Representation of Information
ECE 598 LV – Lecture 11

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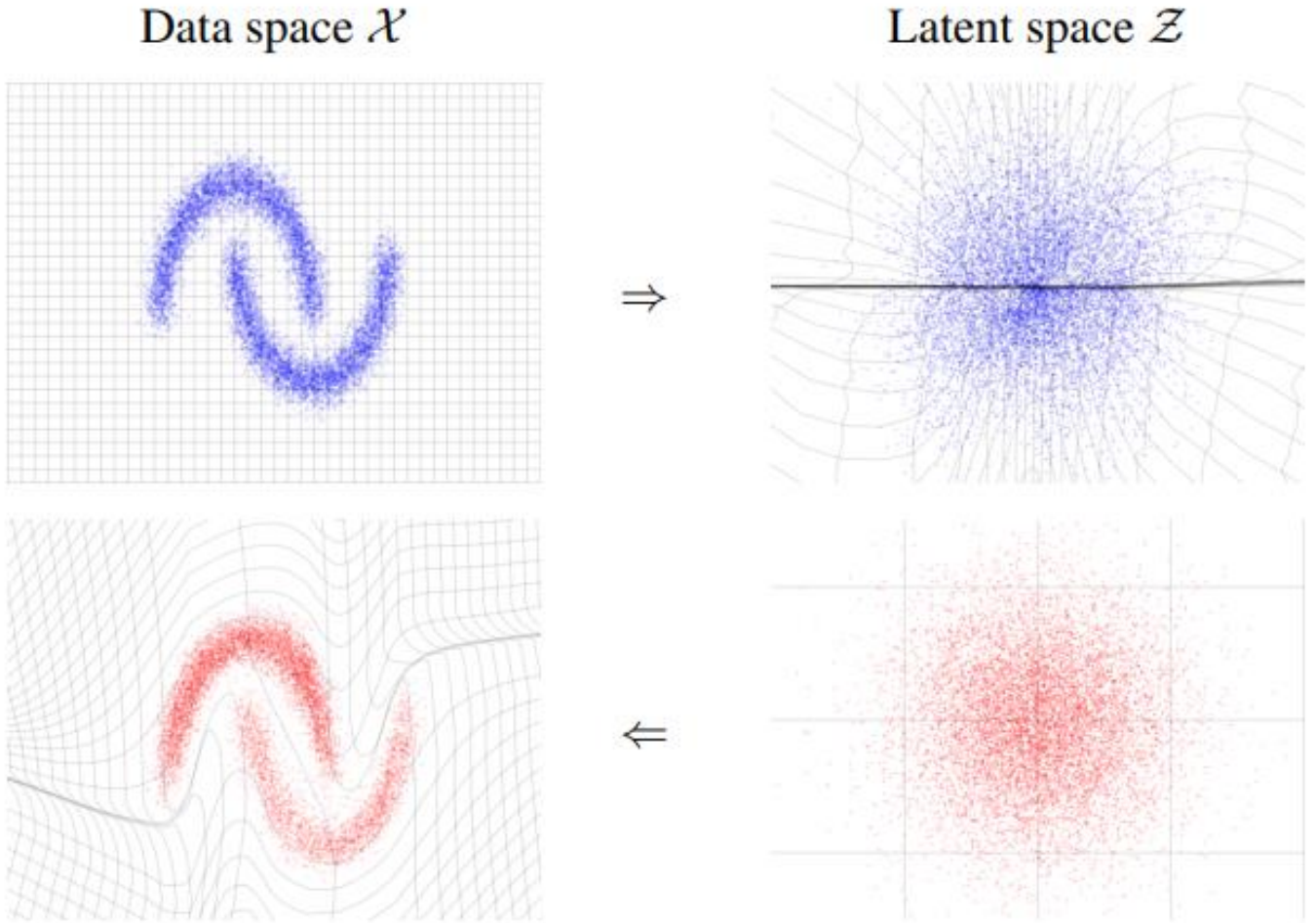
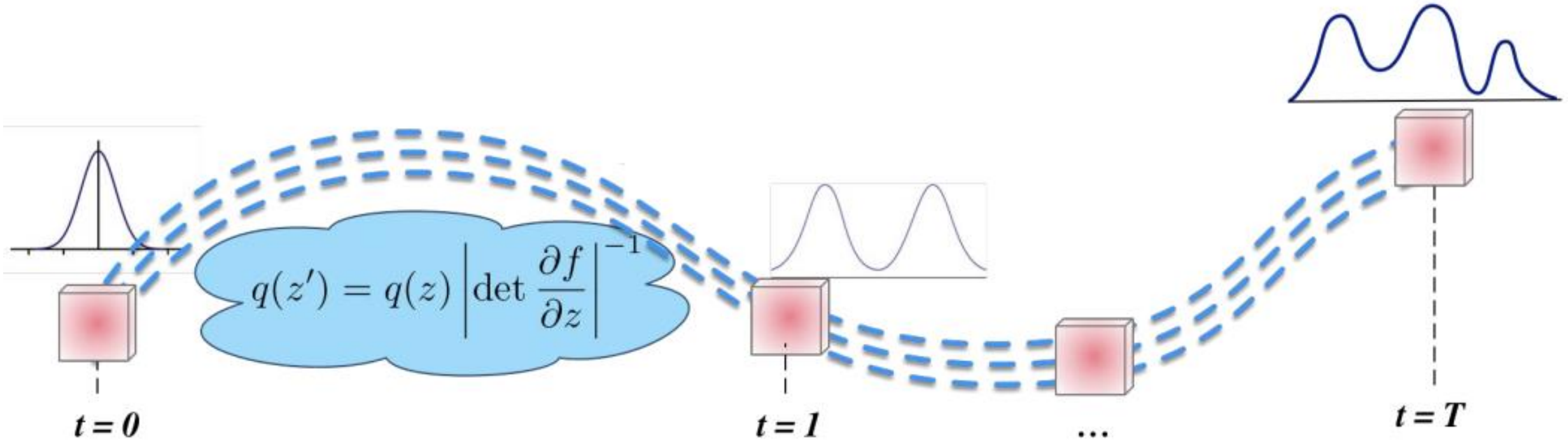


