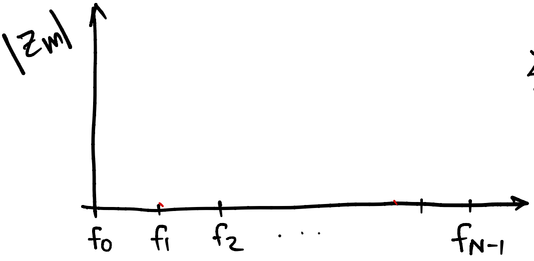
DSP : Discrete Fourier Transform (DFT)

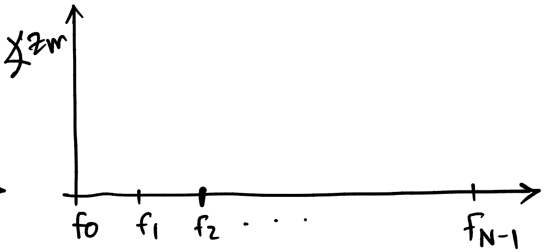
#3

Let's do 3 simple examples.

1. $\mathrm{DFT}(\cos 2\pi f_m n t_s)$
2. $\mathrm{DFT}(e^{j 2\pi f_m n t_s})$
3. $\mathrm{DFT}(\sin 2\pi f_m n t_s)$

Example 2 : DFT ($x[n] = e^{j2\pi f_1 n t_s}$)

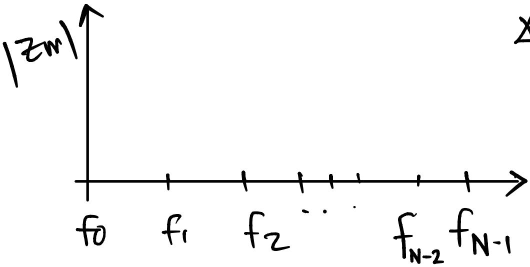


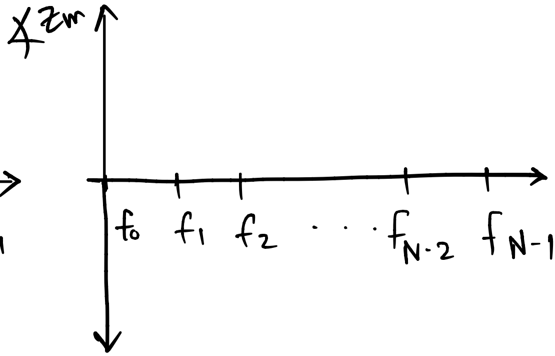


Just one stick rotating at

since it gives $e^{j0.n}$

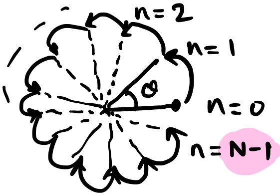
Example 3 : DFT ($x[n] = \sin 2\pi f_1 n t_s$)





Like $\cos 2\pi f_1 n t_s$, you need 2 sticks to cancel out the complex values, but

Translating to real-world frequencies



Now, slowest freq = N samples/cycle

say sampling freq. = f_s

N samples take

1 cycle takes

$\therefore$ slowest frequency =

$\frac{f_s}{N}$ ⇒ Called Fundamental freq.

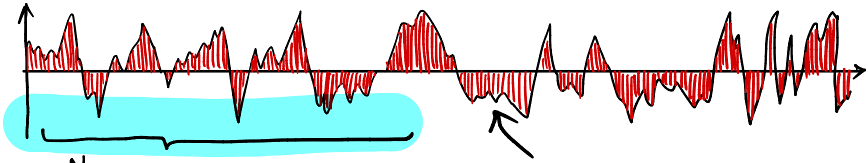
Faster freq. ⇒

For large N, $(N-1) \approx N$,

$\therefore$

Let's put it all together

Consider sampling your voice signal at 8000 Hz and taking a 1000 point FFT (i.e.,)

