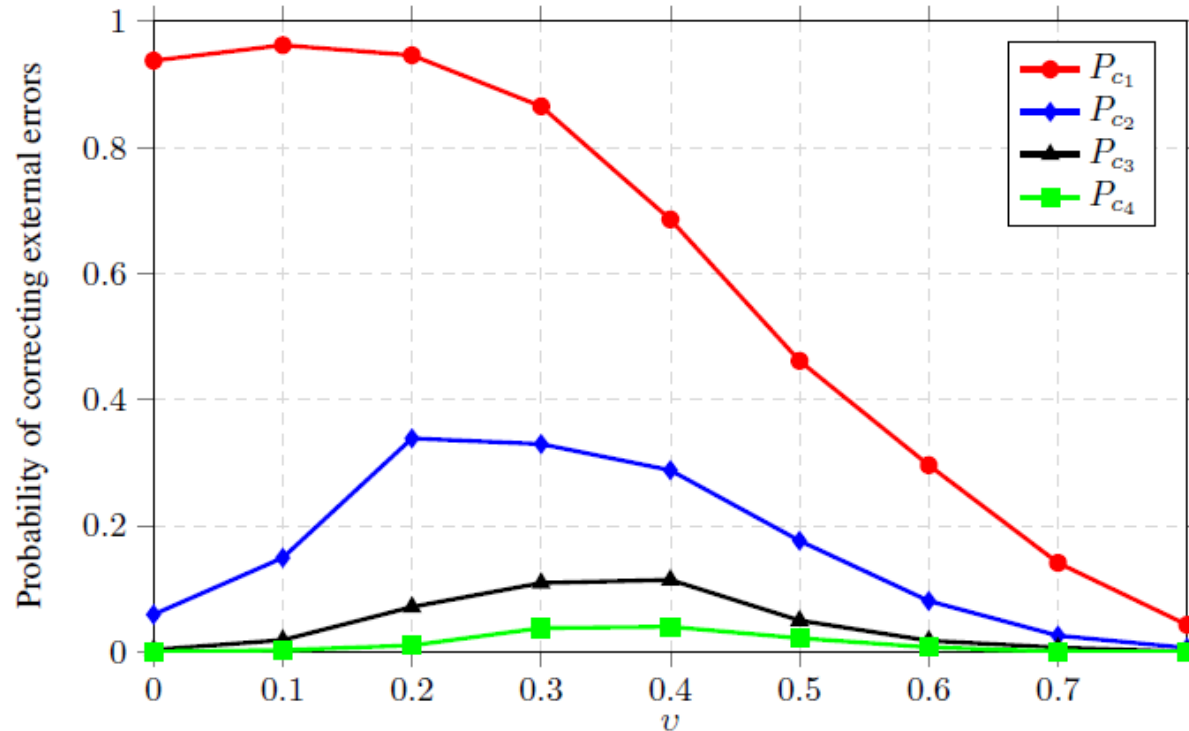
(a) Effect of internal noise at constraint neurons side.



