DSP : Discrete Fourier Transform (DFT)

#4

→ 3 important properties of DFT

① DFT is linear

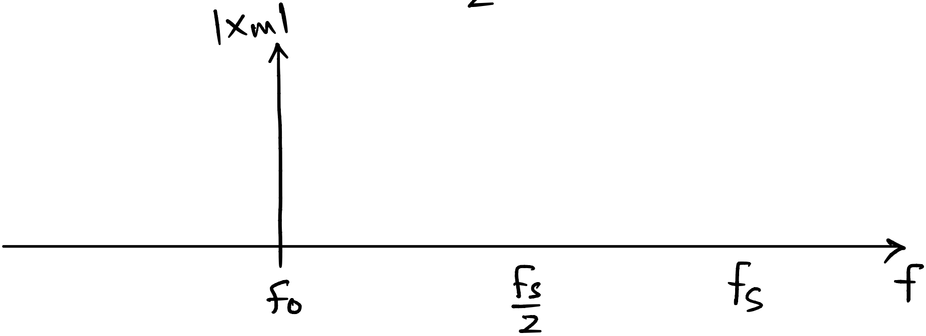
DFT($x[n] + y[n]$) =

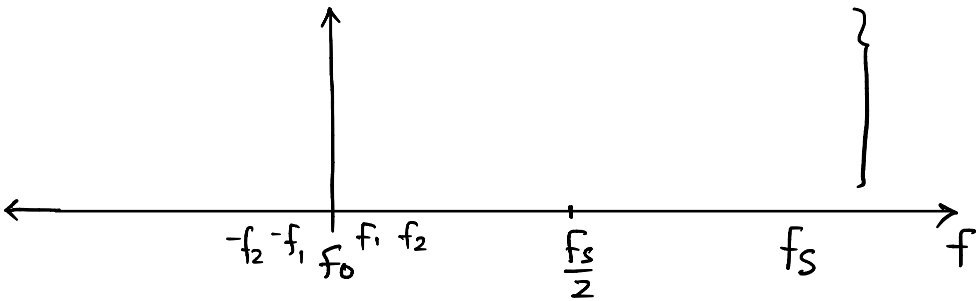
=

So what is DFT of $\cos 2\pi f_1 n t_s + \sin 2\pi f_2 n t_s$?

② For real signals, DFT is symmetric

↳ Not only around $\frac{f_s}{2}$



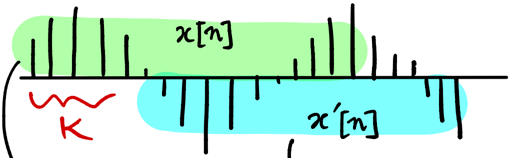


→ So take frequencies in $[\frac{f_s}{2}, f_s]$ and move to the

③ Its like the   sticks are going
clockwise (i.e., (-)ve θ) to cancel the complex part.

③ DFT of a shifted signal is original DFT with a phase shift.

$$X'_m = X_m \cdot e^{j\frac{2\pi}{N}.m.K}$$



DFT $\equiv X_m$ DFT $\equiv X'_m$

Let's shift x by K samples

$$x'[n] = x[n+K]$$

$$X'_m = \text{DFT}\{x[n+K]\}$$

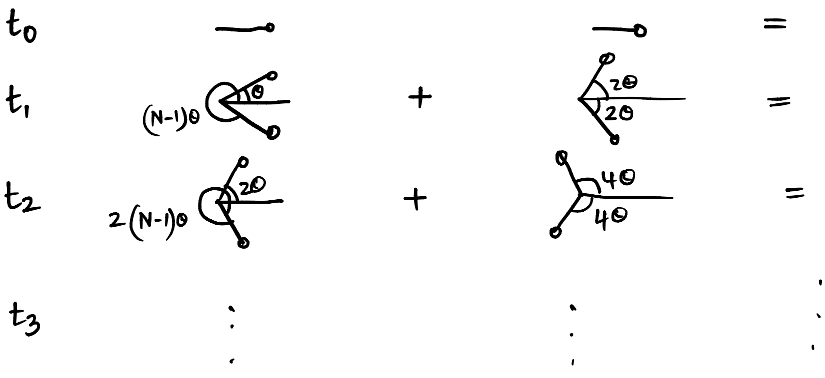
$$\therefore \quad X'_m = \sum_{n=0}^{N-1}$$

