DSP : Discrete Fourier Transform (DFT)

#2

But observe that each, $\bar{f}_i$ vectors are N-Dimensional

And $f_0, f_1, f_2 \cdots f_{N-1}$ are all orthogonal,

thus must be also

$$\Downarrow$$

Vectors $\bar{f}_0, \bar{f}_1, \bar{f}_2 \cdots \bar{f}_{N-1}$ form a for N-dimensional space.

Now, express the original signal X in time & freq. basis

$$I X_t =$$

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ 0 & & 1 & 0 \\ \vdots & & \ddots & \vdots \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

Thus, Fourier transform is the representation of a signal vector in a different (frequency) basis.

$$F . X_f = X_t$$

$$X_f = F^{-1} X_t =$$

$$z_2 = e^{-j0} x_0 +$$

$$z_2 = \sum_{n=0}^{N-1} x_n \qquad \text{where } \theta =$$

$$z_m =$$

What is 'm' in z_m?

Inverse DFT ⇒ From freq back to time domain

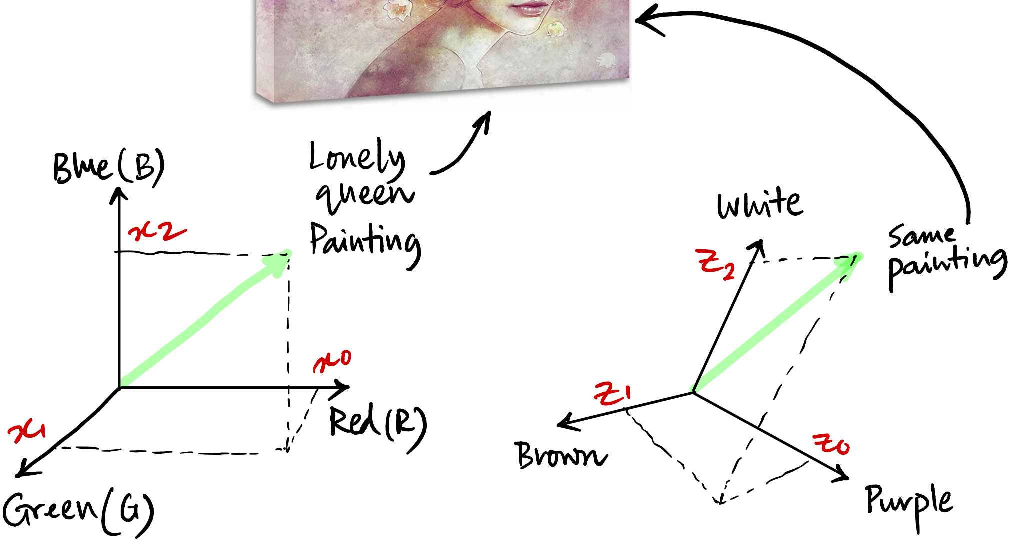
$$IX = F .$$

$$\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \quad \begin{bmatrix} | & | & & | \\ f_0 & f_1 & \cdots & f_{N-1} \\ | & | & & \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_{N-1} \end{bmatrix}$$

$$\boxed{x_n = \sum_{m=0}^{N-1}}$$

↳ Inverse Discrete Fourier Transform (IDFT)