an invertible, stable, mapping between a data distribution $\hat{p}_X$ and a latent distribution p_Z (typically a Gaussian). Here we show a mapping that has been learned on a toy 2-d dataset. The function $f(x)$ maps samples x from the data distribution in the upper left into approximate samples z from the latent distribution, in the upper right. This corresponds to exact inference of the latent state given the data. The inverse function, $f^{-1}(z)$, maps samples z from the latent distribution in the lower right into approximate samples x from the data distribution in the lower left. This corresponds to exact generation of samples from the model. The transformation of grid lines in $\mathcal{X}$ and $\mathcal{Z}$ space is additionally illustrated for both $f(x)$ and $f^{-1}(z)$.

[Dinh, Sohl-Dickstein, and Bengio, 2017]

Normalizing Flows

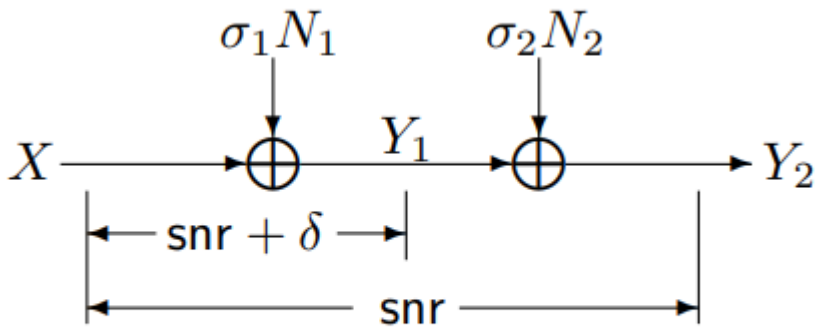


Distribution flows through a sequence of invertible transforms

Rezende and Mohamed, 2015

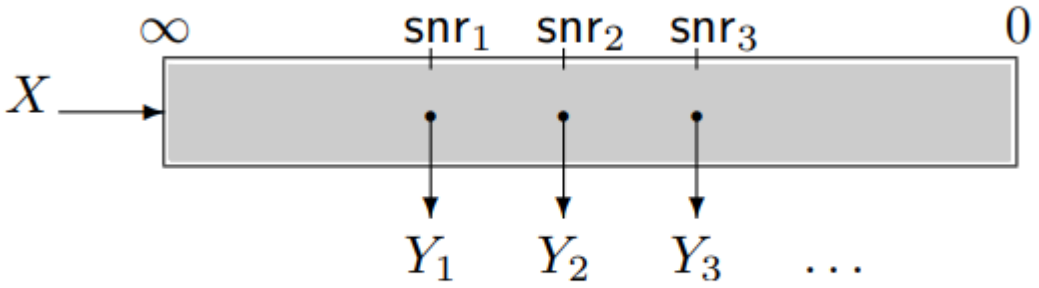
https://www.shakirm.com/slides/DeepGenModelsTutorial.pdf

Not Normalizing Flows



An SNR-incremental Gaussian channel.

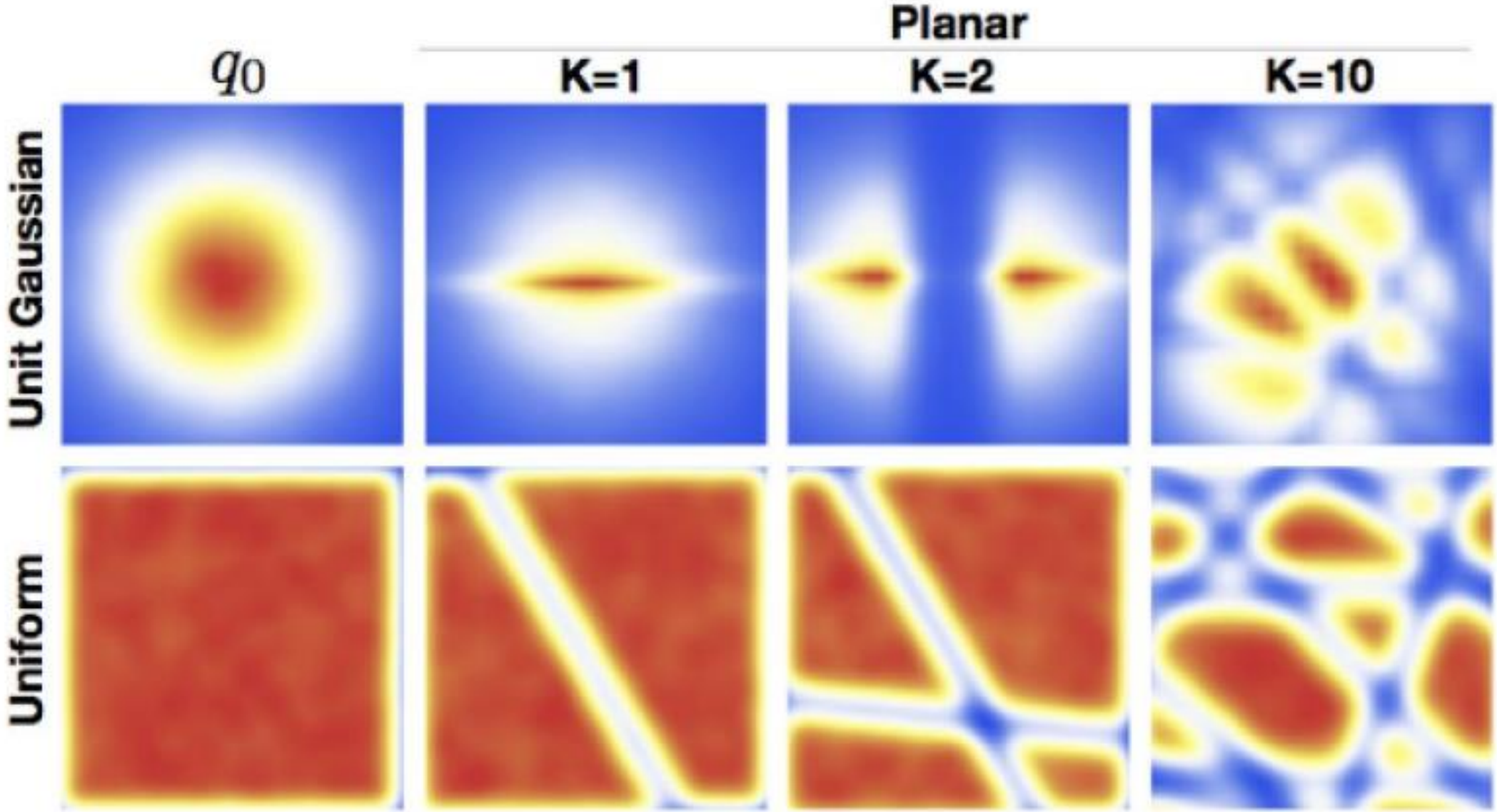
finite



A Gaussian pipe where noise is added gradually.

infinitesimal

Dongning Guo, S. Shamai and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channels," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1261-1282, April 2005.



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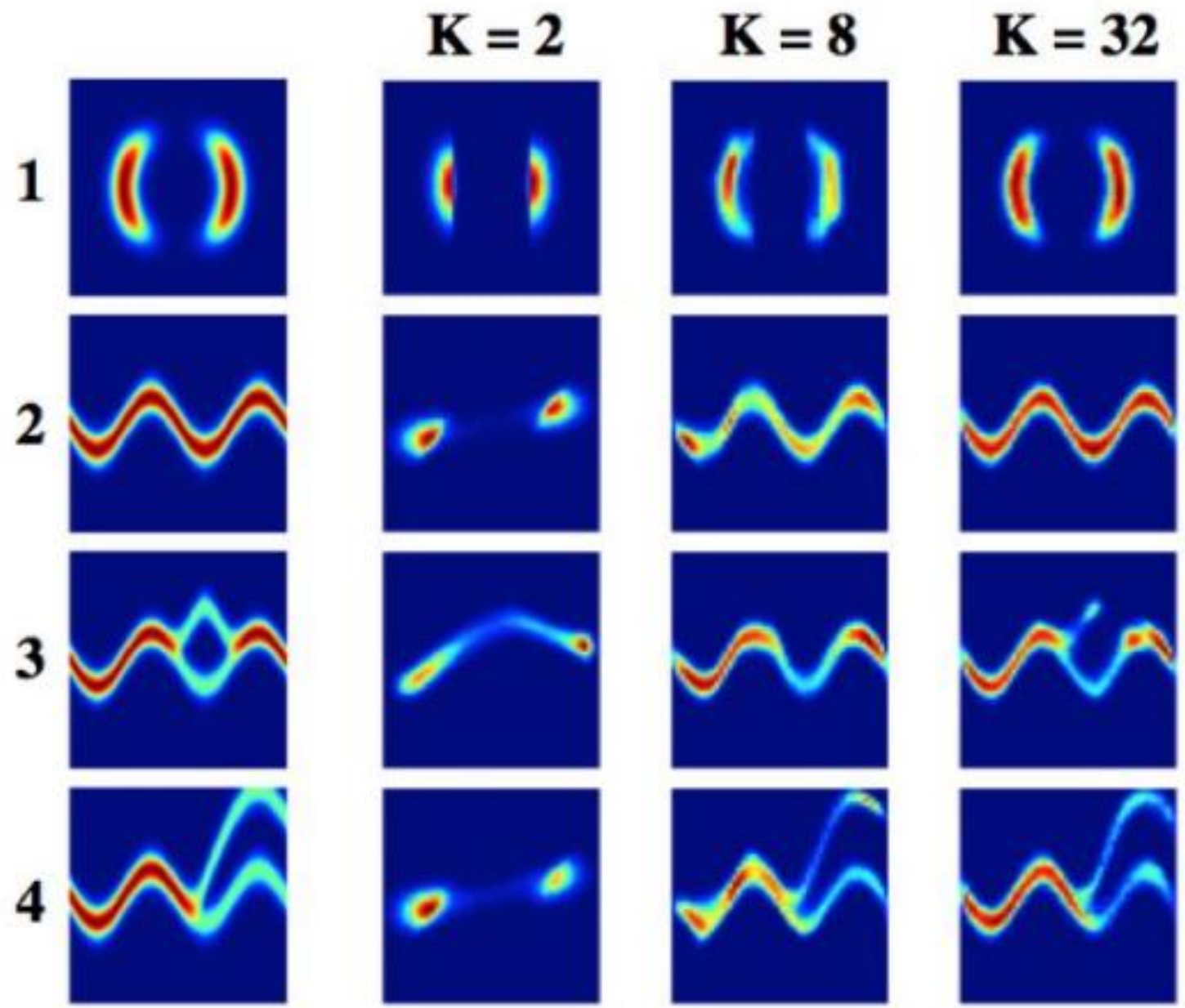

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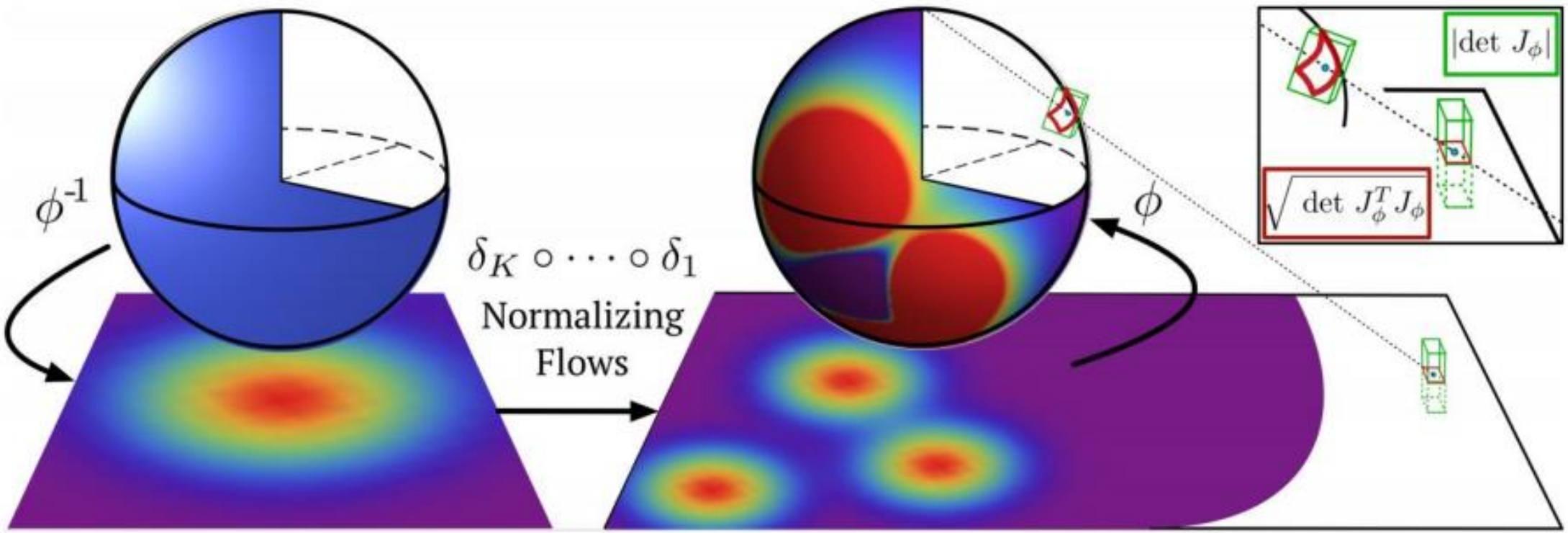
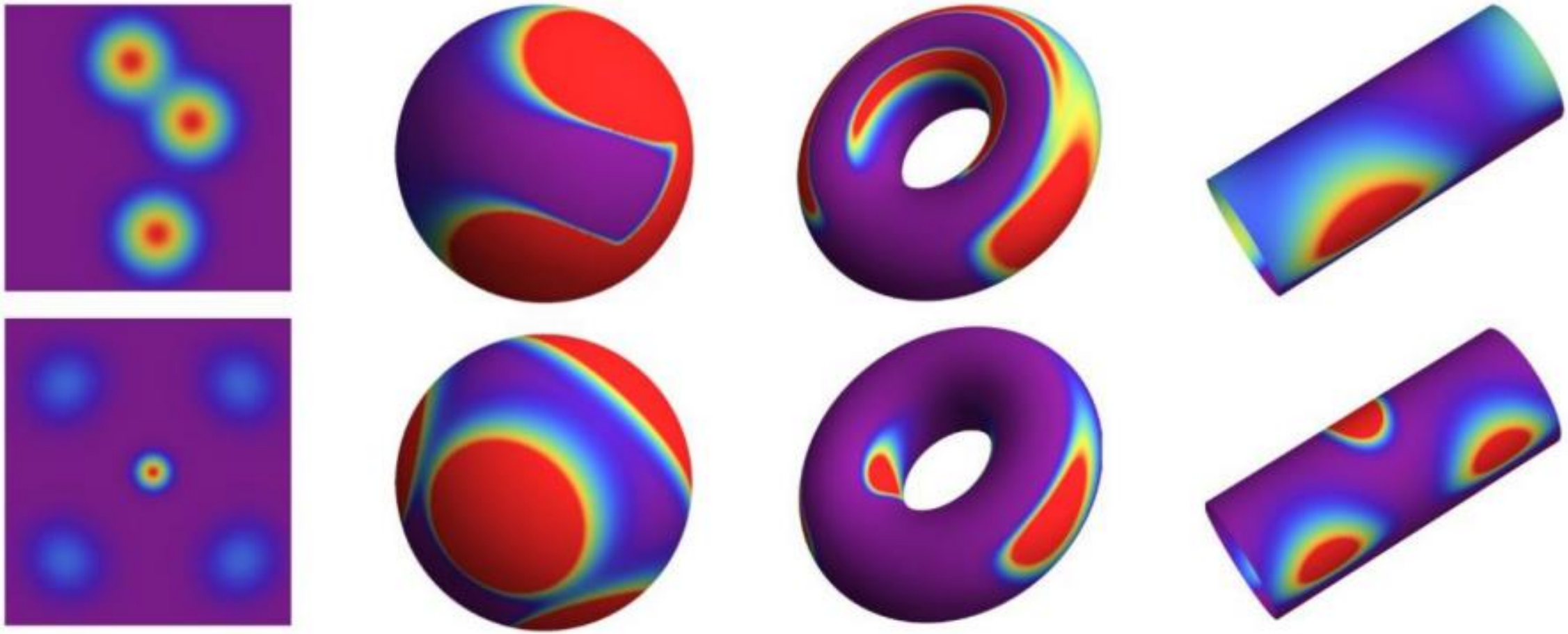


$$\log q_K(\mathbf{z}_K) = \log q_0(\mathbf{z}_0) - \frac{1}{2}\sum_{k=1}^{K} \log \det \left| \mathbf{J}_\phi{}^\top \mathbf{J}_\phi \right|$$

Gemici et al., 2016

https://www.shakirm.com/slides/DeepGenModelsTutorial.pdf



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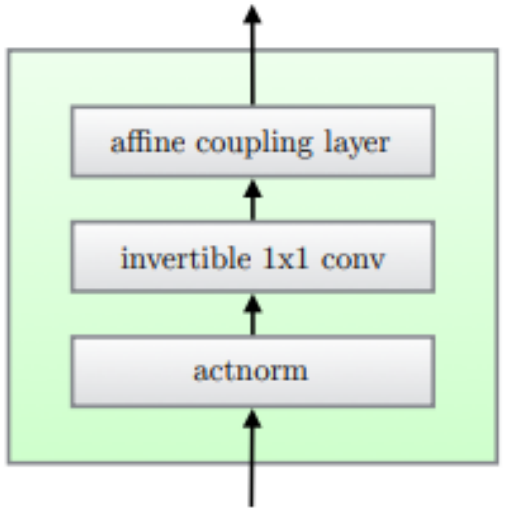
GLOW architecture