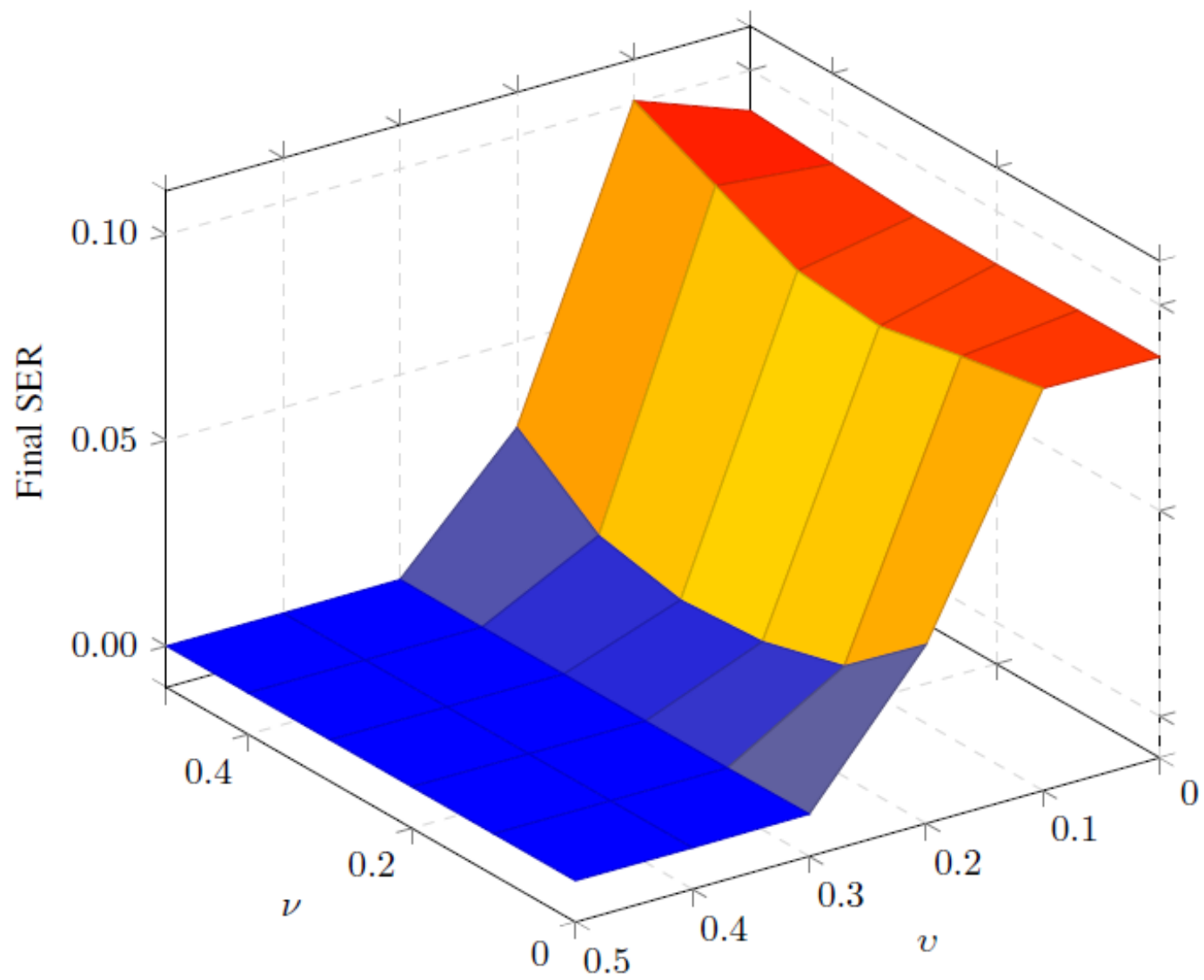
(b) Effect of internal noise at pattern neurons side.

Recall performance analysis: error probability

- Theorem indicates our noisy neural networks outperform noiseless ones, but does not specify the level of errors that such networks can correct
- We have derived a theoretical upper bound on error correction performance



Finitary simulations



Functional benefits of noise

Showed that internal noise actually improves the performance of the recall phase while the pattern retrieval capacity remains exponential in the number of neurons

Cf.