$$= \underbrace{\sum_{n=0}^{N-1}}_{\text{What is this ?}}$$

$$\boxed{X'_m = X_m \, e^{j\frac{2\pi}{N}.m.K}}$$

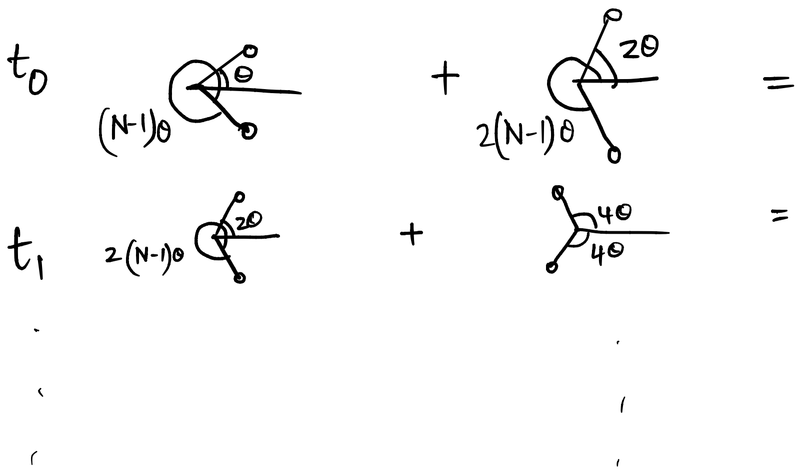
This phase shift is proportional to K, and

$$x_n = \cos 2\pi f_1 n t_s \quad + \quad \cos 2\pi (2f_1) n t_s$$



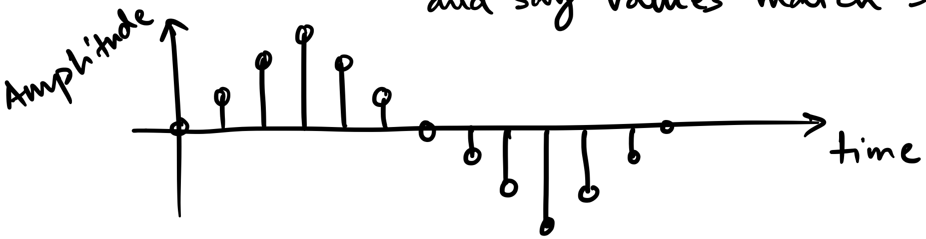
Now $\quad z_1 e^{j\frac{2\pi}{N}\cdot(m=1)(k=1)} \qquad z_2 e^{j\frac{2\pi}{N}\cdot(m=2)(k=1)}$

$\quad z_{N-1} \cdot e^{j\frac{2\pi}{N}(m=N-1)(k=1)} \qquad z_{N-2}\, e^{j\frac{2\pi}{N}(m=N-2)(k=1)}$



NYQUIST SAMPLING

- say you are given the following discrete samples and asked to reconstruct the analog signal... and say values match $\sin(\cdot)$



- We can say freq $f_1 =$

$\therefore$ Analog signal $=$

- But is this the only signal that fits these values?

Observe :

The given values so long as m

i.e.,

fits values

Choosing m as multiple of n $\left(\frac{m}{n} = k\right)$,

we have $\sin$

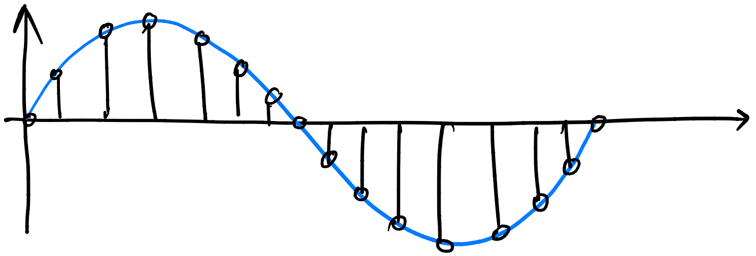
$= \sin$

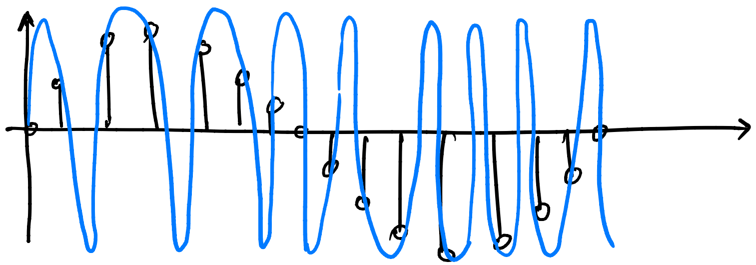
$\because f_s$ is sampling freq.