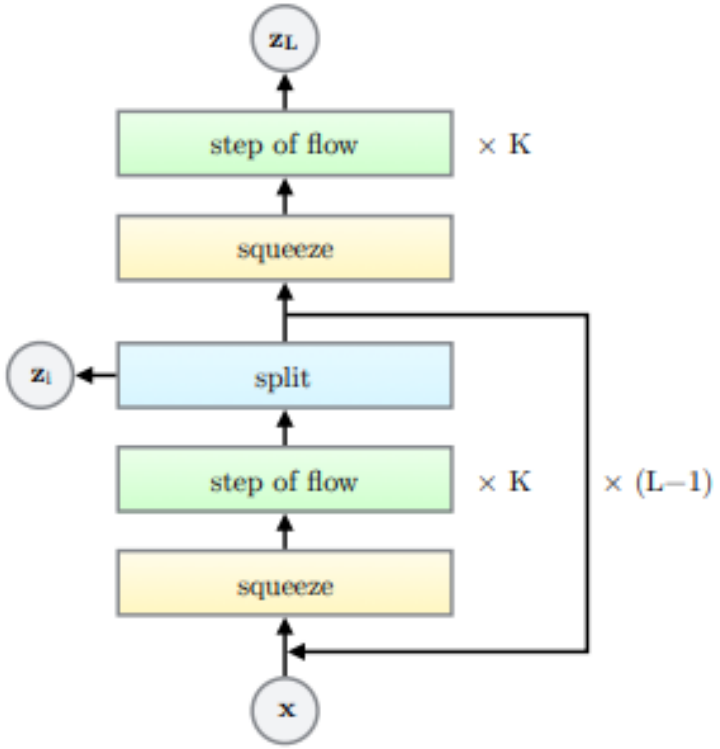
affine coupling layer basically learns a nonlinear independent components representation with easy inverse and easy log det Jacobian, since disentangling is a good representation

invertible convolution is generalization of learned rotation

activation normalization (actnorm) learns affine transformation of activations using a scale and bias parameter per channel



(a) One step of our flow.



(b) Multi-scale architecture (Dinh et al., 2016).

[Kingma and Dhariwal, 2018]

GLOW architecture

Description	Function	Reverse Function	Log-determinant
Actnorm. See Section 3.1.	$\forall i,j : \mathbf{y}_{i,j} = \mathbf{s} \odot \mathbf{x}_{i,j} + \mathbf{b}$	$\forall i,j : \mathbf{x}_{i,j} = (\mathbf{y}_{i,j} - \mathbf{b})/\mathbf{s}$	$h \cdot w \cdot \mathtt{sum}(\log \lvert\mathbf{s}\rvert)$
Invertible 1×1 convolution. $\mathbf{W} : [c \times c]$. See Section 3.2.	$\forall i,j : \mathbf{y}_{i,j} = \mathbf{W}\mathbf{x}_{i,j}$	$\forall i,j : \mathbf{x}_{i,j} = \mathbf{W}^{-1}\mathbf{y}_{i,j}$	$h \cdot w \cdot \log \lvert\det(\mathbf{W})\rvert$ or $h \cdot w \cdot \mathtt{sum}(\log \lvert\mathbf{s}\rvert)$ (see eq. (10))
Affine coupling layer. See Section 3.3 and (Dinh et al., 2014)	$\mathbf{x}_a, \mathbf{x}_b = \mathtt{split}(\mathbf{x})$ $(\log \mathbf{s}, \mathbf{t}) = \mathtt{NN}(\mathbf{x}_b)$ $\mathbf{s} = \exp(\log \mathbf{s})$ $\mathbf{y}_a = \mathbf{s} \odot \mathbf{x}_a + \mathbf{t}$ $\mathbf{y}_b = \mathbf{x}_b$ $\mathbf{y} = \mathtt{concat}(\mathbf{y}_a, \mathbf{y}_b)$	$\mathbf{y}_a, \mathbf{y}_b = \mathtt{split}(\mathbf{y})$ $(\log \mathbf{s}, \mathbf{t}) = \mathtt{NN}(\mathbf{y}_b)$ $\mathbf{s} = \exp(\log \mathbf{s})$ $\mathbf{x}_a = (\mathbf{y}_a - \mathbf{t})/\mathbf{s}$ $\mathbf{x}_b = \mathbf{y}_b$ $\mathbf{x} = \mathtt{concat}(\mathbf{x}_a, \mathbf{x}_b)$	$\mathtt{sum}(\log(\lvert\mathbf{s}\rvert))$

[Kingma and Dhariwal, 2018]

Normalizing Flows

https://openai.com/blog/glow/