ECE 486 (Control Systems) – Homework 5

Due: Oct. 14

Problem 1. (15 points) Consider the following plant:

$$\dot{y} + 5y = 3u$$

Suppose a PI controller is used with P gain K_p and I gain K_i. Assume the closed-loop system is stable. Then what sampling time would you recommend for a discrete-time implementation? Write out Δt as a function of K_p and K_i.

Problem 2. (40 points) Sketch the root loci for the following $L(s)$ by hand by applying rules A–F. (Recall that the root locus plots how the solutions of $1 + KL(s) = 0$ vary as K goes from 0 to $+\infty$.

(a) $L(s) = \dfrac{1}{s^2 + 2s + 20}$

(b) $L(s) = \dfrac{s-3}{s^2 + 2s + 20}$

(c) $L(s) = \dfrac{(s+1)(s+2)}{s(s^2+4)(s^2+5)}$

(d) $L(s) = \dfrac{s+3}{s^5+1}$

Problem 3. (15 points) Consider the feedback system in Figure 1. Determine the transfer function for $Y(s)/R(s)$,

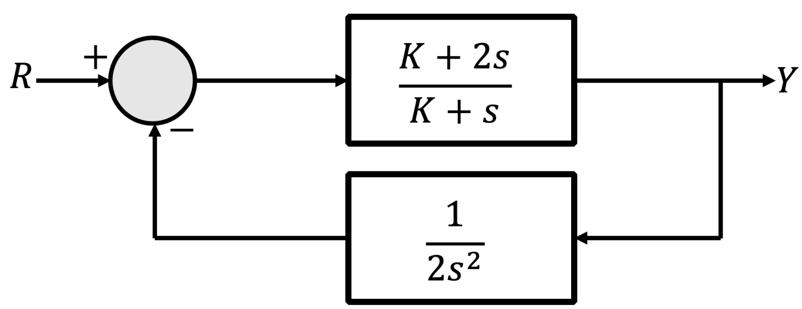

Figure 1: A feedback system.

and write the characteristic equation in terms of K. In other words, find the polynomials $a(s)$ and $b(s)$ such that the closed loop poles are the values of s that satisfy to $a(s) + Kb(s) = 0$.

You do **not** have to do this for this problem, but you should understand how to draw a root locus for where the closed loop pole locations as K goes from 0 to $+\infty$. (This shows that the root locus methods apply to more systems than the constant-gain unity-feedback setting discussed in class.)

Problem 4. (30 points) Suppose the closed loop transfer function is given by:

$$\frac{KL(s)}{1 + KL(s)}$$

where K is some constant control gain.

(a) If $L(s)$ has 3 LHP poles and 1 LHP zero, is the closed-loop system stable for very large values of $K > 0$?

(b) If $L(s)$ has 2 LHP poles, 1 RHP poles, and 3 LHP zeros, is the closed-loop system stable for very large values of $K > 0$?

(c) If $L(s)$ has 5 LHP poles, 4 LHP zeros, and 1 RHP zeroes, is the closed-loop system stable for very large values of $K > 0$?

Give detailed justification for each answer.