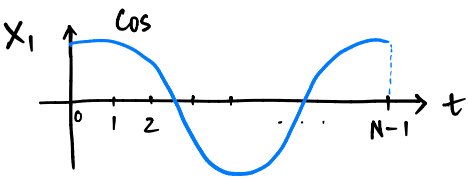


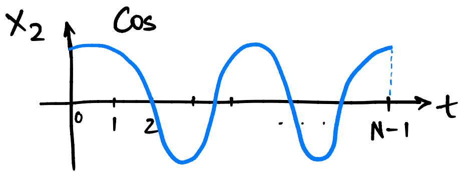
Thus, when analyzing a given signal, we have we can control

Large f_s means

Large N means we have finer Width of bin $= f_s/N$

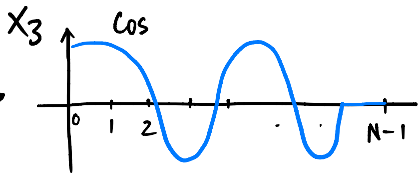
BEWARE OF INCOMPLETE CYCLES.



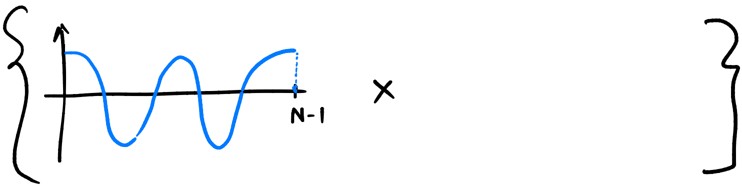


what is the FFT mag. spectrum for these signals?

Now, what is the FFT mag. spectrum of this →



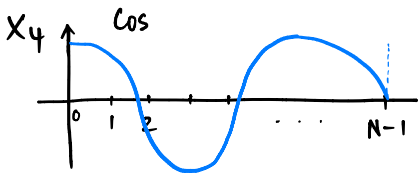
Think of this X_3 signal as →



Thus, DFT(X_3) DFT(X_2) since the DFT ( ) influences results

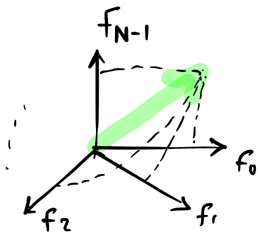
In fact:

is DFT(X_4) = DFT(X_2)

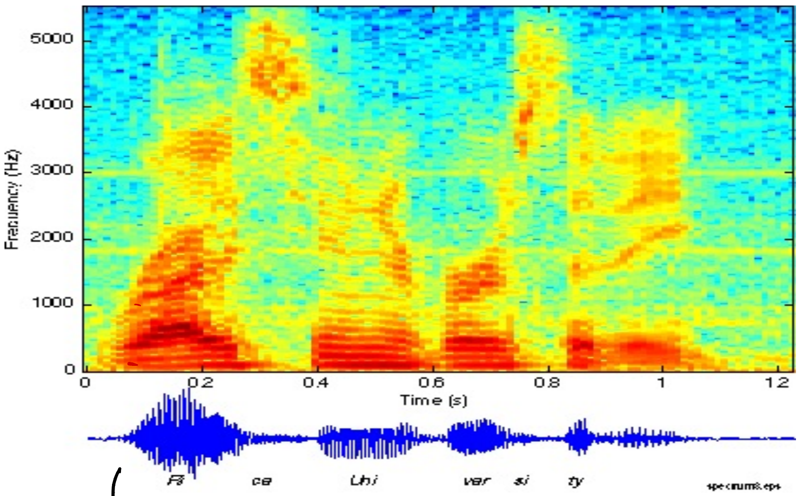


Many different rotations are needed ⇒

Think of this as



REAL SPECTROGRAM : Voice Signal



Shows various freq. components at this time slice.

time domain signal

Shows how a specific freq. comes and goes over time.