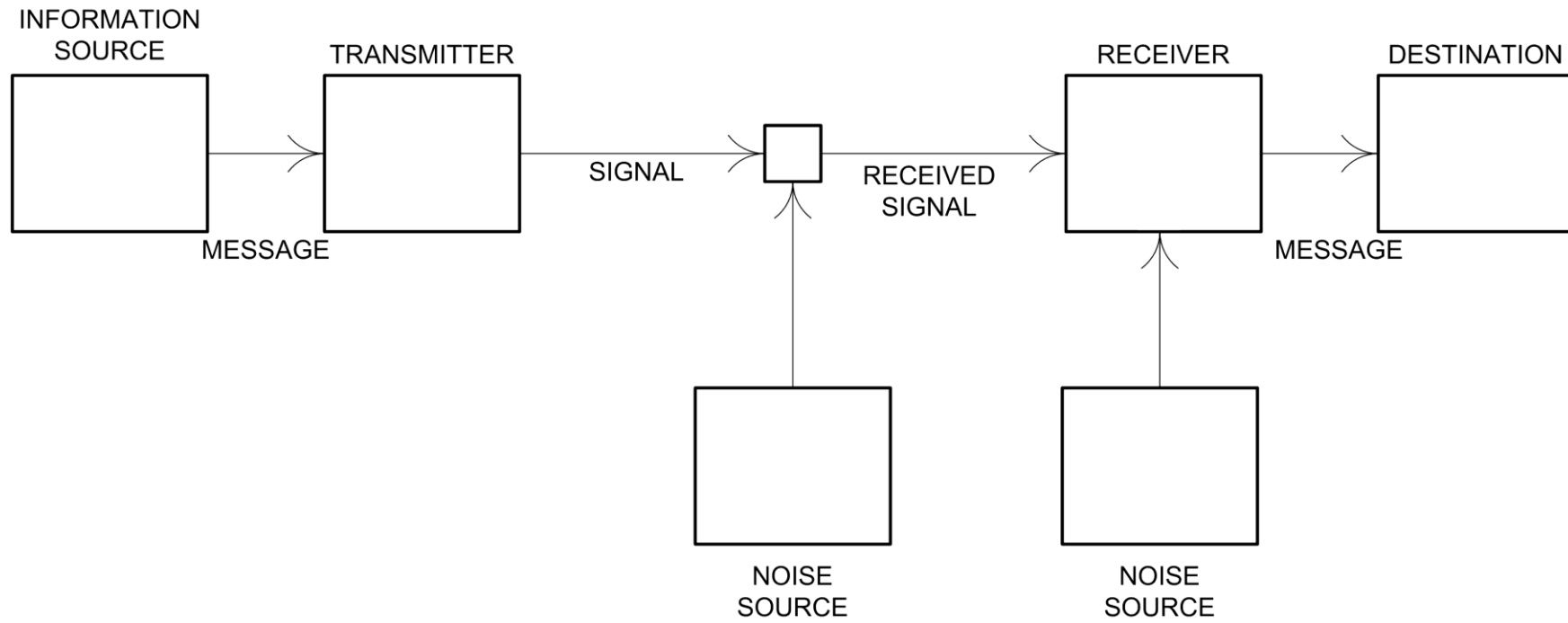
Stochastic resonance in the hippocampal CA3–CA1 model:
a possible memory recall mechanism

Motoharu Yoshida^a, Hatsuo Hayashi^{b,*}, Katsumi Tateno^b, Satoru Ishizuka^b

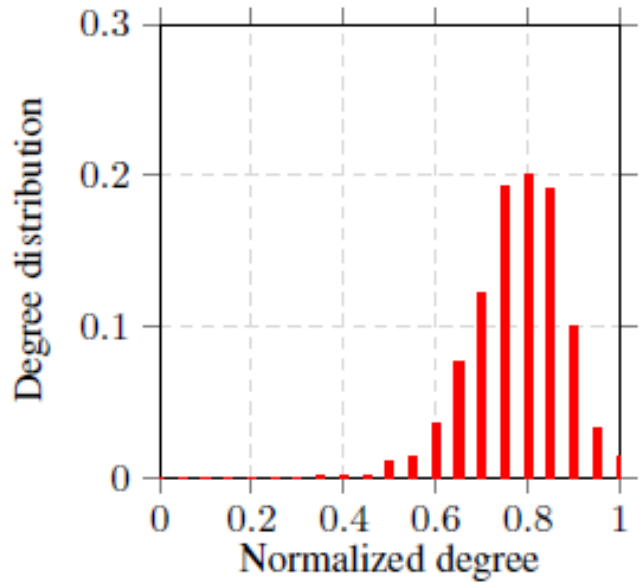
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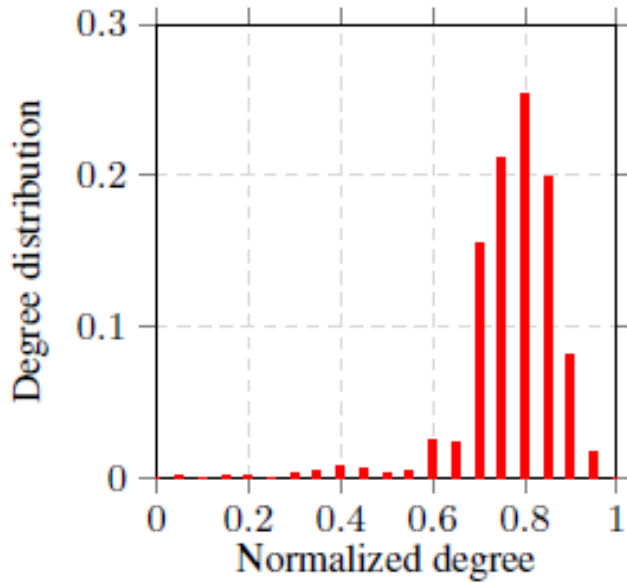
Received 10 September 2001; accepted 24 June 2002



Are there fundamental limits of such computational systems?