→ this means, , separated

by f_1, fits

the given set of points.

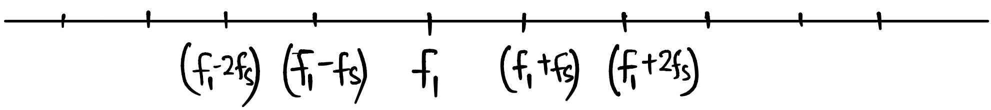




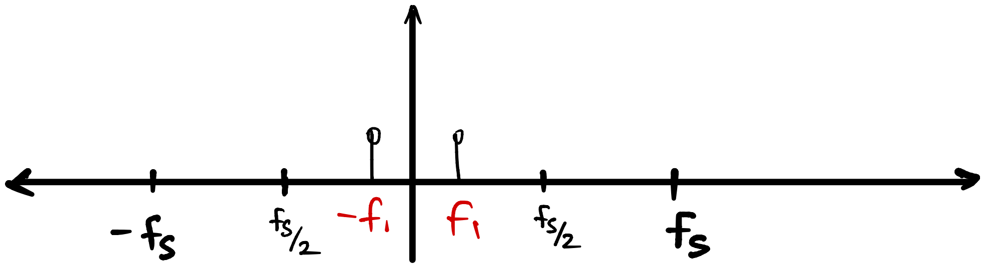
⇓

called frequencies

i.e., of f_0, for all values of k.



→ Observe that higher the sampling freq. f_s, greater is the seperation between the aliased freq.



If I sample a signal at f_s, and therefore get to see freq. components of $\frac{f_s}{N}, \frac{2f_s}{N}, \frac{3f_s}{N} \ldots \frac{N-1}{N} f_s$,

actually, Of these freq. components

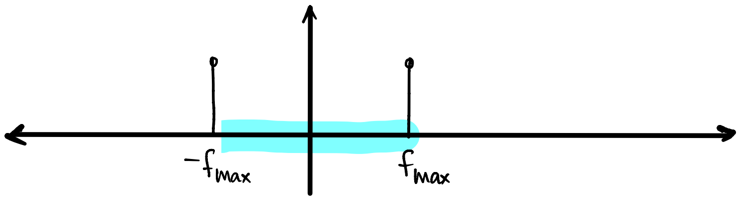
from other freq

contained in the signal.

$\Downarrow$

So what can we do?

③ Now suppose you know

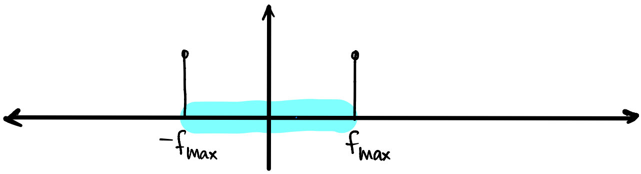


First easy step : Filter out

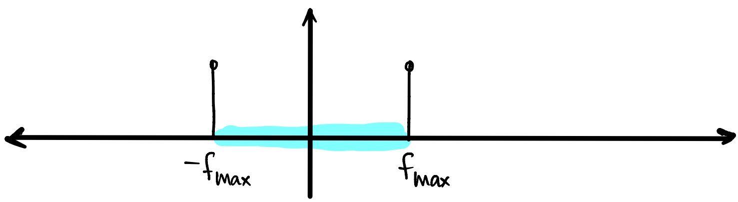
Called filter (it's an analog filter)

But is that enough ?

If sampling freq. , freq. between $[-f_{max}, f_{max}]$ will



Only way to prevent any freq. from $[-f_{max}, f_{max}]$ to alias to f_{max} is when



$\therefore$ $\boxed{f_s \qquad\qquad}$ Nyquist's sampling rate.

→ The signal processing pipeline: