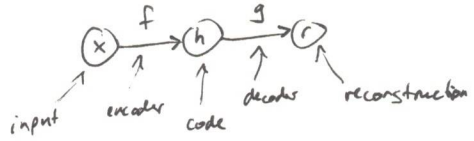
- An autoencoder is a neural network trained to attempt to copy its input to its output
- Internally it has hidden layer h that describes a <u>code</u> or <u>latent space</u> used to represent the input



Tries to set $g(f(x)) = x$ but are designed to be unable to learn to copy perfectly.

→ restricted to prevent exact copying

→ model forced to prioritize which aspects of the input should be copied, so often learns <u>most useful</u> properties of data

Not just deterministic functions f, g, but also stochastic mappings

$$P_{encoder}(h|x) \quad \text{and} \quad P_{decoder}(x|h)$$

<u>undercomplete autoencoders</u>

constrain h to have smaller dimension than x, to force to have preserved most salient properties.

→ reminiscent of data compression and very much a form of approximation theory.

Learning process is minimizing loss function

$$L(x, g(f(x)))$$

where L is loss function penalizing $g(f(x))$ for lack of fidelity with x

when $g(\cdot)$ fixed to be linear and L is mean-squared error, an undercomplete autoencoder recovers principal components analysis (PCA), the Karhunen-Loeve Transform.

why?

when allowing nonlinear (f,g), get generalization of PCA.

One can also regularize, e.g. with sparsity autoencoder

$L(x, g(f(x))) + \Omega(h)$ where $\Omega(\cdot)$ is sparsity penalty on latent space h

in addition to data fidelity term. Hopefully this forces learning of relevant things.

Denoising autoencoders

minimize $L(x, g(f(\hat{x})))$ instead of $L(x, g(f(x)))$

where $\hat{x}$ is noisy version of x.

DAE have to undo the noise corruption rather than simply copying, so again try to get most informative elements. [connection to universal denoising DUDE?]

Contractive autoencoders

different kind of regularization, to force a function that doesn't change much when x changes a little

$L(x, g(f(x))) + \Omega(h, x)$ where $\Omega(h, x) = \lambda \sum_i \|\nabla_x h_i\|^2$

or $\Omega(h) = \lambda \left\| \frac{\partial f(x)}{\partial x} \right\|_F^2$ where penalty is squared Frobenius norm (sum of squared elements)

of Jacobian matrix of partial derivatives associated with encoder function.

AE used for dimensionality reduction, etc. but what about generation?

Stochastic encoder/decoder.

Note that Pencoder, Pdecoder don't have to correspond to same valid joint distribution

$P_{model}(x, h)$ in fact usually don't.

Back up to graphical models before getting to variational autoencoders (VAEs)

Consider conditional model $p_\theta(y|x)$ that approximates underlying conditional distribution, $p^*(y|x)$, a distribution over variables y conditioned on input x.

want to learn $p_\theta(y|x) \approx p^*(y|x)$.

→ graphical models have an interesting calculus, e.g. Forney-style factor graphs ⇔ block diagrams

Parameterizing conditional distributions with neural networks

- differentiable feedforward neural networks are a flexible, computationally-scalable kind of function approximator (universal approximation theorem)

Learning models with neural networks with many hidden layers is deep learning.

Notably NNs can be used to approximate pdfs and pmfs.

→ probabilistic models based on neural networks are computationally scalable since they allow stochastic gradient-based optimization and scaling to large models, large datasets

→ think of them as an operator $NN(\cdot)$

for example, neural net can parameterize categorical distribution $p_\theta(y|x)$ over a class label y, conditioned on image x as

$$p = NN(x)$$
$$p_\theta(y|x) = \text{categorical}(y; p)$$

Consider directed graphical models that have latent variables

→ latent variables are part of model but not observed directly and not part of dataset

denote by z

joint distribution $p_\theta(x, z)$ considers observed variables and latent variables z.

A deep latent variable model (DLVM) denotes a latent variable model $p_\theta(x,z)$ whose distributional properties are parameterized by neural networks

Also conditional $p_\theta(x,z|y)$

Even when each factor (prior or conditional distribution) is relatively simple, marginal $p_\theta(x)$ can be very complex

A main difficulty of maximum likelihood learning in DLVMs is that marginal probability of data under model is typically intractable since $p_\theta(x) = \int p_\theta(x,z)dz$ may not have analytical solution or efficient estimator.

Due to intractability, we cannot differentiate w.r.t. its parameters and optimize, as we can with fully observed models. [Main difficulty is posterior $p_\theta(z|x)$]

There are approximate inference techniques but often computationally intense.

[The framework of VAE provides a computationally efficient way of optimizing DLVMs jointly with corresponding inference model using stochastic gradient descent (SGD)

To turn DLVM's intractable learning problem into tractable problem, introduce a parametric inference model $q_\phi(z|x)$ which is an encoder (recognition model)

ϕ are parameters of inference model, called variational parameters

optimize variational parameters ϕ such that $q_\phi(z|x) \approx p_\theta(z|x)$.