(a) Pattern neuron degrees

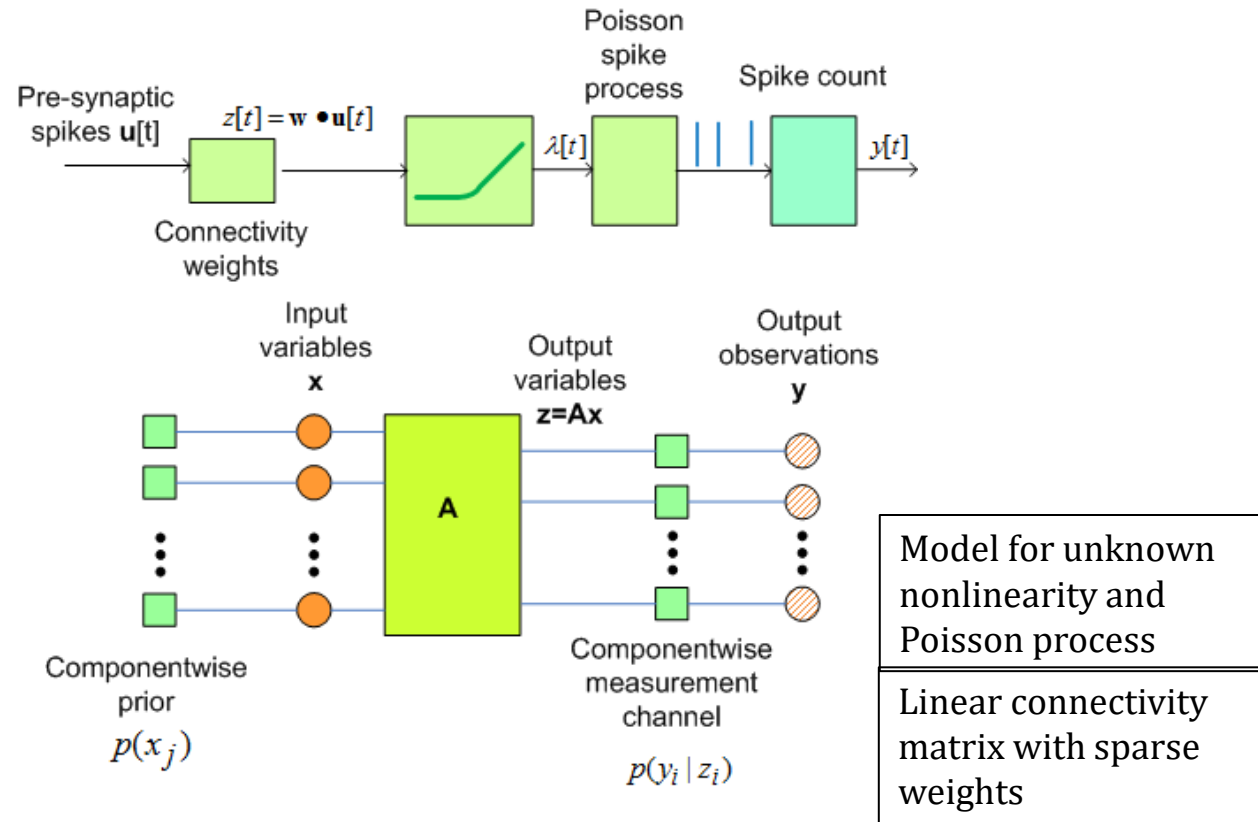


(b) Constraint neuron degrees

Do models explain experimentally measurable properties of neural systems?

Receptive field and connectome reconstruction

Can noise enhance data analysis for efficient receptive field or connectome reconstruction, using multi-neuron excitation?



- Message-passing algorithm for connectivity estimation [Fletcher, Rangan, Varshney, Bhargava, 2011]

Further details

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- L. R. Varshney, “Performance of LDPC codes under faulty iterative decoding,” *IEEE Trans. Information Theory*, vol. 57, pp. 4427–4444, July 2011.
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