Analogy



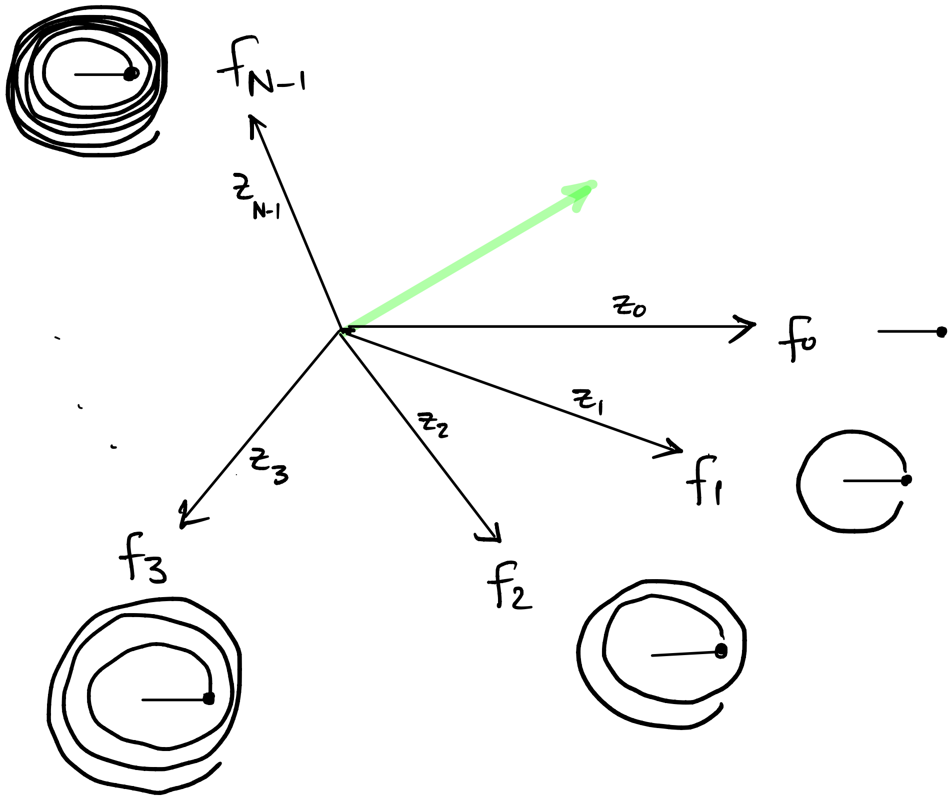
$$\text{Lonely queen} = \begin{bmatrix} | & | & | \\ R & G & B \\ | & | & | \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} | & | & | \\ P & Br & W \\ | & | & | \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix}$$

→ Now when basis is complex, what about the weights?

$$\underbrace{\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}}_{\text{Real}} = \underbrace{\begin{bmatrix} e^{j0} & e^{j0} & \cdots & e^{j0} \\ e^{j0} & e^{j\theta} & & e^{j(N-1)\theta} \\ \vdots & \vdots & & \vdots \\ e^{j0} & e^{j(N-1)\theta} & & e^{j(N-1)^2\theta} \end{bmatrix}}_{\text{complex}} \underbrace{\begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_{N-1} \end{bmatrix}}_{?}$$

DFT is a complex Vector

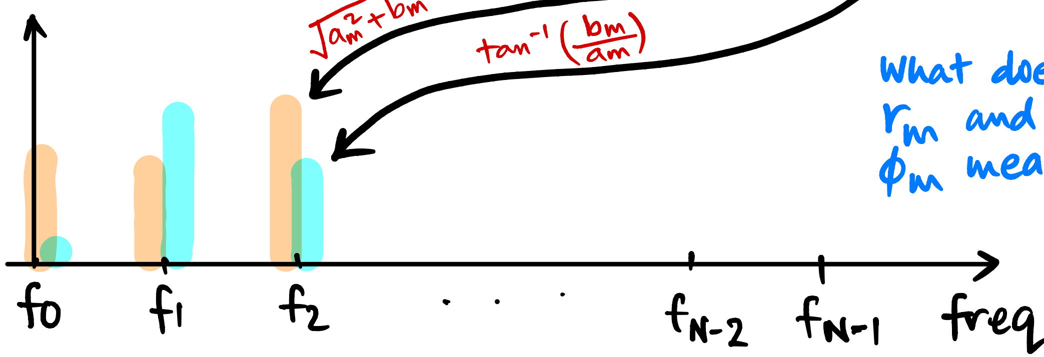
↳ What does that mean?



$$z_m = a_m + jb_m = r_m e^{j\phi_m}$$

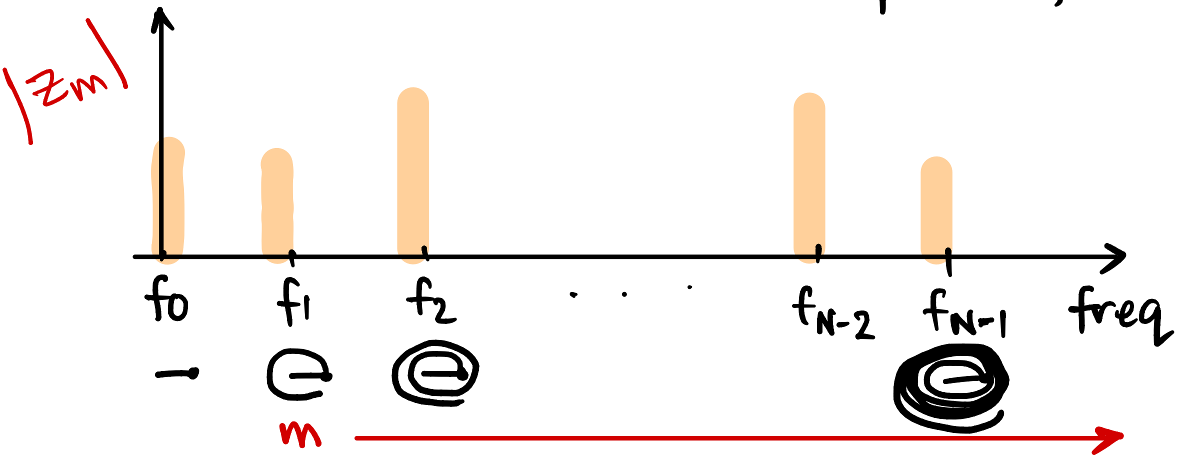
$\sqrt{a_m^2 + b_m^2}$

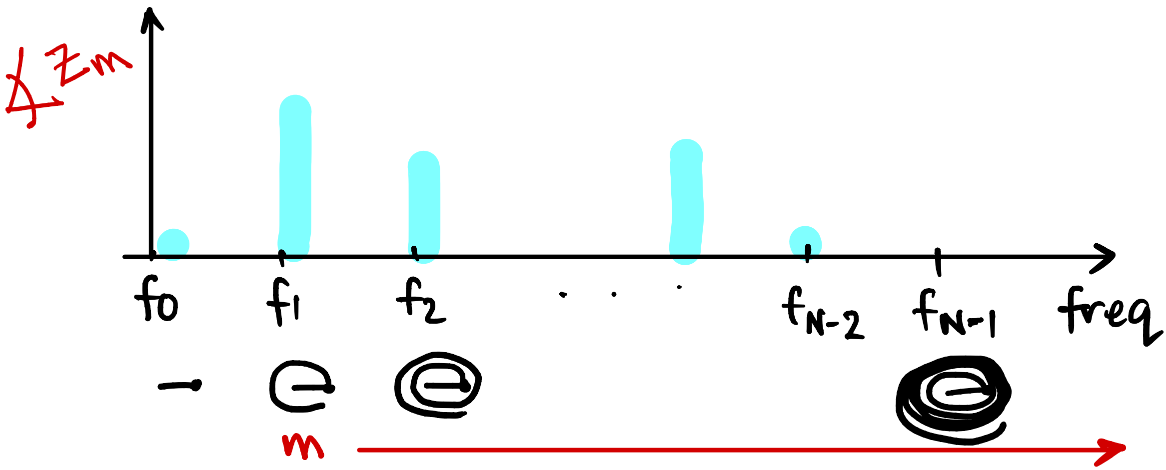
$\tan^{-1}\left(\frac{b_m}{a_m}\right)$



What does r_m and ϕ_m mean?

Magnitude and Phase plots of DFT

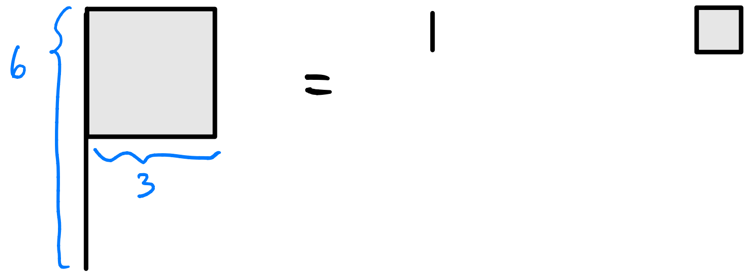


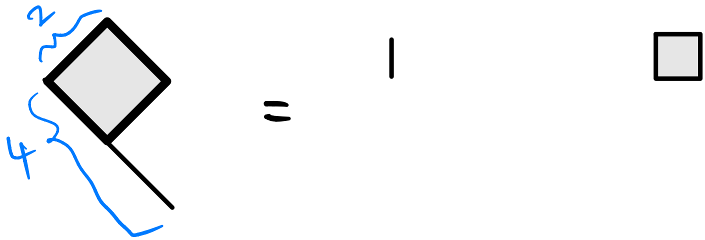


→ Signals will have non-zero magnitudes for some frequencies.

Bandwidth =

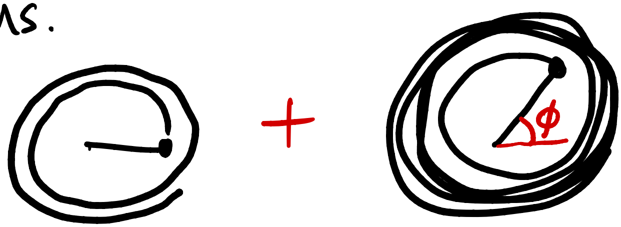
Analogy





→ Similarly, to synthesize any signal with spinning sticks, you may need to start some of the sticks by rotating them with phase ϕ_m, in addition to increasing/decreasing their lengths.

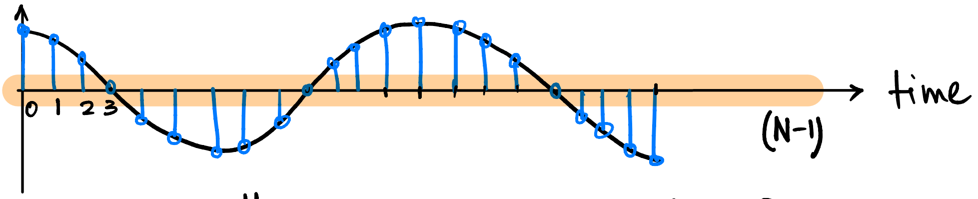
signal we want =



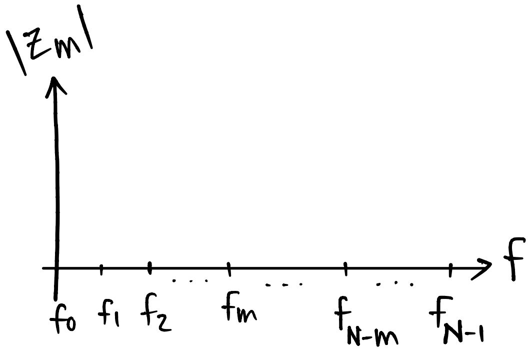
Q: Does a time domain signal have phase?

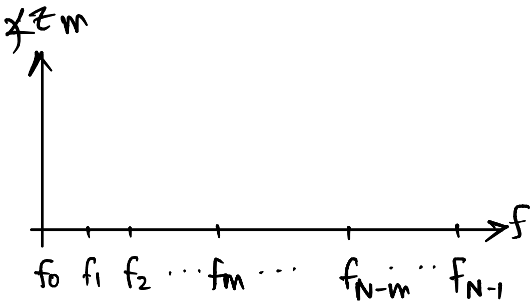
Example 1 : $x(t) = \cos 2\pi f_m t$

Discrete sampled signal $x[n]$ =



⇓ synthesize with frequency basis.



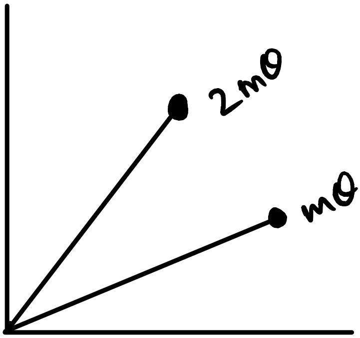


But $\cos 2\pi f_m n t_s$ only has f_m freq.
Why does DFT have components?

Answer:

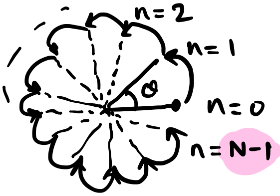
: Note that $\cos 2\pi f_m n t_s$ is not a rotation.
It's only the x-axis shadow of the rotation.
So, to make $\cos 2\pi f_m n t_s$, we need to cancel out the y-axis shadows of the rotating stick.

$$\begin{bmatrix} \cos 0 \\ \cos(m\theta) \\ \cos 2(m\theta) \\ \cos 3(m\theta) \\ \vdots \\ \cos(N-1)(m\theta) \end{bmatrix} = \begin{bmatrix} e^{j0} \\ e^{jm\theta} \\ e^{j2m\theta} \\ e^{j3m\theta} \\ \vdots \\ e^{j(N-1)\theta} \end{bmatrix} \times Z_m$$



- Thus, real signals always around center of
- For $\cos 2\pi f_m n t_s$, all phases are 0

Translating to real-world frequencies



Now, slowest freq = N samples/cycle

say sampling freq. = f_s

N samples take

1 cycle takes

$\therefore$ slowest frequency =

$\frac{f_s}{N}$ $\Rightarrow$ Called Fundamental freq.

Faster freq. $\Rightarrow$

For large N, $(N-1) \approx N$,

$\therefore$