3 important properties of DFT

① DFT is linear

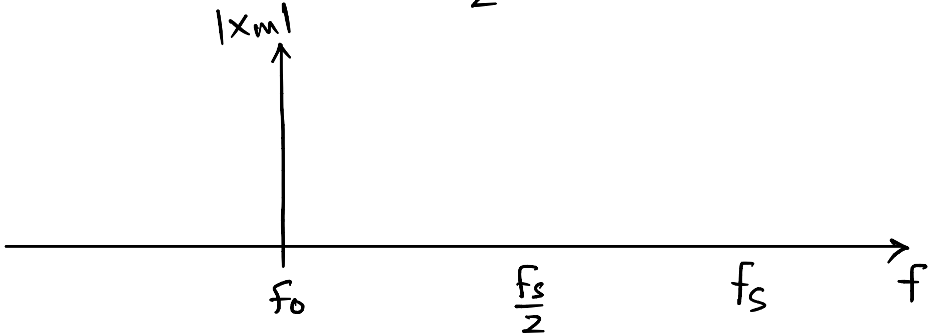
$DFT(x[n] + y[n]) =$

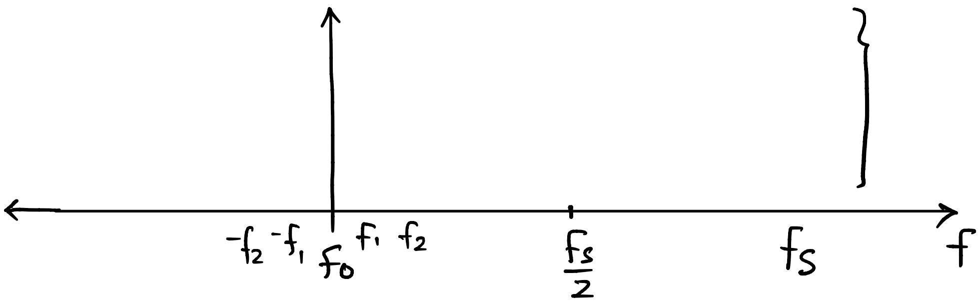
$=$

So what is DFT of $\cos 2\pi f_1 n t_s + \sin 2\pi f_2 n t_s$?

② For real signals, DFT is symmetric

↳ Not only around $\frac{f_s}{2}$



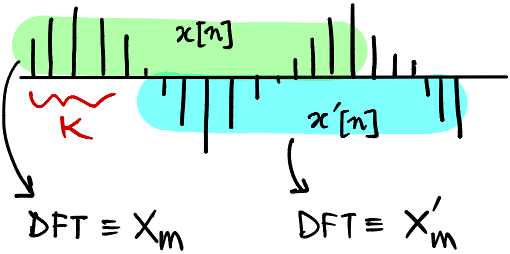


→ So take frequencies in $\left[\frac{f_s}{2}, f_s\right]$ and move to the

③ Its like the sticks are going clockwise (i.e., (-)ve θ) to cancel the complex part.

③ DFT of a shifted signal is original DFT with a phase shift.

$$X'_m = X_m \cdot e^{j\frac{2\pi}{N}.m.k}$$



Let's shift x by k samples

$$x'[n] = x[n+k]$$

$$X'_m = DFT\{x[n+k]\}$$

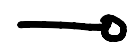
$$\therefore \quad X'_m = \sum_{n=0}^{N-1}$$

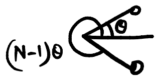
$$= \underbrace{\sum_{n=0}^{N-1} \qquad\qquad\qquad}_{\text{What is this ?}}$$

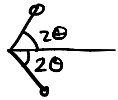
$$\boxed{X'_m = X_m \, e^{j\frac{2\pi}{N}.m.k}}$$

This phase shift is proportional to k, and

$$x_n = \cos 2\pi f_1 n t_s + \cos 2\pi (2f_1) n t_s$$

t_0   $=$

t_1 

$+$ 

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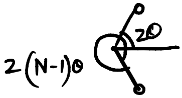
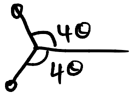
t_2 

$+$ 

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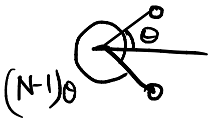
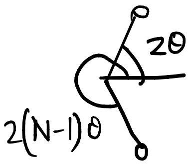
t_3 ⋮ ⋮ ⋮

Now $z_1 e^{j\frac{2\pi}{N}\cdot(m=1)(k=1)}$ $z_2 e^{j\frac{2\pi}{N}\cdot(m=2)(k=1)}$

$z_{N-1} e^{j\frac{2\pi}{N}(m=N-1)(k=1)}$ $z_{N-2} e^{j\frac{2\pi}{N}(m=N-2)(k=1)}$

t_0 

$+$ 

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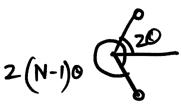
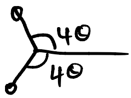
t_1 

$+$ 

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⋮ ⋮